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Analysis of patterns in a precipitation time sequence by ordinary Kalman filter and adaptive Kalman filter

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ABSTRACT This paper proposes a methodology of investigating the patterns in a precipitation time sequence by an ordinary Kalman filter (OKF) and an adaptive Kalman Filter (AKF). Specifically this research aims to investigate the occurrence and characteristics of drought precipitation patterns. The methodology is applied to the 92-year long monthly precipitation time sequence at Fukuoka City in Japan. OKF identifies the periods (in the time sequence) with abnormal precipitation by comparing the observed and average precipitation patterns. AKF detects the changes of the long term precipitation patterns in the time sequence. These changes are associated with the abrupt changes in the parameters of the periodic-stochastic model. The time sequence is divided into several precipitation epochs, where an epoch is uniquely represented by one set of parameter values. The precipitation pattern in each epoch is appropriately characterised and reveals whether the risk of drought occurrence is high or not in an epoch.

Analyse des Courbes de Précipitation Séquentielle dans le Temps par le Filtre Ordinaire de Kalman et le Filtre Adaptable de Kalman

RESUME Cet article propose une méthodologie pour investiguer les courbes d'une précipitation séquentielle dans le temps par le filtre ordinaire de Kalman (OKF) et le filtre adaptable de Kalman (AKF). Plus précisement, cette étude à pour but d'investiguer la survenue et les caractéristiques des courbes de précipitation de sécheresse. Cette méthodologie est appliquée à la précipitation à séquence mensuelle d'une longue période de 92 ans de la ville Japonaise de Fukuoka. OKF identifie les périodes (dans le temps) avec précipitation anormale en comparant les courbes de précipitation observées avec la courbe moyenne. Et AKF détecte les changements à long terme des courbes de précipitation. Ces changements sont associés avec des variations brusques dans les parametres du modèle périodique avec des pointes en crochets. La séquence dans le temps est divisee en plusieurs époques de précipitation une époque étant uniquement représentée par un ensemble de valeurs des parametres. La courbe de précipitation dans chaque époque est bien caracterisée et révèle si le risque de survenue d'une sécheresse est grande ou non dans une époque.

Notation

A_i, B_i	periodic coefficients
f_i	frequency component
G	unknown (nxl) abnormality vector
H	known (mxn) observation matrix
H_o	the hypothesis that no change in precipitation pattern has occurred
H_1	the hypothesis that precipitation pattern has changed at $k=\theta$
I	(nxn) identity matrix
k	time instant
l	innovation cumulative number
m	dimension of observation vector
n	dimension of system state vector; number of syste parameters
q	number of significant frequency components
s	length of moving average (months)
T	return period(years)
u	independent, zero mean, white Gaussian (pxl) system noise vector
w	independent, zero mean, white Gaussian (mxl) observation noise vector
x	(nxl) system state vector
y	(mxl) observation vector ($m \leq n$); smoothed log-transformed monthly average precipitation (log (mm/day
z	monthly average precipitation (mm/day)
α	smoothing coefficient
Γ	known (nxp) system matrix
$\delta_{k\theta}$	Kronecker's delta ($\delta_{k\theta}=1$ if $k=\theta$ and $\delta_{k\theta}=0$ if $k\neq\theta$)
η	threshold value
θ	unknown time instant when abrupt change occurred
Φ	known (nxn) state transition matrix
ϕ_*	abnormality detection index

Introduction

Droughts are often caused by adverse weather (Nemoto, 1974), and their impact on the economy, society, and political situation can be disastrous. The need to limit the undesirable consequences of drought has increased society's interest in long term prediction of adverse weather. However we believe that the evaluation of the possibility of occurrence of adverse weather that would eventually result to drought can be improved if variations in meteorologic and climatologic parameters are fully understood. In this report, the long term variations of precipitation are studied.

Specifically, this report aims to investigate the occurrence of drought by analyzing the dynamic characteristics of the monthly precipitation time sequence using an ordinary Kalman filter (OKF) and an adaptive Kalman filter (AKF). OKF is used to detect periods in the sequence with abnormal precipitation and to determine quantitatively the period's magnitude of abnormality. We classify the periods into three types and evaluate the degree of possibility of drought occurrence in each type. AKF is used to detect changes in the long term precipitation pattern of the time sequence. The shifts in the precipitation pattern divide the sequence into several precipitation epochs, where an epoch represents one precipitation pattern. Each epochs precipitation is characterized, and the risk of drought occurrence is revealed high or not in the epoch. Finally, drought duration curve analysis is performed to evaluate the robustness of the water resources system during the different epochs.

