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Storm Runoff Analysis by Generalized Storage Function Model in a Semi-urban Watershed

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The storage function (SF) models have been extensively used for the rainfall-runoff modeling in different parts of the world due to its simple structure. However, there is a need for an SF model that can be applied in all the watersheds, except urban watersheds with combined sewer system, without requiring the effective rainfall as their input. In this study, therefore, we aim to propose a generalized SF (GSF) model that satisfies the above criteria with a new parameter named as rainfall distribution factor (f). The GSF model without parameter f was also examined in order to check the effectiveness of f in the GSF model. In addition, three different time intervals were considered for the numerical solution method used in this study in order to identify the effect of time intervals on the model performance. The results revealed that the GSF model with f exhibited higher hydrograph reproducibility associated with the lowest error evaluation criteria which emphasize the effect of parameter f. It was also demonstrated that the model performance is changing for the same model at different time intervals which can be interpreted as the strong influence of time intervals on the numerical solution method and subsequent model performance.

Key Words: *GSF model, rainfall distribution factor, time interval, SCE-UA method, watersheds*

1. INTRODUCTION

Among the different lumped rainfall-runoff models, the storage function (SF) models have been widely used in many parts of the world[1), 2), 3), 4)] not only due to its easiness in computation and handling, but also the ease of expressing the nonlinear relationship of the rainfall-runoff process with simple equations[5)]. Kimura invented the first SF model in Japan with two parameters and lag time[6)] and is still widely used in Japan for the flood prediction. Subsequently, Prasad presented a three parameter SF model which has an additional term for the inclusion of the loop effect between storage and discharge[7)]. Successively, Hoshi and Yamaoka added another parameter to incorporate the non-linear unsteady flow effects and improved the robustness of SF model[8)]. However, all these models requiring effective rainfall as their input for the direct runoff prediction. Hence it involves the problems of baseflow separation and effective rainfall estimation which may further affect the value of parameters being estimated and their relative stability.

To cope with this problem, Baba et al.[9)] introduced an SF model with loss mechanism which uses the observed rainfall and runoff and applied to the mountainous river basin in Hokkaido, Japan. However, the use of Baba's SF model for the prediction of runoff in urban areas may be difficult, because the urban area differs completely with the mountainous area in terms of the imperviousness, absence of vegetation, etc[10]. Soon after, Takasaki et al.[11] developed a new urban SF (USF) model considering the urban runoff process which also uses the observed rainfall and runoff for the flood prediction. The performance of different SF models have already been evaluated and it was found that the USF model performs better as compared to conventional SF models[5), 10)]. However, the target area of USF model was the urban watersheds with combined sewer system since many older cities in different parts of the world continue to operate combined sewers instead of the separate sewer system. In the watersheds with combined sewer system, there will be an outflow from the basin to the treatment plant through the combined sewer system rather than discharging into the river as lateral inflow and the USF model considers this effect of storm drainage diverting to the treatment plant through the combined sewer system. On the contrary, the separate sewer system conveys the storm drainage directly to the rivers and the new cities operate under the separate sewer system instead of the combined system.

Hence, there is a need for an SF model that can be applied in all the watersheds, except urban watersheds with combined sewer system, without requiring the effective rainfall as their input. Therefore, this study aims to propose a generalized SF (GSF) model for the storm-runoff analysis to the separate sewer system urban watersheds as well as the non-urban watersheds by considering all the possible inflow and outflow components. The concept of soil moisture parameter tank (SMPT) model[12] was incorporated to account for the groundwater related loss from the basins[11]. Generally, the rainfall is spatially distributed over the watershed and this spatially variable rainfall produces increased surface runoff in comparison with spatially uniform rainfall based on its location of occurrence in the watershed. For example, if a localized rainfall with high intensity is occurring near the watershed outlet, the outlet will receive an immediate high magnitude response without any significant losses. However, in the conventional SF models, the rainfall is spatially averaged over the basin and a uniform rainfall is considered which will further result in the underestimation or overestimation of storm runoff. Therefore, an attempt has been made to address this issue by introducing a new parameter called rainfall distribution factor, hereinafter termed as f, in the proposed GSF model.

