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Performance Evaluation of Urban Storage Function (USF) Model Compared with Various Conventional Storage Function Models for an Urban Watershed

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Various Storage Function (SF) models have been widely used among different parts of the world in which the Urban SF (USF) model is a newly developed SF model mainly for urban watersheds. In this study, we aim to conduct the performance evaluation of USF model and to compare the results with the conventional SF models of Hoshi, Prasad, Kimura, and the linear model. The reproducibility of hydrograph was evaluated using the performance evaluation criteria's of Root Mean Squared Error, Nash Sutcliffe Efficiency (NSE), and other error functions of peak, volume, and time to peak. The results revealed that higher values of NSE for USF model indicate that the hydrograph reproducibility by USF is more reliable than that reproduced by other SF models. Further, AIC and Akaike Weight (AW) were used to compare and identify the best model among all based on the information criteria perspective. The USF model received the lowest AIC score and highest AW in most of the events which indicate that the USF is the most parsimonious model compared with other SF models.

Key Words: *urban storage function model, conventional storage function models, hydrograph reproducibility, performance evaluation, AIC criteria*

1. INTRODUCTION

Modelling the rainfall-runoff transformation process of an urban watershed is essential for the flash flood estimation. Hence, the accurate prediction of hydrograph, which includes the estimation of flood peak, time to peak, volume, etc. is important in order to avoid the losses due to flood plain inundation. Among the different lumped rainfall-runoff models, the Storage Function (SF) models have been widely used in many parts of the world not only due to its easiness in computation and handling, but also the ease of expressing the nonlinear relationship of the rainfall-runoff process with simple equations[1)]. Enormous studies have been conducted using SF models in order to analyse the rainfall-runoff transformation process. Kimura proposed the first SF model in Japan with three parameters[2)]. This nonlinear lumped model is still widely used in Japan for the flood prediction. Later, Laurenson developed a different two parameter model for runoff modelling and tested the method in South Creek, Australia[3]. Subsequently, Prasad presented a three parameter SF model which has an additional term for the inclusion of the loop effect between storage and discharge[4]. Successively, Hoshi and Yamaoka added another parameter and improved the robustness of SF model[5].

However, all these models requiring effective rainfall as their input for the direct runoff prediction. Hence it involves the problems of baseflow and effective rainfall component separation from total discharge and total rainfall respectively, which may further affect the value of parameters being estimated and their relative stability. In order to overcome this problem, Baba et al. introduced an SF model with loss mechanism which uses the observed rainfall and runoff and applied to the mountainous river basin in Hokkaido, Japan[6]. The incorporated loss mechanisms (infiltration and all other outflow components) avoided the need of effective rainfall estimation and baseflow separation.

The use of Baba's SF model for the prediction of runoff in urban areas may be difficult, because the urban area differs completely with the mountainous area in terms of the imperviousness, absence of vegetation, presence of sewage system, etc. Takasaki et al. developed a new Urban SF (USF) model considering the urban runoff process which uses the observed rainfall and runoff directly without effective rainfall estimation and baseflow separation for the flood prediction and compared with Baba's SF model[7]. The model considered all possible inflow and outflow components which include the groundwater inflow as an outflow from the basin.

The various SF models have not been evaluated not only in terms of the prediction accuracy, but also the information criteria aspect point of view, up to the knowledge of the authors. Specifically, there are no studies which describe the performance evaluation of different SF models for an urban area including the USF. Hence, this study aims to conduct the performance evaluation of USF model and compare with the various conventional SF models of Hoshi, Prasad, Kimura, and the linear model for an urban watershed. The Kanda river basin, a typical small to medium sized urban watershed in Tokyo, was selected as the target basin and the five SF models were applied to five flood events. In order to check the performance evaluation in terms of the reproducibility of hydrograph, first, we formulated the SF models with optimal parameters using SCE-UA global optimisation method[8]. The Root Mean Squared Error (RMSE) was chosen as the objective function for optimisation. These optimal SF models were assessed for reproducibility of hydrograph with minimum RMSE and maximum Nash Sutcliffe Efficiency (NSE) and other error functions of peak, volume, and time to peak. Also, for the first time in SF model research, the authors have utilized Akaike Information Criteria (AIC) and Akaike Weight (AW) to identify the best SF model for an urban watershed based on the information criteria perspective[9].

