*E-proceedings of the 38th IAHR World Congress*
*September 1-6, 2019, Panama City, Panama*
doi:10.3850/38WC092019-0367

# A SIMPLIFIED JACKKNIFE APPROACH FOR THE PARAMETER UNCERTAINTY ANALYSIS OF AN URBAN-SPECIFIC RAINFALL-RUNOFF MODEL

SARITHA PADIYEDATH GOPALAN[(1)], AKIRA KAWAMURA[(2)], HIDEO AMAGUCHI[(3)] & GUBASH AZHIKODAN[(4)]

[(1,2,3,4)] Department of Civil and Environmental Engineering, Tokyo Metropolitan University,
1-1 Minami Osawa, Hachioji, Tokyo 192-0397, Japan
charu666@gmail.com; kawamura@tmu.ac.jp; amaguchi@tmu.ac.jp; gubash@tmu.ac.jp

## ABSTRACT

Predictions made by the rainfall-runoff models are inherently uncertain in nature and it is very vital that these models should undergo vigorous calibration and uncertainty analysis. Recent researches relating to hydrologic model uncertainty mostly refers to the identification of parameter uncertainty. The quantitative evaluation of parameter uncertainty of rainfall-runoff models is very important especially in urban watersheds due to the high flood risk in these areas. Therefore, this study aims to analyze the parameter uncertainty of an urban specific rainfall-runoff model, urban storage function (USF) model, using the simplified jackknife approach and its effect on the model simulations. The use of jackknife procedure, a resampling technique, to assess the parameter uncertainty of rainfall-runoff models appear not to have been tried before. The standard rainfall-runoff-model calibration procedure is applied by treating as missing each block of the model residuals to the objective function with a block length of 50. In this study, we scrupulously evaluated the uncertainty of USF model parameters by estimating the 95% confidence interval (CI) of parameters and identified the parameters from the highest to the least uncertainties. Further, the effect of parameter uncertainty on the model simulation uncertainty was investigated by computing 95% CI of the simulated discharge series. The results revealed that the model was able to bracket only 43% of the observations, on average, within the confidence band which further disclosed that the parameter uncertainty has a great impact on the USF model simulation uncertainty.

**Keywords:** Urban storage function model; Block length; Parameter uncertainty; Model simulation uncertainty.

## 1 INTRODUCTION

Rainfall-runoff modeling is the realization of the complex watershed processes that lead to the transformation of rainfall into the runoff, with varying degrees of abstraction (Shrestha, 2009). Rainfall-runoff models are important tools for this purpose and they play a central role, especially in urban watersheds. It is very important to detect the floods in urban watersheds compared to those in rural watersheds because of the increased risks and costs associated with them (Mason et al., 2012). The prominent difference of urban flood flow from the flood in rural watersheds is that the flood flows are higher and more rapid in urban watersheds due to their constant changes in terms of increase in impervious surfaces, expansion of existing sewer systems, etc. (Hollis, 1975). In addition, the hydrologists have been mainly interested in the estimation of watershed hydrological variables such as flood peak, flood volume, etc. with utmost accuracy and reliability in order to carry out appropriate flood prevention measures as well as evacuation actions.

As the rainfall-runoff models are being increasingly used for the runoff simulation, it is very vital that these models should undergo vigorous calibration and uncertainty analysis. The rainfall-runoff modeling process is always associated with uncertainty and the three major sources of uncertainties are data uncertainty (measurement errors associated with the input data), model structure uncertainty (arising from the aggregation of spatially distributed watershed processes into a relatively simple runoff model), and parameter uncertainty (inadequate values of model parameters) (Bates and Townley, 1988; Sivakumar and Berndtsson, 2010). The input data is often contaminated by measurement errors and this inevitably leads to uncertain parameter estimates. As a result, the rainfall-runoff model simulations are far from being perfect, in other words, there always exists a disparity between the model simulation and the corresponding observed data due to this uncertain parameter values (Shrestha, 2009). Consequently, the parameter uncertainty has received a prominent recognition over the other sources of uncertainties and the recent researches relating to hydrologic model uncertainty mostly refers to the identification of parameter uncertainty (Uhlenbrook et al., 1999) and their impact on the model simulation results (Freer et al., 1996; Kuczera and Parent, 1998). Generally, the parameters are either directly measured even though it is too costly to measure them in the field (Nandakumar and Mein, 1997) or estimated through model calibration after the introduction of computer-intensive statistics (Abbott et al., 1986; Refsgaard et al., 1992). The parameters obtained from the calibration process are also not free from uncertainty for various reasons (Duan et al., 1992) and moreover, the high number of parameters included in some rainfall-runoff models will also increase the uncertainty (Brocca et al., 2011). This parameter uncertainty will further contribute to model simulation uncertainties and hence its quantitative evaluation is critical in reducing the uncertainty of these simulations.