The performance evaluation of GSF model was conducted in terms of the hydrograph reproducibility. The GSF model without parameter f was also examined to check the effectiveness of GSF model with f. Both the models were then applied to the five selected flood events of Iga, a small to medium-sized semi-urban watershed in Aichi prefecture, Japan. The Shuffled Complex Evolution-University of Arizona (SCE-UA) global optimization method[13] was used to estimate the model parameters of each event with root mean square error (RMSE) as the objective function. Also, three time intervals (*dt*) of 1, 5, and 10 min were considered in the numerical solution method used in this study to check its effect on model performance. Further, the hydrograph reproducibility of both the models was assessed using different performance evaluation criteria of RMSE, Nash Sutcliffe Efficiency (NSE) and other error functions.

2. MATERIALS AND METHODS

(1) Generalized SF (GSF) model

The SF models are characterized by the relationship between the storage and discharge at the outlet. The storage equation of GSF model is the empirical representation of Hoshi's SF model[8] and is given as:

$$s = k_1 Q^{p_1} + k_2 \frac{d}{dt}(Q)^{p_2} \tag{1}$$

where s: storage (mm), Q: observed river discharge (mm/min), t: time (min), dt: change in time (min), and k_1, k_2, p_1, p_2: model parameters. In order to avoid the effective rainfall separation process, we used a continuity equation which can include all the possible inflows and outflows of a conceptual watershed and is given as:

$$\frac{ds}{dt} = fR + I - E - O - Q - q_l \tag{2}$$

where f: rainfall distribution factor, R: observed rainfall (mm/min), I: inflows from other basins (mm/min), E: evapotranspiration (mm/min), O: water intake from the basin (mm/min), and q_l: groundwater related loss. The rainfall should consider as a fraction of the observed rainfall and f will represent this fraction. Even though f partially represents the runoff coefficient in its expression, the main purpose of its incorporation is to consider the spatial distribution of basin rainfall. Hence, in this study, it is named as the rainfall distribution factor (f) rather than the runoff coefficient. The inclusion of f enables us to incorporate the effect of spatially distributed rainfall in the basin. The groundwater related loss (q_l) was defined by considering the infiltration hole height (z) in the SMPT model and is given by[12]:

$$q_l = \left\{ \begin{array}{ll} k_3(s-z) & (s \geq z) \\ 0 & (s < z) \end{array} \right\} \qquad (3)$$

where k_3 and z are the parameters. Combining the expression of storage (Eq. 1) with the continuity equation (Eq. 2) yields a second-order ordinary differential equation (ODE) as follows:

$$k_2 \frac{d^2}{dt^2}(Q+q_R)^{p_2} = -k_1 \frac{d}{dt}(Q+q_R)^{p_1} + fR + I - E - O - (Q+q_R) - q_l \qquad (4)$$

In order to solve this second-order ODE, the change of variables is performed as follows:

$$x_1 = Q^{p_2} \qquad (5)$$

$$x_2 = \frac{dx_1}{dt} = \frac{d}{dt}(Q)^{p_2} \qquad (6)$$

Substituting Eq. (3) into Eq. (4) and performing the change of variables will lead to the emergence of two first-order ODEs concerning two conditions as shown in Eq. (3). When $s \geq z$, the first-order ODE is as follows:

$$\frac{dx_2}{dt} = -\left(\frac{k_1}{k_2}\right)\left(\frac{p_1}{p_2}\right) {x_1}^{(p_1/p_2-1)} x_2 - \left(\frac{1}{k_2}\right) {x_1}^{(1/p_2)} - \left(\frac{k_1 k_3}{k_2}\right) {x_1}^{(p_1/p_2)} - k_3 x_2 + \left(\frac{1}{k_2}\right)(fR + I - E - O + k_3 z) \qquad (7a)$$

In the case of $s < z$, the first-order ODE concerning the same processes are given as,

$$\frac{dx_2}{dt} = -\left(\frac{k_1}{k_2}\right)\left(\frac{p_1}{p_2}\right) {x_1}^{(p_1/p_2-1)} x_2 - \left(\frac{1}{k_2}\right) {x_1}^{(1/p_2)} + \left(\frac{1}{k_2}\right)(fR + I - E - O) \qquad (7b)$$

By solving the non-linear ODE of (7a) and (7b) numerically, we obtain the simulated river discharge Q. In order to solve the first-order ODE, we used the Runge-Kuta-Gill (RKG) method which is one of the most efficient numerical solution methods[14]. Therefore, the GSF model is a 7-parameter model with parameters $k_1, k_2, k_3, p_1, p_2, z, f$. Additionally, in order to analyze the effect of parameter f in the model, the same GSF model was considered without parameter f $(f = 1)$ which is referred to as GSF model without f hereafter in the study.