2. MATERIALS AND METHODS

(1) Storage function models

The SF models are characterized by the relationship between the storage and discharge. The different existing models are given by the following equations:

$$s = k_1 Q \qquad \text{Linear model}^{1)} \qquad (1)$$

$$s = k_1 (Q)^{p1} \qquad \text{Kimura's model} \qquad (2)$$

$$s = k_1 (Q)^{p1} + k_2 \frac{dQ}{dt} \qquad \text{Prasad's model} \qquad (3)$$

$$s = k_1 (Q)^{p1} + k_2 \frac{d}{dt} (Q)^{p2} \qquad \text{Hoshi's model} \qquad (4)$$

where s: storage (mm), Q: observed river discharge (mm/min), t: time (min), k_1, k_2, p_1, p_2: model parameters. Among the models, Hoshi's model has been found to be superior[5]. Some simplifications to the Hoshi's storage model can lead to Prasad, Kimura, and linear storage models. In this paper, the authors used the Kimura's SF model with one storage tank which is widely used as a special case of Kimura's model with the delay time (third parameter) equal to zero. Because the delay time is a function of effective rainfall, and basin characteristics and thereby its estimation is an extremely difficult process. USF model is the empirical representation of Hoshi's SF model in which the observed river discharge Q is replaced by the discharge, including the drainage from the sewage system $(Q + q_R)$ for the urban area, where, q_R (mm/min) is drainage from the basin through the combined sewer system. The USF model is given by the following equation[7]:

$$s = k_1 (Q + q_R)^{p1} + k_2 \frac{d}{dt} (Q + q_R)^{p2} \qquad (5)$$

Combining the above expression of storage with the following continuity equation yields the nonlinear expression of USF model.

$$\frac{ds}{dt} = R + I - E - O - Q - q_R - q_l \qquad (6)$$

where q_l: loss to the groundwater (mm/min), R: observed rainfall (mm/min), I: inflow from other basins (mm/min), E: evapotranspiration (mm/min), O: water intake from the basin (mm/min). Further, the loss to groundwater (q_l) was defined by considering the infiltration hole height (z) and is given by[7]:

$$q_l = \begin{cases} k_3(s-z) & (s \geq z) \\ 0 & (s < z) \end{cases} \tag{7}$$

where k_3 and z are the parameters. The drainage q_R from the combined sewage system discharged into the river is controlled by the carrying capacity of the sewer. Hence, the maximum volume of q_R cannot exceed maximum carrying capacity $q_{R\,max}$. The q_R is calculated as[7],

$$q_R = \begin{cases} \alpha(Q + q_R - Q_0) & \alpha(Q + q_R - Q_0) < q_{R\,max} \\ q_{R\,max} & \alpha(Q + q_R - Q_0) \geq q_{R\,max} \end{cases} \tag{8}$$

where α is the slope of the linear relationship between total discharge $Q + q_R$ and the drainage q_R; and Q_0 is the initial river discharge just before the rain starts [7]. Equating (5) and (6) after differentiating (5) with respect to time will lead to a second order Ordinary Differential Equation (ODE). This second order ODE is transformed into the first order ODE and can be numerically solved. The river discharge Q will obtain from the solution.

The USF is a seven parameter model with parameters $k_1, k_2, k_3, p_1, p_2, z, \alpha$ used in the rainfall-runoff modelling. Generally, the conventional Hoshi's SF is a four parameter model with parameters k_1, k_2, p_1, p_2 . However, in order to incorporate the loss to groundwater q_l and to consider the observed discharge as a whole in urban watersheds, we added two more parameters k_3 and z. Hence Hoshi's SF model can be designated as a 6-parameter model. In a similar way, the Prasad, Kimura and Linear models were transformed into 5, 4, and 3-parameter models respectively. The SCE-UA method proposed by Duan[8] was used with RMSE as the objective function and it was minimised to identify the optimal parameters of all the above models. The search range of parameters for SCE-UA is set as, k_1(10-500), k_2(100-5000), k_3(0.001-0.05), p_1(0.1-1), p_2(0.1-1), z (1-50), and α (0.1-1)[7].