Modeling of the precipitation sequence

The 92 year long (1890-1981) precipitation sequence at Fukuoka City forms the basis for the analysis in this investigation. The use of monthly average precipitation provides a suitable approach to the investigation of long term variations in precipitation pattern.

The 92 annual totals are plotted on normal paper using the Weibull plotting method as shown in Figure 1. In this figure, the annual precipitation appears normally distributed, however the abnormally wet and dry years deviate from the normal line. From this result, one may think that the years with abnormally high and low precipitation belong to a population different from those on or close to the normal line.

Logarithmic transformation of the 1104(92 years) monthly average data (in mm/day) was done to normalize the data. The log-transformed precipitation data was smoothed using the recursive low-pass filter shown in eq.1 (Bendat and Piersol, 1976).

$$y(k) = (1-\alpha)\log\{z(k)\} + \alpha y(k-1) \tag{1}$$

We chose $\alpha = 0.6$. The smoothed log-transformed monthly average precipitation $y(k)$ at time instant k is modeled by a periodic function as follows.

$$y(k) = M_y + \sum_{i=1}^{q}(A_i \sin 2\pi f_i k + B_i \cos 2\pi f_i k) + w(k) \tag{2}$$

The purpose of smoothing and the reasons for choosing and for modeling the sequence by a periodic function are mentioned in detail by Kawamura <u>et al</u>. (1985). We set q=5 in eq.2. The dominant frequency components <u>used</u> in this study are $f_1=1/48$, $f_2=5/72$, $f_3=1/12$, $f_4=1/3$ and $f_5=5/12$ (cycles/month), having periods of 4 years, 1.2 years, one year,3-months and 2.4 months respectively, which were obtained by MEM spectral analysis of the smoothed transformed data (Kawamura <u>et al</u>. 1985). Comparison of the power spectra of z(k) and y(k) shows similar dominant peaks, indicating that the periodic properties of z(k) are retained in y(k).

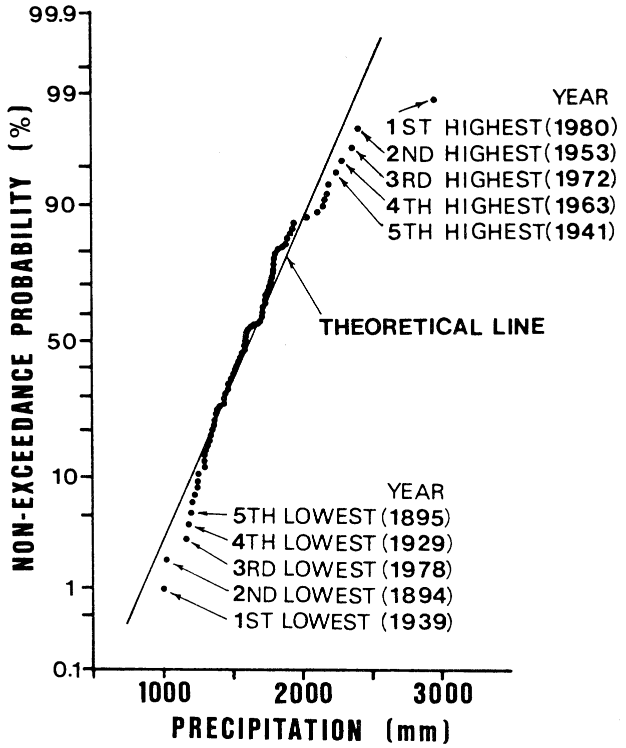

Figure 1 Plot of the annual precipitation data on normal distribution paper.