(2) Parameter estimation

The SCE-UA method proposed by Duan et al.[13] was used to identify the optimal parameters of both the models (GSF model with and without f). The search range of model parameters in the SCE-UA method was set as the physical minimum and maximum values of parameters based on the previous studies[5), 11)]. It is given as k_1(0-500), k_2(0-5000), k_3(0-1), p_1(0-1), p_2(0-1), z (0-300) [5), 11)], and f (0-10). Sometimes, the basin average rainfall will be very low even though high magnitude rainfall occurs near the basin outlet. Therefore, the basin average rainfall should consider as doubled, tripled, etc. in order to represent a high magnitude rainfall near the watershed outlet. Consequently, the maximum possible value of f was set as ten in order to incorporate the effect of a ten times higher magnitude rainfall resulting from the spatial distribution of rainfall in the basin. The hydrograph reproducibility of models with the observed discharge was assessed using the RMSE, NSE, and other error functions of percentage error in peak (PEP), percentage error in volume (PEV) and error in time to peak (ETP)[15].

(3) Study area and data used

The target basin is the Iga watershed, tributary of the Yahagi River, with an area of about 9.6 km^2 at Iga Bridge as shown in **Fig.1**. The rainfall and water level data at ten-minute interval were collected from the Okazaki City Government during 2013-2016 for the study. The *dt* values of 1, 5, and 10 min were considered in the RKG numerical solution method since the data was observed in every 10 min. Five target events were selected from the data which can produce a peak discharge value greater than 15 m^3/s for the application of the proposed models and are shown in **Table 1**. The rainfall data from the three rain gauges were used to compute the catchment average rainfall by using the Thiessen polygon method. The inflow component I was assumed as

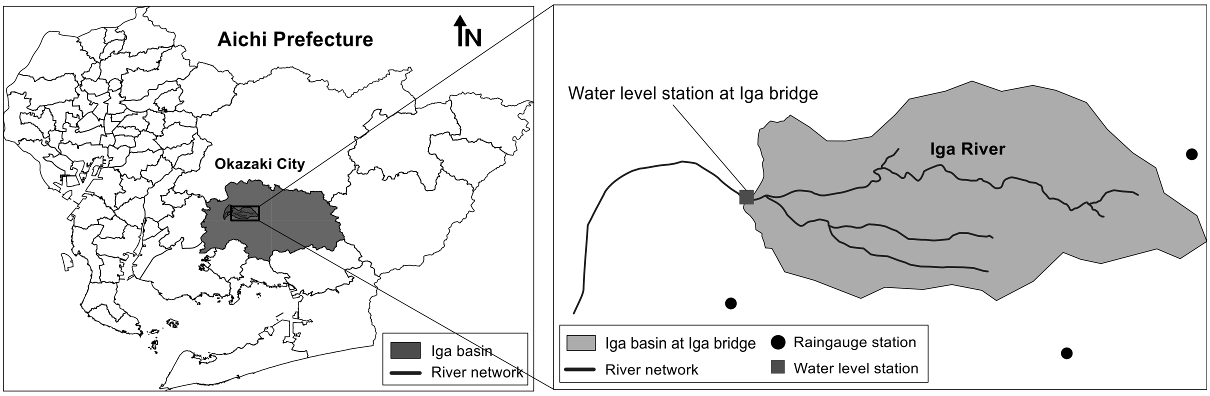

Fig.1 Index map of the Iga basin at Iga Bridge.

Table 1. Characteristics of target events.

Event No.	Event date	Peak Q (m^3/s)	Total R (mm)	Meteorological factors
1	10/9/2015	27.5	40.3	Typhoon
2	7~8/9/2013	22.8	44.8	Frontal event
3	8~9/9/2015	22.5	134.2	Typhoon
4	15~16/10/2013	15.1	138.7	Typhoon
5	26~27/5/2014	15.1	68.9	Frontal event

zero for the basin since there was no inflow to the Iga basin. The outflow components O and E were also set at zero because there is no intake from the considered basin and the evapotranspiration during heavy rainfall is trivial. The effect of I, O and E components will vary from basin to basin based on the inflow to the basin, outflow from the basin, etc. and these effects were neglected in this study based on the prevailing conditions in the target basin.