(2) Performance evaluation

The river discharge computed for each event by the different SF models was compared in order to assess the reproducibility with the observed hydrographs using five error functions[10] of RMSE, NSE, Percentage Error in Peak (PEP), Percentage Error in Volume (PEV), and Percentage Error in Time to Peak (PETP). Further, AIC was also used in order to identify the best model by comparing the different models for each event[9]. The best model is then the model with the lowest AIC score and is given by the following expression.

$$AIC = 2K - 2\log(\mathcal{L}(\hat{\theta}|y)) \tag{9}$$

where K: number of parameters to be estimated and $\log(\mathcal{L}(\hat{\theta}|y))$: log likelihood at its maximum point of the model estimated. Later, this concept was refined to correct for small data samples as[11]:

$$AIC_C = AIC + \frac{2K(K+1)}{n-K-1} \tag{10}$$

where n: sample size. A better way of interpreting the AIC_C score is to normalize the relative likelihood values as Akaike Weight (AW). The weight of all models summed together equals one and the model with highest AW is considered to be the best one. The AW is considered as the weight of evidence that the model i is the best-approximating model for the given data and candidate models and is given as,

$$\text{AW} = \frac{\exp(-0.5\Delta AIC_{c,i})}{\sum_{r=1}^{R}\exp(-0.5\Delta AIC_{c,r})} \tag{11}$$

where the ΔAIC_C is calculated as,

$$\Delta AIC_C = AIC_{C,i} + AIC_{C,min} \tag{12}$$

where $AIC_{C,i}$: individual AIC_C score for each model, $AIC_{C,min}$: minimum AIC_C score of the models tested, R: number of models, r: model being considered.

(3) Study area and data used

The target basin area at Koyo Bridge is about 7.7 km^2 shown in **Fig.1**. The rainfall and water level data at one minute interval were collected from the Bureau of Construction, Tokyo Metropolitan Government (TMG) during 2003-2006 for the present study. The basin average rainfall was determined using the Thiessen polygon method from the eight rain gauges scattered over the basin. Five target events were selected from the data, whose 60-minute maximum rainfall (R_{60}) is greater than 30 mm, for the application of SF models and is given in **Table 1**. The inflow component I other than precipitation was fixed at 0.0012 mm/min based on the business annual report of the TMG. The water intake from the basin O and evapotranspiration E were set at 0 since there is no water intake from the target basin and the evapotranspiration during heavy rainfall is insignificant. The maximum drainage, $q_{R\,max}$ was estimated at 0.033 mm/min using the Manning's equation[7].

3. RESULTS AND DISCUSSIONS

(1) Hydrograph reproducibility

The SCE-UA method was applied for the parameter estimation of the five models for the five selected flood events of the target basin. The parameters were optimally estimated with a minimum value of RMSE. The SF models with these optimally estimated

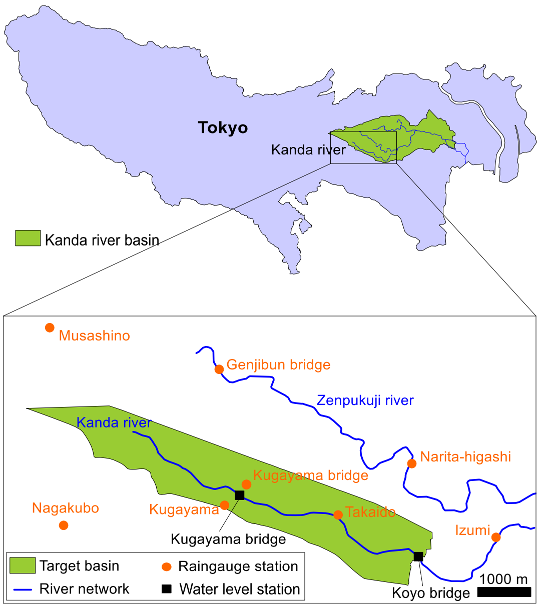

Fig.1 Index map of study area.

Table 1. Characteristics of target events.

Event No.	Event date	R_{60} (mm)	Total R (mm)	Climatic factors
1	13/10/2003	53.9	57.5	Intensive localized storm
2	24~25/6/2003	42.8	55.5	Frontal event
3	8~9/10/2004	42.0	261.1	Typhoon
4	11/09/2006	32.7	37.9	Frontal event
5	15/07/2006	31.5	31.5	Frontal event

parameters were further used to estimate the river discharge in order to evaluate the hydrograph reproducibility. **Fig.2** shows the reproduced hydrograph by the different SF models for the two selected flood events out of five due to the space constraints. The selected events are Event 1 and 3 where Event 1 is single-peaked and Event 3 is multi-peaked and the rest of the events are single-peaked. It can be envisaged from **Fig.2 (a)** and **(b)** that the 7-parameter USF model almost overlaps with the observed river discharge and accurately reproducing the peak. Even though the 6-parameter model shows a slight deviation in the reproduced hydrograph at the rising and recession limbs in both events, the model accurately reproducing the peak discharge. The 5-parameter model performs well in the reproduction of discharge peak for Event 1. However, the model failed to reproduce the peak discharge with Event 3 and lower predicted the peak. Both the 4 and 3-parameter models were unable to reproduce the observed hydrograph exactly, especially the 3-parameter model, and lower predicted the peak. The results exhibited that the USF model can more precisely reproduce the shape of the observed hydrograph as well as the peak discharge compared with other SF models irrespective of the number of peaks. On the other hand, the 6-parameter model not preserving the shape of the hydrograph particularly in the multi-peak event even though it predicts the peak accurately. The 5-parameter model failed to reproduce not only the shape of the hydrograph but also the peak in the multi-peak event. The 4 and 3-parameter models failed to conserve the shape and peak discharge regardless of the number of peaks.