Any analysis with a calibrated model must include parameter uncertainty because calibration without uncertainty is meaningless and misleading (Abbaspour, 2015). Many uncertainty analysis techniques have been developed and applied to different catchments in the past decades (Yang et al., 2008). Most of these techniques rely on either parametric methods or Bayesian methods (Gallagher and Doherty, 2007; Kuczera, 1988; Selle and Hannah, 2010; Yang et al., 2008). However, in the parametric method, the structure of the model is specified a priori and the number and nature of the parameters are generally fixed in advance, and there is a little flexibility (Sivakumar, 2017). The Bayesian techniques also necessitate the assumption of the prior distribution of model parameters (Selle and Hannah, 2010). Hence, the nonparametric method has gained popularity over the above methods because they make no prior assumptions on the model structure and is more flexible.

The jackknife approach, one of the nonparametric technique, was introduced by Quenouille (1949) for resampling the original data to develop replicate samples and to find the distribution of a statistics such as bias, standard deviation, etc. Subsequently, Tukey (1958) used the jackknife method to provide an estimate of the variance of the statistic. The delete-1 jackknife, in its simplest form, evaluates the statistics of interest by leaving out each observation at a time from the sample set. It has been used to solve many problems in various fields such as hydrology and soil science (Donnelly-Makowecki and Moore, 1999; Gaume et al., 2007; Lilly et al., 2008), environment and finance (Gomes et al., 2008), etc. in which it has been successfully used in hydrological modeling for the frequency analysis of extreme events, estimation of the sampling variability of reconstructed runoff, identification of dynamic models, etc. (Duchesne and MacGregor, 2001; Sun et al., 2013; Takara, 2009) by utilizing the non-time series data. However, the use of jackknife for time series data was limited because the classical jackknife method assumes that the data set is independent and identically distributed (*iid*) (Efron, 1982). Therefore, the direct resampling is not feasible for a time series data which exhibits strong temporal correlation, and the dependence cannot be preserved (Li et al., 2010).

The outcome from the rainfall-runoff models is hydrograph time series data, and hence the time series application of jackknife method is necessary. In order to overcome the problem of dependence of time series, the delete-d jackknife approach has been developed in which a block of observation is deleted instead of single observation at a time in order to preserve the serial dependence in the time series (Shao and Wu, 1989) and later this approach has been used for many applications. However, the use of jackknife technique for the parameter uncertainty analysis by employing the time series data appears to be very narrow until recently. Only one study has been conducted to quantify the uncertainty arising from the parameter calibration of rainfall-runoff models in ungauged catchments using this technique as far as the authors know. In the reported study, the parameter uncertainty was estimated using the jackknife procedure in which a re-calibration was carried out multiple times for each gauged catchment, leaving out a year of record on each pass (Jones and Kay, 2007). However, the main objective of the study was to provide a mechanism for the estimation of uncertainty bounds of generalized flood frequency curves for ungauged catchments using two conceptual rainfall-runoff models and not the parameter uncertainty. Therefore, it is essential to employ this technique for the assessment of parameter uncertainty and its subsequent effect on the simulation uncertainty of rainfall-runoff models as a primary objective.

The selection of block length is very critical for time series in delete-d jackknife method and each block should be effectively independent for the application. Therefore, a simplified jackknife method is demonstrated in this study in which the existing delete-d jackknife method has been modified in order to overcome the problems associated with block length selection. Also, to the best of our knowledge, no studies have been reported in the watersheds for the parameter uncertainty analysis of urban-specific rainfall-runoff models using the jackknife approach. The rainfall-runoff modeling and associated uncertainty studies are highly dependable on the nature of the study area as well as the model being used (Mockler, et al., 2016). Therefore, there is a need to conduct such studies in different types of watersheds worldwide with different rainfall-runoff models. Hence, this study explores the use of jackknife approach to analyze the parameter uncertainty and its effect on the discharge simulation of urban storage function (USF) model (Takasaki et al., 2009), a relatively new storage function (SF) model specially developed for the urban watersheds where combined sewer system is in use, using a case study in the upper Kanda river basin, a typical small to medium sized urban watershed in Tokyo, Japan. The authors have already evaluated the performance of different SF models and found that the USF model has higher performance compared with conventional SF models for a typical small to medium sized urban watershed in Tokyo, Japan (Padiyedath et al., 2018) and having a future role in urban watersheds. Therefore, it is very essential to address the parameter uncertainty issue associated with the model in order to reduce its effects on the model simulation uncertainty.