3. RESULTS AND DISCUSSIONS

(1) Parameter estimation

The SCE-UA method was applied for parameter estimation of both the models for the five selected flood events in the target watershed. The convergence of parameters was also checked and it was found that the parameters converged before the 100th generation in each SCE-UA application run. Therefore, the best parameter set at the 100th generation was used for further hydrograph reproduction. **Fig.2** shows the estimated model parameters by both the models for each selected event with three different time intervals used during the RKG solution method. It is clear from **Fig.2 (a)** that the parameter k_1 varies between models as well as among events in the Iga basin. The variation between models was observed only for events 4 and 5. The $k_2$values were almost similar for the models among the events except for some as shown in **Fig.2 (b)**. The k_1 and k_2 values of Iga basin were quite small compared with the search range in most of the events because these parameters represent the physical watershed characteristics like watershed area, stream length, shape of the basin, etc.[3), 7)]. Further, Park et al.[4)] reported that an increase in the basin area, as well as the stream length, will lead to an increase in parameter k_1. Iga basin is a small basin which further resulted in relatively smaller k_1 and k_2 values for the Iga basin. There is high variability in parameter k_3 between the models in the Iga basin as illustrated in **Fig.2 (c)**. The p_1 values in **Fig.2 (d)** were almost close for all the models even though they vary among the events. Parameter p_2 exhibited variation in values for the models among the events in the Iga basin as shown in **Fig.2 (e)**. The parameter z portrayed high variability between models in the Iga basin as shown in **Fig.2 (f)**. Further, the parameter f shows values greater than one during events 1, 4 and 5 in the Iga basin whereas the values were close to one in the remaining events as depicted in **Fig.2 (g)**. It can be envisaged from **Fig.2** that the estimated parameter values are varying from event to event even for the same model which may be possibly due to the difference in meteorological factors that caused the rainfall event. Moreover, the results revealed that the parameter values are changing in each event with respect to the change in dt values in each of the model for most of the time.

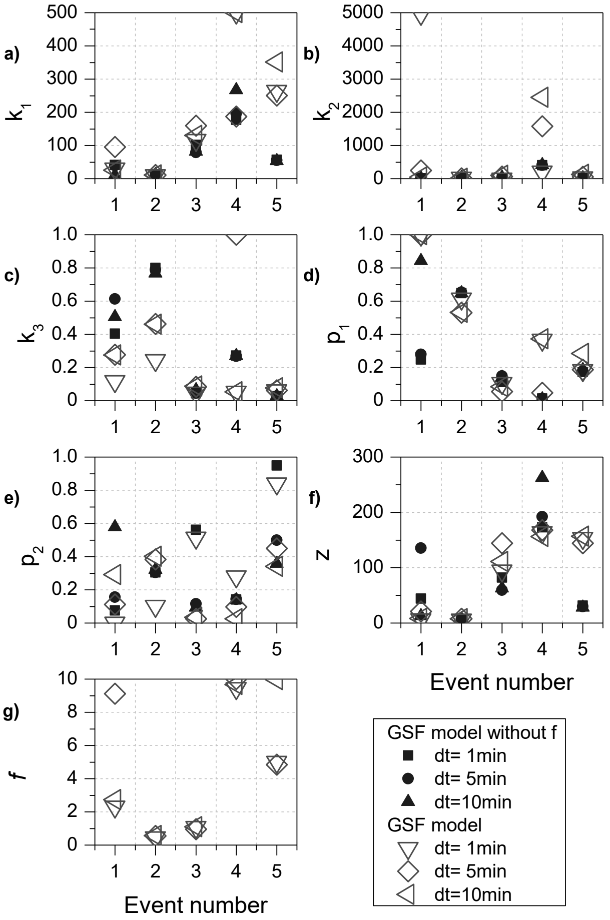

Fig.2 Event-based optimal parameters for the GSF model with and without f at different dt values.