Fig.3 shows the values of various error functions, i.e., RMSE, NSE, PEP, PEV, and PETP for the five events by the five models. From **Fig.3 (a)** and **(b)**, we can see that the USF model generates low RMSE close to zero and high NSE close to 100%, followed by the 6 and 5-parameter models in which 6-parameter model values are adjacent to USF model for all the events. The low RMSE and high NSE of USF model can be interpreted as the high hydrograph reproducibility. However, the 4 and 3-parameter models have the highest RMSE and least NSE compared with other models. This is because of the absence of parameters that describes the loop effect between the storage and discharge during the rising and recession limbs. It is evident that the model with a large number of parameters will have the least

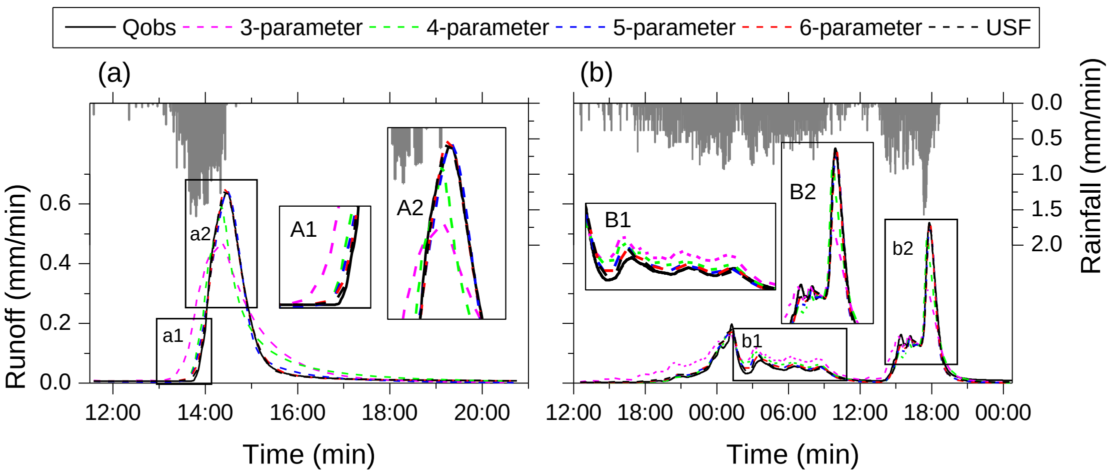

Fig.2 Reproduced hydrograph by each model for (a) Event 1 and (b) Event 3. The rectangles A1, A2, B1, and B2 represent the enlarged view of the sections a1, a2, b1, and b2 of the hydrograph.

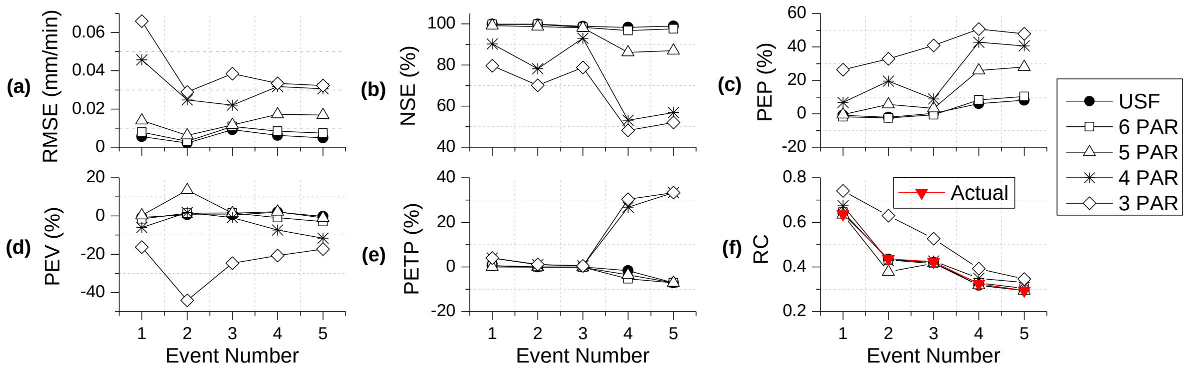

Fig.3 Comparison of RMSE, NSE, PEP, PEV, PETP, and Runoff Coefficient (RC) by the different SF models (* PAR represents parameter).