## 2 METHODOLOGY

### 2.1 Simplified jackknife approach

Jackknife is a resampling technique which is less dependent on model assumptions and does not need the theoretical formula required by the traditional approaches (Shao and Tu, 1995). The method requires the recalculation of parameters using the resampling method to provide a specified number of evaluations of the parameters (Cover and Unny, 1986). However, the jackknife requires repeatedly computing the statistic by deleting each observation at a time, which may be computationally intensive for large data samples (Mousavi et al., 2011). An idea for computational saving is to use the delete-d jackknife by dividing the n data points into d blocks of length g (n=dg) and compute the statistics of interest by deleting each block (Shao and Tu, 1995) rather than deleting each observation at a time. When the length of block d is large, the blocks will be approximately independent and the serial correlation between the blocks is overcome in the case of time-series data (Jones and Kay, 2007). However, the block length selection is still a challenging task and therefore a new methodology has been applied in this study to overcome this problem as follows.

Consider the original data set $\{X(t), Q(t)\}$; where $X(t)$ is the input data at time $t$, $Q(t)$ is the observed discharge data at time $t$, and $t$ is the time from $1, \ldots, N$ where $N$ is the data length. The observed discharge can be written as a function, $Q(t) = F(X, \theta) + \varepsilon(t)$ where $X = X(1), \ldots., X(t)$, $\theta$ is the parameter vector $\theta_1, \ldots. \theta_p$ with $p$ being the number of model parameters, and $\varepsilon(t)$ is the model residuals. Initially, the model was calibrated to obtain the calibrated parameter vector $\hat{\theta}$ which was further used along with the input data to compute the model calibrated discharge data, $\hat{Q}(t)$. Now, the observed discharge can be demonstrated as $Q(t) = F\left(X, \hat{\theta}\right) + \hat{\varepsilon}(t)$. Thereafter, the model residuals were computed using the following equation.

$$\hat{\varepsilon}(t) = Q(t) - \hat{Q}(t) = Q(t) - F(X, \hat{\theta}) \qquad [1]$$

The model residuals, $\hat{\varepsilon}(t)$, were assumed to be *iid* for $t = 1, \ldots, N$, which is the only assumption made for the proposed jackknife method. Now, the delete-d jackknife method is applied to the model residuals and the detailed jackknife procedure is outlined as follows:

1. Divide the model residual series into d blocks of length g. Then leave out the first block of residual data set and add the remaining jackknifed residual series to the calibrated discharge series, $Q_1^*(t) = \hat{Q}(t) + \varepsilon_1^*$.
2. Calibrate the jackknifed discharge series $Q_1^*(t)$ with input data set, $X$ and obtain the jackknifed parameter vector $\hat{\theta}_1^*$ and the associated simulated discharge series, $\hat{Q}_1^*(t) = F\left(X, \hat{\theta}_1^*\right)$.
3. Repeat steps (1) and (2) by deleting each block and obtain the jackknifed parameter vector, $\hat{\theta}_d^*$ and associated simulated discharge series, $\hat{Q}_d^*(t)$. In the present study, the residual data series is divided into a block length of 50 for the jackknife analysis (g =50).
4. Derive the ordered jackknife estimates, $\hat{\theta}^* = \hat{\theta}^*(1), \ldots, \hat{\theta}^*(p)$ obtained after the jackknife resampling method. Then, the 95% confidence interval (CI) for $\hat{\theta}^*$ was estimated from the ordered jackknife samples.

In the present context, jackknife resampling has been applied to the model residuals rather than applying to the observed discharge series data. Here, we are leaving out the model residuals only, and keeping other inflow and outflow components and thereby considering their effect over the subsequent data blocks during the calibration process.

### 2.2 USF model

The USF model is a conceptual storage function model in which the catchment processes are represented on a lumped basis. The model is based on the relationship between the rainfall over the basin and the runoff at the outlet point. It is given by the following equation (Takasaki et al., 2009):

$$s = k_1(Q + q_R)^{p_1} + k_2 \frac{d}{dt}(Q + q_R)^{p_2} \qquad [2]$$

where $s$ is watershed storage (mm); $Q$ is river discharge (mm/min); $q_R$ is storm drainage from the basin through the combined sewer system (mm/min); $t$ is time (min); and $k_1, k_2, p_1, p_2$ are model parameters. Combining the above expression of storage with the following continuity equation yields the nonlinear expression of the USF model:

$$\frac{ds}{dt} = R + I - E - O - (Q + q_R) - q_l \qquad [3]$$

where $R$ is rainfall (mm/min); $I$ is urban-specific and groundwater inflows from other basins (mm/min); $E$ is evapotranspiration (mm/min); $O$ is water intake from the basin for intended purposes (mm/min), and $q_l$ is groundwater-related loss (mm/min). The groundwater-related loss was defined by considering the infiltration hole height, $z$ and is given by the following equation (Takasaki et al., 2009):

