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Parameterization of rain cell properties using an advection–diffusion model and rain gage data

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Abstract

To reduce flooding risks and improve urban drainage management, there is a need to increase the forecasting accuracy for rainfall models on small typical urban time and space scales. Increased rainfall forecasting accuracy will in turn improve runoff prediction and thus, prevent flooding hazards, decrease pollution discharge through combined sewers, increase waste water treatment efficiency, etc. For this purpose, we analyzed the parameters of a two-dimensional stochastic advection–diffusion model including a Fourier domain method and an extended Kalman filter algorithm for investigation of motion, shape, size, and intensity distribution of convective rainfall. The resulting set of model parameters (advective velocity, apparent turbulent diffusion, and development/decay of rainfall rate) is used to study convective rainfall variability. It appears that the speed at which the rainfall cell is advected is not dependent on the cell development stage or apparent diffusion. Instead, there is a dependence between the source/sink term and apparent diffusion. This can be explained by the turbulent updraft of warm air which results in large rainfall intensity increase. This strong turbulence results in larger diffusion (and vice versa). The behavior of the model parameters is therefore physically explainable and relevant. The results can be used as first choice of parameter values when modeling convective rainfall over ungaged areas.

1. Introduction

There is a need to improve the forecasting accuracy of rainfall models for use in small-scale urban areas (e.g., Schilling, 1990). Forecasted rainfall from these models can be used as input to runoff models and, consequently, lead to improvement of overall management of urban drainage systems. This in turn may lead to less flooding problems, reduced pollution discharge, improved efficiency at waste water treatment stations, etc.

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Many models for space–time rainfall have been suggested during the last years (Georgakakos and Kavvas, 1987). Most rainfall generating models (e.g., Kavvas and Puri, 1983) are based on a stochastic description using Poisson processes. Sivapalan and Wood (1987) developed a rainfall generator based on nonstationary space–time correlations that exhibits storm movement, cellular structure, and birth and decay of cells. For forecasting purposes, however, models need to include a high degree of determinism, i.e., size, shape, and direction of actually observed rainfall. Data input may be from radar and/or rain gages. Models for short-term rainfall forecasting in most cases, use radar data as input (Bellon and Austin, 1984; Einfalt and Denoeux, 1987; Browning and Collier, 1989). Urban catchments, however, are often not equipped by radar but by rain gages instead. Also, for most urban applications, rain gages supply sufficiently detailed data for small-scale catchments. Radar may, however, provide information on a larger scale embedding the rain gage system (e.g., specifying boundary and initial conditions for the forecasting area).

Jinno et al. (1990, 1993 introduced a model based on a two-dimensional stochastic advection–diffusion equation in combination with a Fourier domain method and an extended Kalman filter algorithm for short-term, small-scale rainfall forecasting. The model can be used to forecast motion, shape, size, and intensity distribution of convective rainfall patterns. Since a Fourier domain method represents the rainfall field, any irregular shaped intensity pattern can be reproduced in the model.

A model like this, can be used to parameterize the variation of rainfall structures and, thus, be of help in more fundamental research. This, in turn, may be practically utilized, e.g., for choice of model parameters in ungaged areas or when studying effects of parameter variability on the rainfall intensity in numerical studies (see e.g., Kawamura et al., 1992, 1996).

The objective of the present paper is to use the described model to study variability patterns of convective rainfall as observed in a dense rain gage network. Especially, we are interested to parameterize rainfall in terms of advective velocity, apparent turbulent diffusion, and development/decay of rainfall rate. We use ten high-intensive rainfall events observed by Niemczynowicz (1984) to quantify the above parameters. The results can be used as first choice of parameter values when modeling convective rainfall over ungaged areas.

2. Properties of convective rainfall

As mentioned above, the model is based on an advection–diffusion equation. Thus, it is assumed that main forces acting on the rain drops during short periods of time are advection and diffusion. Fig. 1 shows a typical example of convective rain cell behavior that supports this assumption. The figure shows a horizontal view of convective rainfall intensity distribution over the city of Lund during 7 min. A cell-shaped pattern is formed over the gaged area (12 rain gages) that is advected in a south–east direction. The broken lines in Fig. 1 indicate the average direction of movement and the cross displays the location of the advected cell center.