Kalman filter formulation

Now we consider the problem of identifying M_y and A_i and B_i by OKF and AKF, assuming f is known. The system equations for OKF and AKF are given respectively in eqs. 3 and 4, while the observation equation for both OKF and AKF is given in eq. 5.

$$x(k+1) = \Phi(k)x(k) + \Gamma(k)u(k) \qquad (3)$$

$$x(k+1) = \Phi(k)x(k) + \Gamma(k)u(k) + \delta_{k\theta}\, G(k) \qquad (4)$$

$$y(k) = H(k)x(k) + w(k) \qquad (5)$$

Here n is 11 and m is one; $x=[M_y\ A_1\ \ B_1 \ldots A_5\ \ B_5]$; $\Phi(k)=I$; the observation eq.5 corresponds to eq.2; and $H(k) = (1\ \sin 2\pi f_1 k\ \cos 2\pi f_1 k\ , \ldots\ \sin 2\pi f_5 k\ \ \cos 2\pi f_5 k)$. Here the state variables are the system parameters M , A , and B . Details of parameter estimations by OKF and AKF and the various properties of the two filters for the analysis of time series expressed by a periodic function and the definition of $\phi_*(k,1)$ which expresses quantitatively the system abnormality at time instant k are given by Ueda, et al. (1984), Kawamura, et al. (1984) and Kawamura et al. (1986).

Analysis of abnormal precipitation

Since the plot of the one-step ahead predictions by OKF are almost the same as the average precipitation pattern (Ueda et al. 1984; and Kawamura et al. 1984), we can calculate recursively $\phi_*(k,l)$ using the 1-month series of residuals between the predicted and observed values. Thus we can determine the magnitude of abnormality of an 1-month observed precipitation period from the magnitude of ϕ_*. Here we set $l = 15$. From the information on how a 15-month observed precipitation differs from the average precipitation pattern, we can decide whether the observed precipitation of the period is abnormal or not.

Using the above procedure, the numbers of peak ϕ_* above $\phi_* = 6.0$, 5.0, 4.0 and 3.0 are determined to be 6,10, 19 and 44 respectively. The respective recurrence intervals of the periods with abnormal precipitation identified by these peak ϕ_* are 15,9,5 and 2 years on the average. In this report, we analyze the 19 abnormal precipitation periods arranged in descending order of magnitude of peak ϕ_* in Table 1. Figure 2 illustrates the first five most abnormal precipitation periods (ranked 1-5 in Table 1).

Table 1 The abnormal precipitation periods detected by OKF

	Peak ϕ_*		
Rank	Magnitude	Time of occurrence	Type
1	7.90	Apr. 1894	A
2	7.85	July 1904	C
3	6.14	July 1956	C
4	6.09	May 1980	B
5	6.08	Feb. 1939	A
6	6.02	Oct. 1944	C
7	5.68	Oct. 1971	C
8	5.16	Aug. 1933	C
9	5.15	Oct. 1962	B
10	5.08	June 1902	C
11	4.93	July 1899	C
12	4.90	June 1948	B
13	4.72	May 1953	B
14	4.52	Jan. 1951	C
15	4.45	July 1977	A
16	4.21	Apr. 1913	C
17	4.19	Aug. 1936	C
18	4.19	Apr. 1941	B
19	4.15	July 1976	C

We illustrate the characteristics of the abnormal precipitation in the 19 periods detected by OKF. Each period whose length is 15 months, starts from the time of occurrence of peak ϕ_*. In Figure 3(a), the abnormal precipitation period ranked I is characterized by abnormally low precipitation in May, July, August and October of 1894 and April, May, August and September of 1895 and below average