RMSE and highest NSE which further reveals that the SCE-UA method has successfully identified the optimal parameters for each model in each event. The PEP and PEV become positive for underestimation and PETP becomes positive for early prediction of peak time. **Fig.3 (c)** depicts that the PEP estimated by USF and 6-parameter models are very low and not greater than 10% in any of the events, even though the 6-parameter model shows slight deviations. On the contrary, the 5, 4 and 3-parameter models largely vary in their PEP values and always lower predicts the peak flow. Likewise the PEP, the USF and 6-parameter models show the best ranges of PEV and PETP in **Fig.3 (d)** and **(e)** respectively, which is close to zero compared with the other models. Simultaneously, the 5, 4 and 3-parameter models generate higher values of PEV and PETP. They overestimated the volume with an early peak time estimation. **Fig.3 (f)** additionally shows the Runoff Coefficient (RC) and we can see that the RC value of USF, 6, 5, and 4-parameter models are very close to the actual value except for 5-parameter model in Event 2. The 3-parameter model exhibits greater discrepancies with the actual values of RC in all events. The higher values of NSE coupled with the lower values of RMSE, PEP, PEV, and PETP for USF indicated that the hydrograph reproducibility by USF is the highest among the SF models.

(2) AIC aspect

In addition to the hydrograph reproducibility, AIC aspect was used in order to determine the best model for the each selected event. **Fig. 4 (a)** shows the AIC_C values for each model in each event. It can be seen from the figure that the 6-parameter model has the lowest AIC_C in Event 1 and the 5-parameter model has the lowest AIC_C in Event 2. However, USF received the lowest AIC_C in Events 3, 4 and 5. Even though the USF model has not received lowest AIC_C in the first two events, it was close to the lowest value. Hence the USF model is suitable for both single and multi-peak events, although the 6-parameter is good for only single-peaks. There is almost zero support for 4 and 3-parameter models from **Fig.4 (a)** because they generate a far higher AIC_C score, which indicates the necessity of more parameters in order to describe the storage characteristics of the urban watershed. From **Fig.4 (a)**, it is not easy to clearly distinguish the difference between the AIC_C values of USF, 6, and 5-parameter models. Hence, we analysed the AIC_C values using an associated statistic known as AW to depict the differences distinctly. **Fig.4 (b)** shows the AW for each event by different SF models. The weight exhibits an opposite trend of AIC_C values and the model with the highest weight will be the best one[11)]. Likewise the AIC_C score, the 6 and 5-parameter models received highest weights in Event 1 and 2 respectively. During the rest of the events, USF model is having the highest weight.

As a general rule of thumb, the AW of the candidate models in each event should be higher than 10% of the highest AW of that event[12)] so that we can easily exclude models with a weight lower than 10% of the highest AW. Based on this rule, we can exclude the 4 and 3-parameter models. In Event 3, 4 and 5, the USF model is having highest AW followed by the 6-parameter model. Even though the 6-parameter is followed by USF, the difference between the AW values of these models are quite high, significantly higher for Event 3 and 5. Therefore USF is much more effective than the 6-parameter model in such

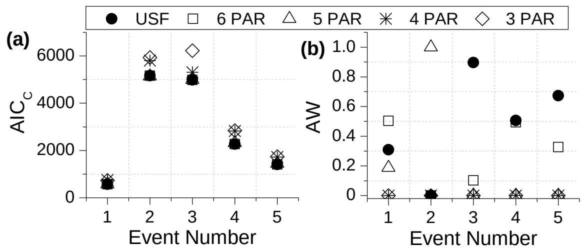

Fig.4 The summary of AIC results, (a) corrected AIC (AIC_C) and (b) AW values for the five events.

multi and single-peak events. In Event 1, the 6-parameter model received highest AW followed by USF model. However, the difference in AW values is not so high as compared with that of in Event 3 and 5. Consequently, the USF model is found to be more suitable over the 6-parameter model for the use in urban watersheds as per the AIC aspect. From now on, we will explain in detail the exceptional behaviour of Event 2. In Event 2, the 5-parameter model was found to be the model with highest AW and all other models failed to attain a weight higher than 10% of that highest AW. This can be explained by the long duration of low flow before the rising limb starts, which resulted from the long duration of intermittent rainfall. Thus, it can be inferred that the 5-parameter model is enough for the representation of this long duration low flow.