By adopting a Lagrangian view and displaying the cell center at different time steps

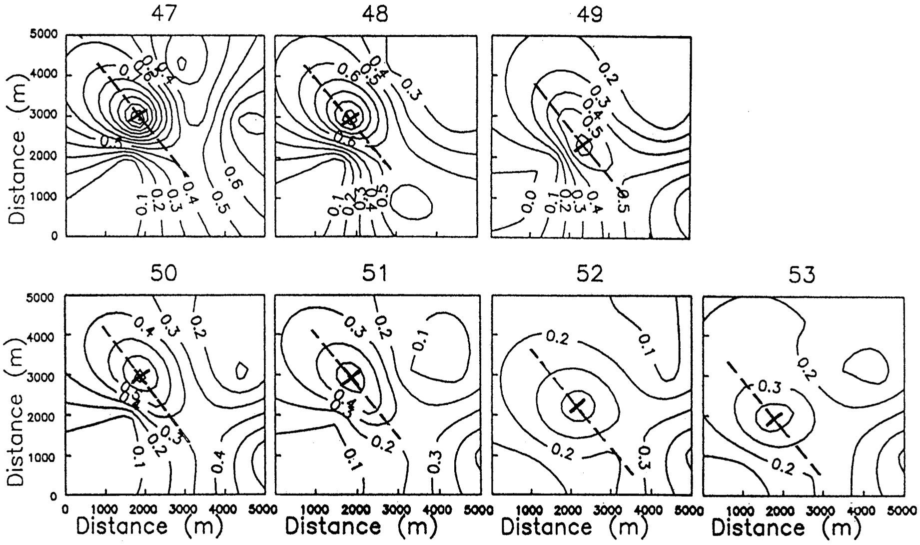

Fig. 1. Example of an observed convective rainfall cell during 7 min (time steps 47–53 min; data from Niemczynowicz, 1984). The broken line and cross denote the average direction of movement of the cell and location of cell center, respectively (unit is in mm/min).

in the same diagram, the relevance of the advection–diffusion assumption can be inferred. Fig. 2 shows this for the observed cell in Fig. 1. The figure displays the change in intensity distribution for the different time steps along vertical transects (correspond-

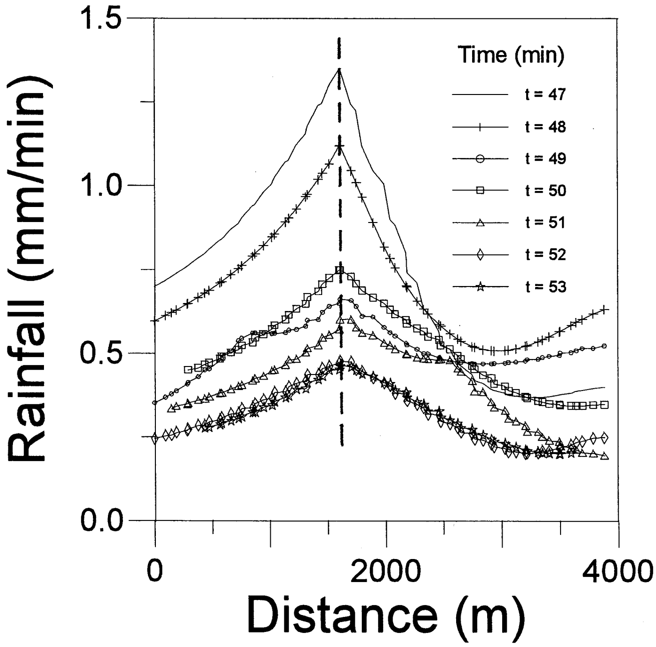

Fig. 2. Lagrangian view through the cell center in Fig. 1. The rainfall intensity is plotted along the transects (broken lines) in Fig. 1. The vertical broken line indicates the cell center according to the crosses in Fig. 1.