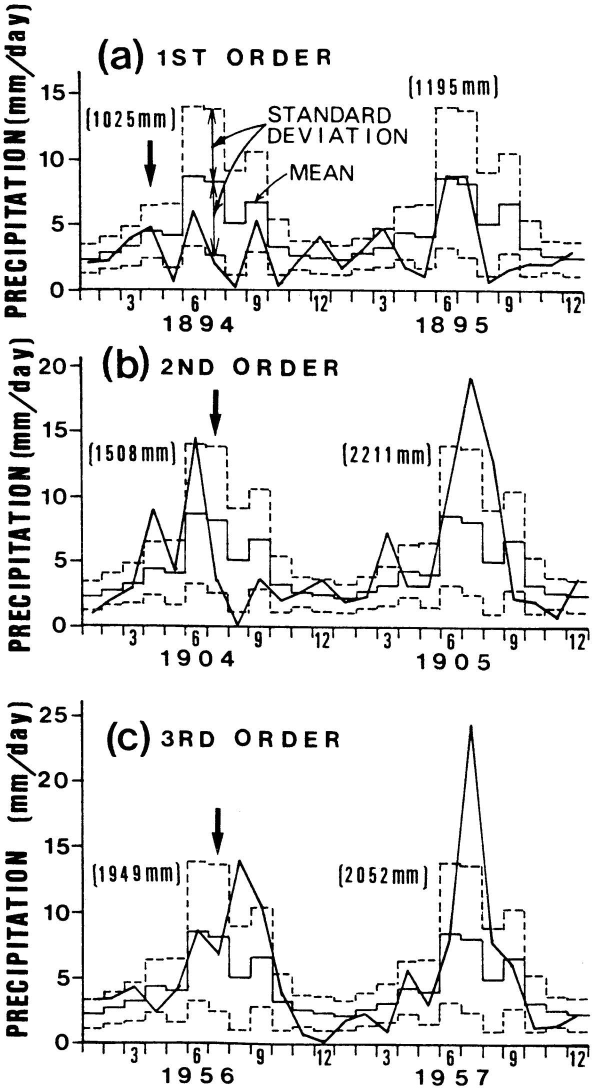

Figure 2 Abnormal precipitation periods (up to rank 5) detected by OKF. The figures in parentheses are annual precipitation amounts; downward-arrow indicates the time of occurrence of peak ϕ_* as detected by OKF.

precipitation in the rest of the months. As a result, the second and fifth lowest annual precipitation totals were recorded in 1894 and 1895 respectively.

The detection of abnormal precipitation periods by OKF, through ϕ_*, results in the identification of three types of abnormal precipitation periods: Type A, Type B and Type C. Type A is characterised by months with below average precipitation depths. Type B is typified by months with above average precipitation amounts. Type C is characterised by both extremely high and low precipitation amounts, which occur alternately at an interval of two or more months. These three types of abnormal precipitation periods could span for one or two years. The 19 abnormal precipitation periods are classified according to these types, as shown in Table 1. Of the 19 abnormal precipitation

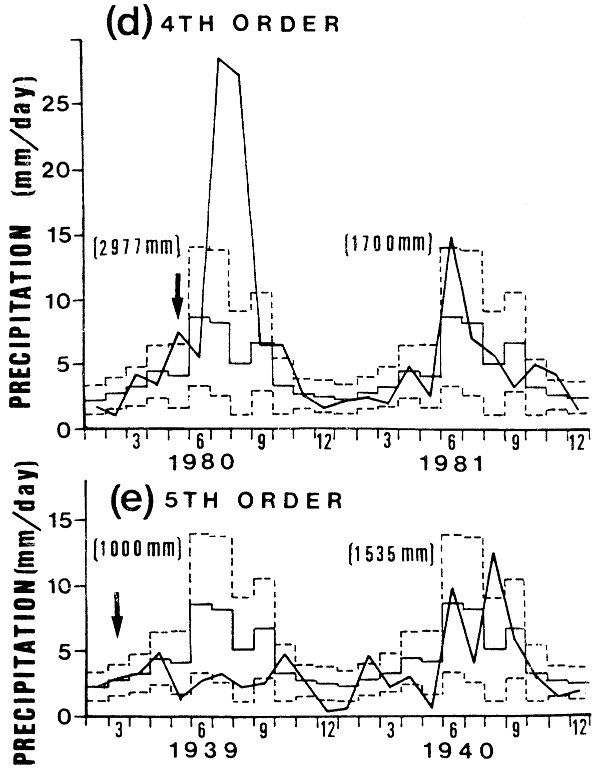

Figure 2 Continued.

periods, three, five and eleven are Type A, Type B, and Type C respectively (as shown in Figure 3), suggesting that Type C is most likely to occur whenever there is an increase of ϕ_*. This increase of ϕ_* should warn us of the possible occurrence of both abnormally high and low precipitation.