According to the above discussion, the 7-parameter USF model is found to be good not only for the hydrograph reproducibility, but also the most parsimonious in most of the flood events in an urban watershed. This implies that all the parameters of USF are important in estimating the urban river discharge, especially the parameter α which considers the effect of storm drainage that is diverted to the waste water treatment plant instead of going to the river. The other SF models with a reduced number of parameters received higher AIC_C scores which indicate that the parameters included are not sufficient to explain the urban river discharge.

4. CONCLUSIONS

The five SF models with optimal parameters were applied to five flood events in an urban watershed in Tokyo for the performance evaluation with minimum RMSE. First, the models were assessed in order to check the hydrograph reproducibility using five error functions of RMSE, NSE, PEP, PEV, and PETP. The results revealed that USF has least RMSE (high NSE) among all models for all events which further shows that the SCE-UA method has successfully identified the optimal parameters. The lower values of PEP, PEV, and PETP received by USF model further indicate that the hydrograph reproducibility by USF is the highest among the SF models. In addition, the summary of AIC results shows that the USF received the highest AW in most of the events compared with other SF models which make it the most parsimonious model. The other SF models have the lower AW scores that indicated the necessity of the addition of more parameters which describe the storage characteristics of an urban watershed.

As a conclusion, the USF model can be considered as the best not only for the hydrograph reproducibility, but also the most parsimonious based on the AIC perspective in most of the flood events in an urban watershed when compared with the conventional ones, if the optimal parameters are successfully identified for the events. However, in this study, we didn't consider the parameter uncertainty and their relative stability even though it is an important aspect. Hence the same will be carried out in the follow-up studies.

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REFERENCES

1) Kawamura, A., Morinaga, Y., Jinno, K. and Dandy, G. C.: The comparison of runoff prediction accuracy among the various storage function models with loss mechanisms, *Proc. 2nd APHW Conference*, Vol. 2, pp. 43-50, 2004.
2) Kimura, T.: The Flood Runoff Analysis Method by the Storage Function Model. The Public Works Research Institute, Ministry of Construction, 1961 (In Japanese).
3) Laurenson, E. M.: A catchment storage model for runoff routing, *J. Hydrol.*, Vol. 2 (2), pp. 141-163, 1964.
4) Prasad, R.: A nonlinear hydrologic system response model, *Proc. ASCE*, Vol. 93, HY4, pp. 201-221, 1967.
5) Hoshi, K. and Yamaoka, H.: A relationship between kinematic wave and storage routing models. *Proc. 26th Japanese Conf. on Hydraulics, JSCE*, pp. 273-278, 1982 (In Japanese).
6) Baba, H., Hoshi, K. and Hashimoto, N.: Synthetic storage routing model coupled with loss mechanisms. *Annual Journal of Hydraulic Engineering, JSCE*, Vol. 43, pp. 1085-1090, 1999 (In Japanese).
7) Takasaki, T., Kawamura, A., Amaguchi, H. and Araki, K.: New storage function model considering urban runoff process, *Journal of JSCE, B*, Vol. 65 (3), pp. 217-230, 2009 (In Japanese).
8) Duan, Q., Sorooshian, S., and Gupta, V.: Effective and Efficient Global Optimization for Conceptual Rainfall-Runoff Models, *Water Resources Research*, Vol. 28 (4), pp. 1015-1031, 1992.
9) Akaike, H.: Information theory and an extension of the maximum likelihood principle", In: Petrov, B.N., Csaki, F. (Eds.), *2nd International Symposium on Information Theory*, Akademiai Kiado, Budapest, pp. 267-281, 1973.
10) Padiyedath, G. S., Kawamura, A., Sahoo, B. and Amaguchi, H.: Effect of evapotranspiration on the discharge estimation in Baitarani watershed, India in the context of climate change, in: World Environmental and Water Resources Congress 2017, ASCE, Reston, VA, pp. 577-588, 2017.
11) Hurvich, C. and Tsai, C. L.: Regression and time series model selection in small samples, *Biometrica*, Vol. 76, pp. 297-293, 1989.
12) Royall, R. M.: Statistical Evidence: a likelihood paradigm. Chapman and Hall, New York, 1997.

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