Further, to check the significance of the optimized value of f, the spatial rainfall distribution of total rainfall in the basin was plotted as shown in **Fig.3**. The first two events were plotted out of five from the basin due to the page constraints. It can be seen from **Fig.3 (a)** that a heavy rainfall of 58mm is occurring near the watershed outlet compared with the basin average rainfall of 40.3 as shown in **Table 1**. Therefore, the basin average rainfall should be increased to incorporate this actual rainfall effect near the outlet which further resulted in an f value

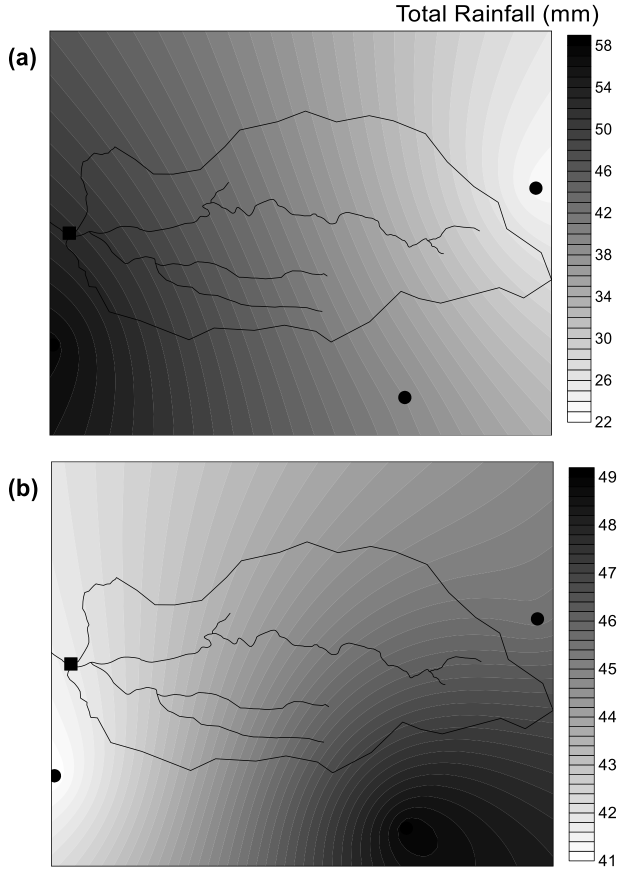

Fig.3 Spatial distribution of total rainfall during (a) event 1, and (b) event 2 (circle and square represent rain gauge and water level stations respectively).

greater than one as shown in **Fig.2 (g)**. From the rainfall distribution of event 2 as shown in **Fig.3 (b)**, it can be envisaged that the spatial variability is very low in the basin as well as compared with the basin average rainfall of 44.8 mm given in **Table 1**. Hence, the *f* values close to one will be sufficient to represent this low variability. The results revealed that the parameter *f* have physical significance in relation to the spatial rainfall distribution in the basin. The GSF model with parameter *f* can adjust the basin average rainfall in order to cope with the spatial variation of rainfall and its value can be either less or greater than one.

(2) Hydrograph reproducibility

Fig.4 shows the reproduced hydrographs by both the models at different time intervals for the five selected flood events. It is clear from **Fig.4** that the simulated discharge by both the models at different time intervals almost overlaps with the observed river discharge except for events 1 and 4 in the Iga basin. During event 1, both the models failed to reproduce the shape as well as the peak discharge as shown in **Fig.4 (a)**. They overestimated the first peak and underestimated the second peak of event 1. The GSF model without *f* highly deviated from the observed