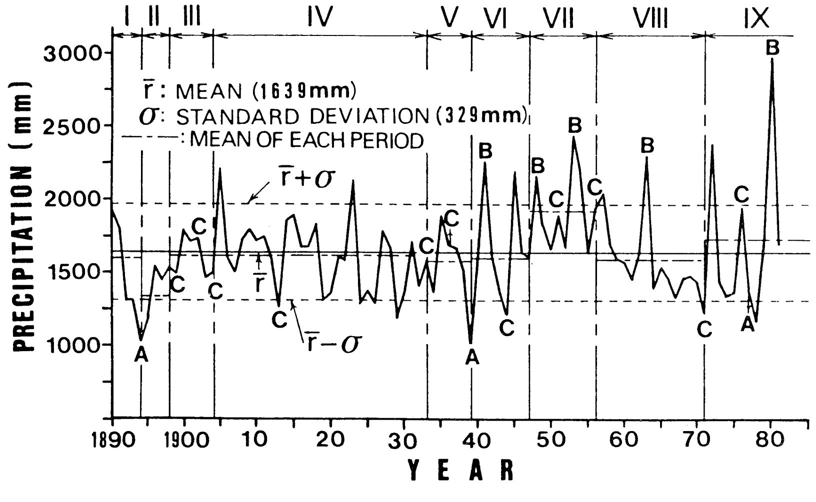

Figure 3 The annual precipitation sequence divided by AKF into nine different epochs. A,B and C are the types of the abnormal precipitation detected by OKF.

As shown in the above discussion, the risk of drought occurrence in each period can be evaluated using OKF.

Analysis of long-term precipitation patterns

AKF detects whether or not an abnormality is present in x by evaluating the variation of the innovations using a generalized likelihood ratio test (GLRT). If such abnormality is detected as an abrupt change in the system parameters, the time instant θ and its magnitude $G(\theta)$ are estimated quantitatively, and the state variables and the error covariance matrix are compensated according to the magnitude of abnormality at the time instant when the abnormality is detected (Ueda *et al.* 1984 and Kawamura *et al.* 1986). In this report, θ is the time when the change in precipitation pattern occurs. An equivalent GLRT is performed to detect whether or not a change in precipitation pattern has taken place, as follows:

$$\phi_*(\theta,1) \underset{H_0}{\overset{H_1}{\gtrless}} \eta \qquad (6)$$

The detection of the time occurrences of the changes of precipitation patterns leads to the division of the precipitation sequence into several epochs, where each epoch represent one precipitation pattern.

According to the statistical divisions made by the World Meteorological Organization (WMO), the interpretation of climatic change differs by time scale. A phenomenon regarded as abnormal in one time scale may be a normal event in a longer time scale. Since the 92 year precipitation sequence is analyzed using a monthly step, the changes in the precipitation pattern correspond to time scales from several years to several decades, which conform to the modern time scale defined by WMO (Nemoto, 1974).

In this study we aim to detect the change of precipitation pattern which recurs at an interval of about 10 years on the average; thus we set $\eta=5.0$. Ten peak ϕ_* corresponding to ten abnormal precipitation periods (ranked 1-10 in Table 1) are above this level of η. AKF yields finally eight changes in precipitation pattern in the 92 year sequence. This divides the sequence into nine precipitation epochs. Table 2 lists the parameters identified at the last time instant k by AKF for each epoch, and by OKF for the whole sequence. Figure 3 exhibits the nine precipitation epochs identified by AKF and the time of occurrences of the 19 abnormal precipitation periods identified by OKF.

The drought duration curve (DDC) of the monthly precipitation is drawn for the 92 year record (Figure 4) and epochs II, IV, VII and IX (Figure 5). Each curve is prepared by first taking the s-month moving average of the precipitation sequence, abstracting the annual minima for that s-month interval and estimating the return period for each annual minimum. The DDC is the plot of the annual minima of the different s-month intervals (s=1 to 24), having the same return period (Takeuchi 1986).