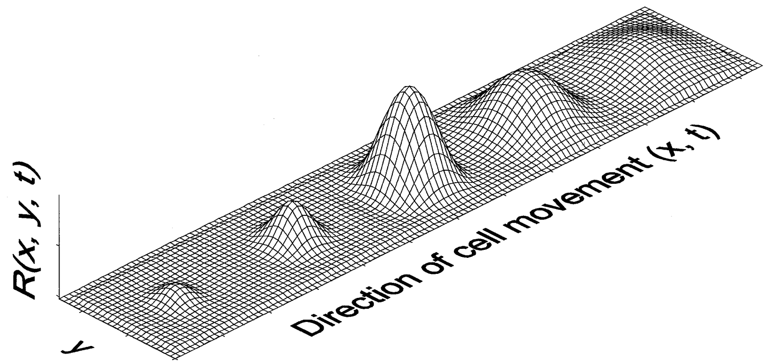

Fig. 3. Schematic of how a rainfall intensity pattern evolves during 5 time steps using the advection–diffusion model of Jinno et al. (1993), Eq. (1). Note that the model is not confined to only regular gaussian shapes but instead can reproduce any irregular pattern due to the Fourier representation.

ing to the broken lines in Fig. 1) through the cell center. The vertical broken line in Fig. 2 indicates the location of the cell center as displayed by the crosses in Fig. 1. The change in intensity pattern with time according to Fig. 2, suggests a diffusive behavior of the convective rain cell. The sharp peak in rainfall intensity during the first minutes is rapidly evened out in a diffusive manner during the 7-min observation period.

Fig. 3 shows schematically how the model (Eq. 1), can reproduce the behavior of the cell in Figs. 1 and 2. The x-axis is taken in the main direction of movement of the convective rainfall and the two-dimensional stochastic advection–diffusion equation may be expressed as (assuming spatial homogeneity regarding rainfall speed, diffusion, and development/decay):

$$\frac{\partial R(x,y,t)}{\partial t} + u\frac{\partial R(x,y,t)}{\partial x} =$$

$$D_x\frac{\partial^2 R(x,y,t)}{\partial x^2} + D_y\frac{\partial^2 R(x,y,t)}{\partial y^2} - \gamma R(x,y,t) + \epsilon(x,y,t)$$

where $R(x,y,t)$ = rainfall intensity in space (x,y) and time (t) (m/min), u = advective speed of the rainfall cell in the direction of movement (m/min), D_x, D_y = diffusion coefficients in the x- and y-direction, respectively (m^2/min), γ = development/decay coefficient of the rainfall intensity (min^{-1}), $\epsilon(x,y,t)$ = stochastic component with zero-mean Gaussian white noise in time and space (m/min^2).

The diffusion coefficients D_x and D_y describe the rate at which the cell expands horizontally (Fig. 3). The development/decay coefficient γ describes the development (cumulus stage; the first three stages in Fig. 3) and decay (dissipating stage; the two last stages in Fig. 3) of the cell in terms of change in rainfall intensity. Negative values of γ represent a situation when the cell develops or expands (cumulus stage), i.e., the rainfall intensity increases. Similarly, positive values of γ, represent a decaying situation (dissipating stage) when the rainfall intensity decreases. The last term in Eq. (1) is a stochastic component to consider a specified error and uncertainty level for the rainfall observations. The model is further described by Jinno et al. (1993).

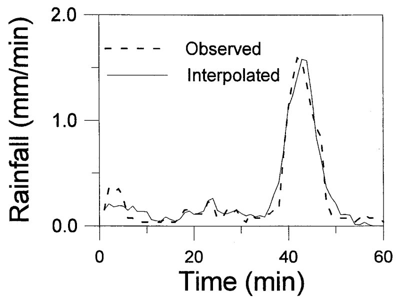

Fig. 4. Comparison between observed and spatially interpolated rainfall intensities using the model in Fig. 3.

Fig. 4 gives an example of comparison between model output and observations. In this case, the noise level was set to zero and the interpolation error calculated as the root-mean-square error for all gages during the entire model period (60 min) was 0.12 mm/min. Consequently, the model is flexible and quickly adjusted to variations in rainfall intensity.

3. Parameterization of convective rainfall

The described model above was used to study variability patterns and parameterize observed convective rainfall as observed by a dense rain gage network. We used the four parameters of the model; advective velocity u, apparent turbulent diffusion D_x and D_y, and development/decay γ of rainfall rate for this purpose. Ten high-intensive rainfall events observed by Niemczynowicz (1984) were used to quantify these parameters.

The 10 chosen rainfall events represent individual rain cells similar to the one shown in Fig. 1. The maximum recorded intensity during the events varied between 0.8 and 2.4 mm/min (Table 1). The duration of the analyzed events was between 15 and 40 min. For all events, one-minute data from 10 to 12 rain gages were used.