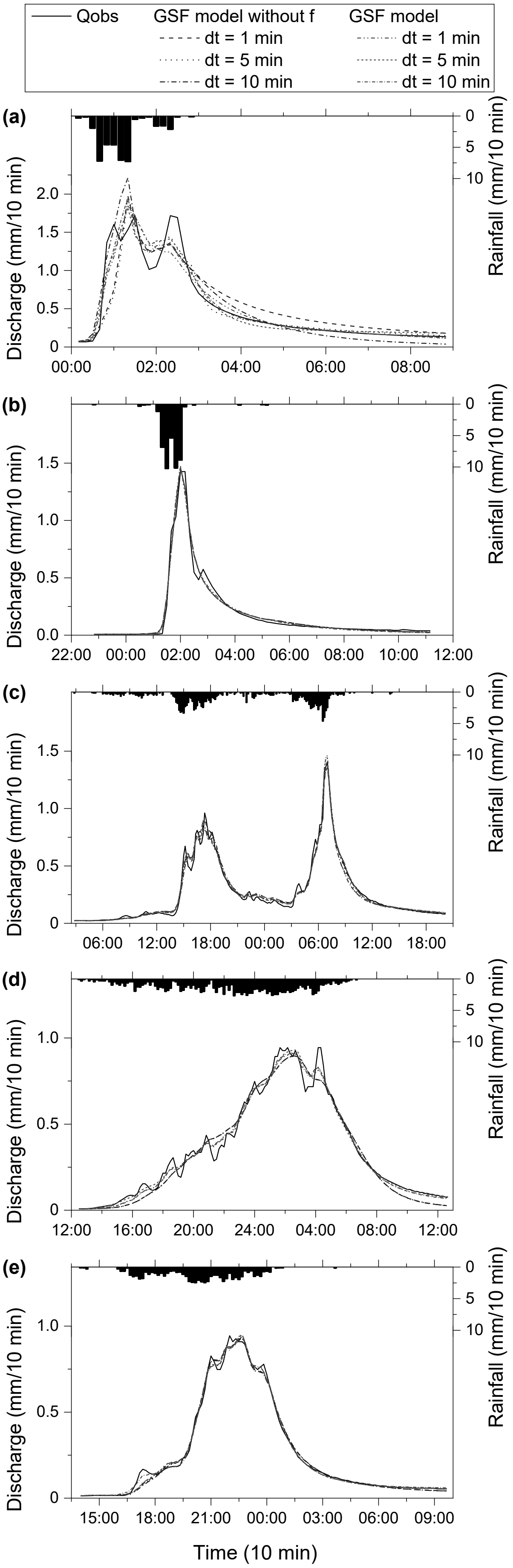

Fig.4 Reproduced hydrographs for a) event 1, b) event 2, c) event 3, d) event 4, and e) event 5.

discharge at the rising and recession limbs, whereas the simulated discharge by the GSF model was comparable with the observed discharge at the rising and recession limbs. It can be envisaged from **Fig.4 (d)** that the GSF model at different time intervals was able to reproduce the shape and peak discharge of the hydrographs with slight discrepancies compared with the GSF model without f. Conversely, the GSF model without f shows considerable deviations at the beginning and end of the hydrograph. Therefore, the results revealed that the GSF model can more accurately reproduce the shape of the observed hydrograph as well as the peak discharge compared with the GSF model without f at different *dt* values.

It is clear from **Fig.4** that sometimes there is a lower prediction of the second peak in multi-peak events in the basin. This can be attributed to the objective function of RMSE which was minimized during the SCE-UA optimization method. The SCE-UA method attempts to reduce the RMSE by following the recession limb of the first peak and rising limb of the second peak, in which the model failed to exactly reproduce the peak discharge and underestimated it. The RMSE value will be quite high if the model tries to reproduce the second peak by skipping the recession limb of the first peak and the rising limb of the second peak. It may be possible to accurately reproduce the peak discharge by calibrating the model using an objective function other than RMSE which can initiate the type of simulation that the model is required to make.

From **Fig.4**, it is not easy to clearly portray the difference between the simulated discharge hydrographs of both the models at different time intervals. Hence, we evaluated the performance of these models using RMSE, NSE, and other error functions of PEP, PEV, and ETP as shown in **Fig.5**. From **Fig.5 (a)** and **(b)**, we can see that the GSF model at different time intervals generates low RMSE values close to zero and high NSE values close to 100% in all the events, especially in events 1 and 4, compared with GSF model without f. This can be attributed to the presence of parameter f in the model that describes the spatial distribution of rainfall in the watershed. The low RMSE and high NSE of GSF model can be interpreted as to its high hydrograph reproducibility. It can also be inferred that there could be a high variation in the spatial distribution of rainfall during events 1, 4, and 5 that resulted in f values greater than one during these events. **Fig.5 (c)** depicts that the PEP values estimated by the GSF model were close to zero and not greater than 5% except event 1. The model received the lowest PEP in all the events except event 3. However, the PEP values at different *dt* values were quite different even for the same model. Likewise the PEP, the GSF model shows PEV values close to zero in all the considered time intervals as illustrated in **Fig.5 (d)**, while the GSF model without f exhibited great variation in the values reaching a maximum PEV value of -10% during event 1. The PEV values of GSF model were relatively stable at different *dt* values compared with the GSF model without f. The predicted ETP values were similar in both the models as demonstrated in **Fig.5 (e)**. There was no significant difference in the overall shape of reproduced hydrographs at different *dt* values for the same model which resulted in almost same RMSE and NSE values at different time intervals in each model. However, the time interval has the influence on the peak discharge as well as the flood volume which resulted in different degrees of deviations of