Table 2 Identified system parameters by AKF and OKF

		Mean	Frequencies(cycle/month) and their amplitudes									
			$f_1=1/48$		$f_2=5/72$		$f_3=1/12$		$f_4=1/3$		$f_5=5/12$	
	Epoch	M_y	A_1	B_1	A_2	B_2	A_3	B_3	A_4	B_4	A_5	B_5
AKF	I	0.53	.06	.02	.06	-.00	-.10	-.09	.01	.00	-.01	.00
	II	0.43	-.15	-.01	.00	-.12	.01	-.06	.01	.04	-.04	.02
	III	0.55	.03	-.02	-.02	.02	-.12	-.13	.00	-.01	-.01	.01
	IV	0.53	.03	-.01	.03	.02	-.10	-.10	.03	.02	-.03	-.00
	V	0.50	-.03	-.01	-.01	-.02	-.18	-.04	-.00	.01	-.02	.01
	VI	0.51	.03	.13	-.05	.02	-.19	-.09	.01	.03	-.03	-.00
	VII	0.61	-.01	.03	-.03	.04	-.11	-.12	.02	.02	-.04	.00
	VIII	0.53	.02	.01	.01	-.00	-.14	-.10	.01	.00	-.02	-.01
	IX	0.54	-.12	-.04	-.02	-.04	-.14	-.11	.01	.01	-.01	-.01
OKF	Total	o.54	-.00	.00	.01	.01	-.12	-.10	.02	.01	-.02	-.00

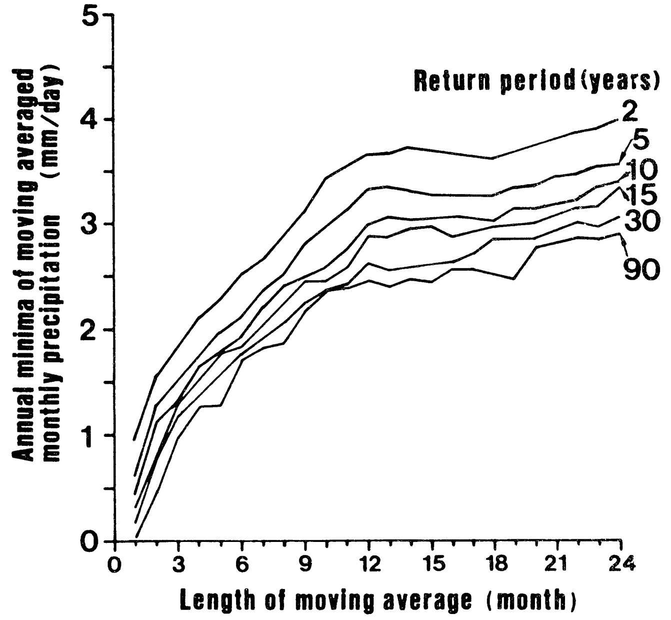

Figure 4 DDC of the monthly precipitation for the 92-year record.

Discussion

Characterization of the long term precipitation patterns

We have confirmed that the parameters identified by AKF for each epoch were almost the same as those by the least-square method for the same epoch, and the parameters identified by OKF and by the least

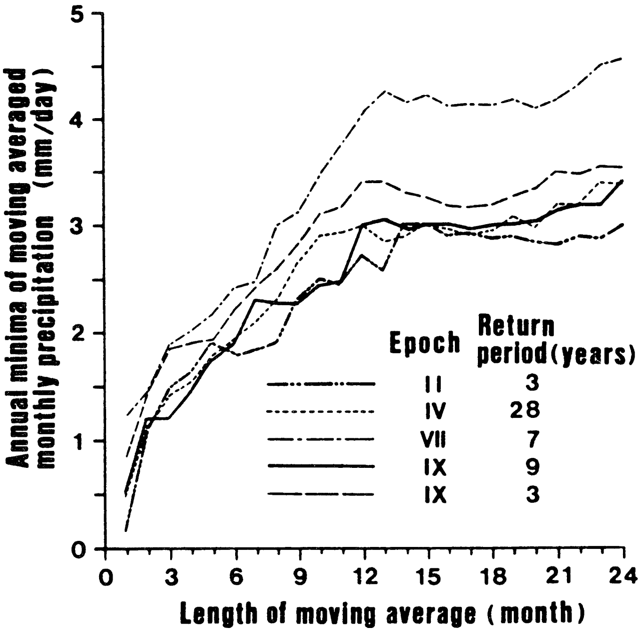

Figure 5 DDC of the monthly precipitation for epochs II, IV, VII and IX.

square method for the whole sequence are equal. The average precipitation pattern defined by the parameters estimated by OKF (shown in Table 2) has a very dominant one year cycle, as indicated by amplitudes A_3 and B_3; these amplitudes A_3 and B_3 are far higher than the rest of the amplitudes which are almost equal to zero. The change from one epoch to another is mainly caused by the mean M_y and frequency components lower than or equal to one year (f_1, f_2, and f_3).