Table 1
Intensity characteristics and identified final parameters of Eq. (1) for the 10 high-intensive rainfall events

Rainfall event	Obs. max intensity, mm/min	u, m/min	D_x, $\times 10^4$ m^2/min	D_y, $\times 10^4$ m^2/min	γ, $\times 10^{-2}$ min^{-1}	Interpol. error, mm/min
1	1.5	290	1.4	0.93	1.8	0.08
2	2.2	490	4.1	3.3	4.0	0.06
3	1.7	0	2.3	1.5	2.5	0.07
4	1.3	300	1.6	0.85	1.7	0.04
5	2.2	500	2.4	1.3	2.6	0.06
6	2.1	230	1.5	1.0	1.4	0.07
7	0.8	640	0.68	0.34	0.67	0.04
8	2.4	280	5.0	2.9	4.7	0.05
9	1.0	510	1.4	0.76	1.4	0.05
10	1.4	190	1.1	0.87	1.2	0.03
Mean	1.66	343	2.15	1.38	2.20	0.055
St. dev.	0.55	190	1.38	0.96	1.28	0.016

Table 1 summarizes the model calibrations and parameter variability. From the interpolation error it is seen that the model error in general is very small and thus, the model is able to modify to very rapid and fluctuating rainfall intensities. The average speed of the convective rainfall was 343 m/min ($u = 5.7$ m/s). The diffusion coefficient in x-direction (direction of movement) was consistently larger ($D_x = 2.15 \times 10^4$ m^2/min) than the one in the y-direction ($D_y = 1.38 \times 10^4$ m^2/min). An elongation of the cells in the direction of movement is often observed in experimental studies (e.g., Berndtsson et al., 1994; see also Fig. 1).

A correlation analysis of the parameters displays that advective velocity u is uncorrelated with other parameters. The apparent turbulent diffusion coefficients D_x and D_y are highly inter-correlated, though. The parameters D_x and D_y have a correlation coefficient of 0.959. A regression equation could be fitted according to: $D_x = 0.392 + 1.24\ D_y$.

The minimum value of γ_{min}, indicating the strength of the cell activity (small values indicate a strong cell development), during the modeled rainfall event was compared to maximum rainfall intensity and other model parameters. As expected there is an inverse relationship between γ_{min} and the maximum rainfall intensity and diffusion coefficients D_x and D_y. On average the correlation is -0.620 between γ_{min} and diffusion coefficients. This indicates that if the cell development is strong (small value of γ_{min}), the turbulence and thus, the diffusion are large. This hypothesis is further strengthened by the large negative correlation between maximum rainfall intensity and γ_{min} (-0.610), indicating similar physical relationship.

Consequently, it appears that the speed at which the rainfall cell is advected has no dependence on the cell development stage or apparent diffusion. Instead, there is a dependence between the source/sink term and apparent diffusion. This can be explained by the turbulent updraft of warm air which results in large rainfall intensity increase. This strong turbulence logically results in larger diffusion (and vice versa). The dependence between D_x, D_y, and γ_{min} is therefore physically relevant.

4. Summary and discussion

The present analysis has shown that a two-dimensional stochastic advection–diffusion equation in combination with a Fourier domain method and an extended Kalman filter algorithm can be used to analyze motion, shape, size, and intensity distribution of convective rainfall. The main conclusions can be summarized according to:

1. the diffusion coefficient in the x-direction (direction of movement) is consistently larger than the one in the y-direction (about 55% larger). This shows that the diffusion is larger in the direction of movement in analogy with sub-surface transport phenomena;
2. the advective velocity u is uncorrelated with other model parameters. This means that the speed at which the rainfall cell is advected has no dependence on the cell development stage or apparent diffusion;
3. there is a dependence between the minimum source/sink term (γ_{min}) and apparent diffusion. This depends probably on the turbulent updraft of warm air which results

in large rainfall intensity increase. The turbulent updraft consequently also means larger diffusion (and vice versa).

4. The maximum recorded rainfall intensity is also correlated to the minimum source/sink term (γ_{min}). This means that intensive cell activity is closely connected to mass changes in the rainfall.

In general, it has been shown that the model parameters behave in a way that is physically explainable and relevant to the rain cell development stages. Relationships between parameter values can be used when modeling convective rainfall over ungaged areas. Average parameter values in this study can be used as first choice of parameter values in new modeling areas.

Acknowledgements

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