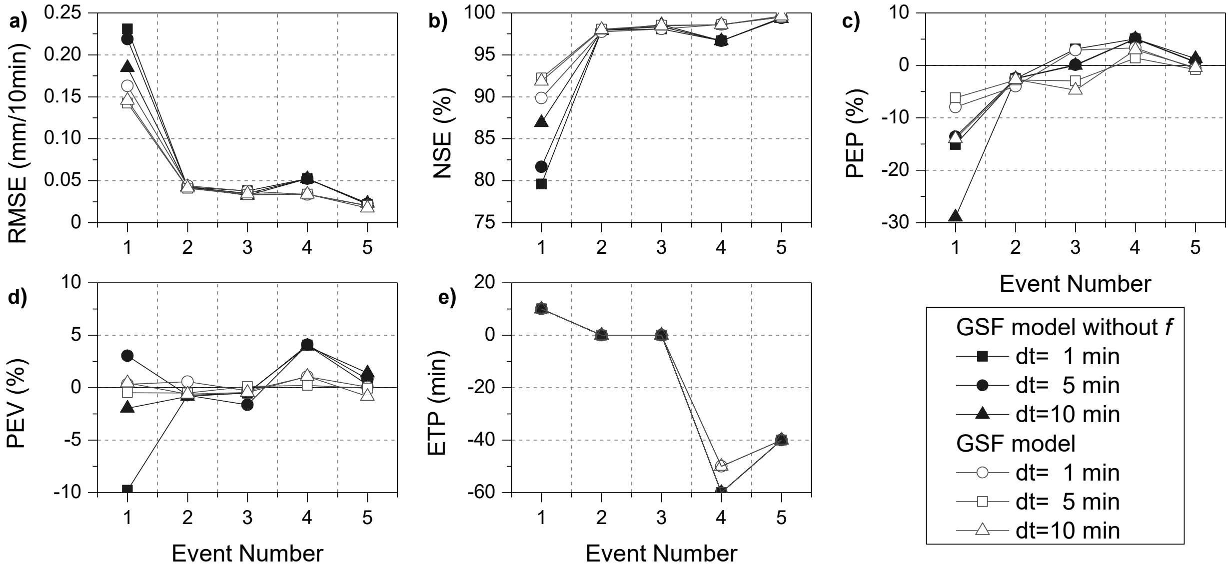

Fig.5 Comparison of RMSE, NSE, PEP, PEV, and ETP by the GSF model with and without f at different time intervals.

simulated discharge at the rising and recession limbs with respect to the time intervals.

The higher values of NSE coupled with the lower values of RMSE, PEP, PEV, and ETP in all the events for the GSF model indicated its high reproducibility compared with the GSF model without f. This results further showed the relevance of parameter f in the GSF model. The rainfall distribution is temporally as well as spatially varying and the value of f will depend on meteorological factors, basin geology and geomorphology, etc. Not only parameter f is subject to change, but the remaining parameters of GSF model will also vary in each event based on the meteorological factor and hence the real-time application of the model using the calibrated parameters is a challenging task. However, one solution to tackle this issue is the real-time prediction of the model parameters using data assimilation techniques which will improve the model effectiveness in an operational context.

4. CONCLUSIONS

A generalized SF (GSF) model was proposed that can be applied in urban watersheds with separate sewer system as well as in the non-urban watersheds by considering all the possible inflow and outflow components. A new parameter, rainfall distribution factor, f was introduced in the model to account for the effect of spatially distributed rainfall in the watershed. The GSF model was applied to five selected flood events of the Iga basin, Japan along with GSF model without f in order to evaluate the hydrograph reproducibility of the proposed model as well as the effectiveness of f in the GSF model. Three different time intervals were also considered in the RKG numerical solution method in order to check its effect on model performance. The results revealed that the GSF model has the least RMSE (high NSE) compared with the GSF model without f for all events. The lower values of PEP, PEV, and ETP received by GSF model in most of the events further indicate its higher hydrograph reproducibility. This can be attributed to the presence of parameter f in the GSF model that describes the spatial distribution of rainfall in the watershed. In addition, it was also demonstrated that the performance, as well as the estimated parameters, are changing for the same model at different time intervals which can be interpreted as their influence on the model performance.

There is a need for the improvement of the model for the application in different types of catchments in an operational context using data assimilation approaches to check its applicability and hence the same will be carried out in our future studies.

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