As shown in Figure 1, the second and the fifth lowest years occurred in epoch II. This epoch is typified by less precipitation, as indicated by the mean of the epoch being very much below the mean of the whole sequence (Figure 3). As shown in Table 2, the estimate of M_y of epoch II is the lowest in the nine epochs. Amplitudes of the one year cycle are relatively dominant, and the amplitudes A_1 and B_2 are very dominant, suggesting that the epoch's precipitation pattern differs much from the average precipitation pattern. Hence the risk of drought occurrence is certainly the highest among the nine epochs.

The precipitation patterns in epochs III, IV, V and VIII are similar to the average precipitation pattern, so that the risk of drought occurrence is not high.

The highest value of M_y in Table 2 occurred in epoch VII which is characterized by much precipitation, as indicated by the very high mean of the epoch, well above the mean of the sequence (Figure 3). However the amplitudes of the five frequency components in this epoch are practically the same as those identified by OKF. Thus the epoch's precipitation pattern is very similar to the average precipitation pattern, except for the abrupt change in the mean level. Hence we can safely say that drought is least expected in this kind of precipitation pattern.

In epochs VI and IX (present epoch), the amplitudes A_1 and B_1 for the 4-year cycle are very dominant (shown in Table 2). The three types of abnormal precipitation periods (A, B, and C) tend to happen at the same epoch characterized by a very dominant 4-year cycle

(Figure 3). The first lowest (1939) and the fifth highest (1941) annual precipitation totals appeared in epoch VI, whereas the third highest (1972), the third lowest (1978), and the first highest (1980) yearly precipitation totals were observed in epoch IX. Although abnormal precipitation type B appeared in these epochs, droughts also happened in these epochs. Hence the risk of drought occurrence is regarded high in these epochs.

Drought duration curve analysis

In this section, the robustness of the water resources system in the different epochs is evaluated using the drought duration curves shown in Figure 4 and 5. These figures show that the ascending slope of each DDC suddenly flattens at s equal to 12 months. In Figure 5, the DDC of epoch II for T=3 years is lower than the DDC of epoch IV for T=28 years. (As explained in the previous section, epoch II has a drought precipitation pattern, epoch IV has a normal precipitation pattern, epoch VII has a flood precipitation pattern and epoch IX a drought-flood precipitation pattern.) This means that the drought with a return period of three years in epoch II is more severe than the drought with a 28-year recurrence interval in epoch IV. In addition, this DDC of epoch II is lower than the DDC of the 92-year record for T=15 years and the former is also less than or equal to those portions between s=6 and 9 months and between s=20 and 24 months of the DDC of the record for T=30 years and T=90 years.

Also, the DDC of epoch VII for T=7 years is higher than the DDC of the 92 year record for T=2 years. This includes that, if a water resource system is planned using data belonging to epoch VII, its robustness is diminished if that epoch is terminated abruptly by another epoch characterized by a lower DDC. Moreover, the DDC of epoch IX for T=9 years is nearly equal to the DDC of epoch IV for T=28 years. It indicates that a drought in an epoch characterized by epoch IX is more severe than the drought in an epoch having a normal precipitation pattern. As presented here, the DDC which expresses the robustness of the water resources system, differs significantly among epochs, and the DDCs of some of the epochs are more critical than those of the 92-year record. Hence in order to have a more accurate evaluation of its robustness, a planned water resources system should be analysed not only by record but also by epoch.

Conclusions

In this report, we have used two filtering techniques (OKF and AKF) to study the patterns of a precipitation time series. OKF has been used successfully to detect periods with abnormal precipitation, which are usually indiscernable by visual inspection of the precipitation sequence. AKF has been applied effectively to divide the precipitation sequence into several epochs, where each epoch is characterised by one precipitation pattern. The characteristics of the precipitation pattern in each epoch can be used as a basis for predicting the future behaviour of the precipitation sequence and evaluating the possibility of the occurrence of a drought.

Finally the DDC analysis has shown that the robustness of water resources system would differ significantly among epochs.

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