$$q_l = \begin{cases} k_3(s-z) & (s \geq z) \\ 0 & (s < z) \end{cases} \qquad [4]$$

where $k_3$ and $z$ are the parameters. The expression for storm drainage $q_R$ from the combined sewer system discharged out of the basin is developed by assuming a linear relationship between total discharge $Q + q_R$ and the storm drainage $q_R$ immediately after the rainfall. The $q_R$ is defined (Takasaki et al., 2009) as follows:

$$q_R = \begin{cases} \alpha(Q + q_R - Q_0) & \alpha(Q + q_R - Q_0) < q_{R\,max} \\ q_{R\,max} & \alpha(Q + q_R - Q_0) \geq q_{R\,max} \end{cases} \qquad [5]$$

where $\alpha$ is the slope of the linear relationship between total discharge $Q + q_R$ and the drainage $q_R$; and $Q_0$ is the initial river discharge just before the rain starts (Takasaki et al., 2009). The maximum volume of $q_R$ cannot exceed the sewer maximum carrying capacity $q_{R\,max}$. Substituting Eqs. [2] and [4] into [3] will lead to a second-order Ordinary Differential Equation (ODE) and can be numerically solved after transforming into a first-order ODE. The river discharge $Q$ is obtained as the solution after subtracting the $q_R$, which is calculated using Eq. [5], from total discharge. The USF is a seven-parameter model with parameters $k_1, k_2, k_3, p_1, p_2, z, \alpha$ and for a detailed description of the USF model, see Takasaki et al. (2009) and Padiyedath et al. (2018).

### 2.3 Parameter estimation

The Shuffled Complex Evolution-University of Arizona (SCE-UA) method proposed by Duan et al. (1992) was used to estimate the optimum parameter values of the USF model. It is a well-known, global optimization strategy developed for effective and efficient optimization for calibrating the watershed models. The algorithmic parameters of SCE-UA were selected as per the recommendations of Duan et al. (1993). As a first step of the method, it generates an initial population by random sampling from the feasible parameter space, which was defined by setting the lower and upper search range of each parameter, for $p$ number of parameters to be optimized. The population is partitioned into several complexes, each of which is permitted to evolve independently. The number of complexes, $C$, was set equal to 20 and the number of populations in each complex, $r = 2p + 1$. The objective function to be minimized using the SCE-UA method was selected as the root mean square error (RMSE) between the observed and computed discharge.

### 2.4 Parameter and model simulation uncertainty quantification

Generally, the parameter uncertainty is expressed by estimating the CI of the parameters. However, the CI gives the uncertainty range of each parameter from which it is difficult to identify the parameter with the highest and least uncertainty due to the different ranges of parameter values. Therefore, in addition to the CI of parameters, we have computed different statistical estimators of the mean ($\bar{\theta}$), standard deviation ($\sigma_\theta$), and coefficient of variation (CV) of the jackknifed parameter sets. The CV can establish the parameters with the highest and lowest uncertainties. Further, the model simulation uncertainty which occurs due to the parameter uncertainty is illustrated as the 95% CI band which was derived at the 2.5% and 97.5% levels of the cumulative distribution of the simulated model outputs. The 95% CI is not a single solution, but a range of good solutions generated by different combinations of parameter ranges. It should envelope most of the observations, at the same time it is desirable to have a small envelope (Swain and Patra, 2017). P-factor is a statistical term used for the assessment of simulation uncertainty from the 95% CI associated with the parameter uncertainty. It is the percentage of observed data lies within the 95% CI (Yang et al., 2008) and theoretically, its value ranges between 0 and 100%. The goodness of calibration and simulation uncertainty is judged on the basis of the closeness of the P-factor to 100% (i.e., all observations bracketed within the 95% CI) which further indicate that the simulated results are identical to the observed values.

## 3 STUDY AREA AND DATA USED

The selected urban watershed for the particular study was the upper Kanda River basin, having an area of 7.7 km$^2$ at Koyo Bridge, and is shown in Figure 1. The urbanization rate of the basin was more than 95%. The drainage pattern follows the combined sewer system and the sewer installed population rate is 100%. The computed time of concentration of surface runoff from the upstream reaches to the watershed outlet was about 30 min which further indicated that the river discharge will occur immediately after the rainfall within a short period. Therefore, the rainfall and water level data at one-minute intervals from the Bureau of Construction, Tokyo Metropolitan Government (TMG), during 2003–2006 were used for the present study. The average rainfall of the basin was determined using the Thiessen polygon method from the eight rain gauges scattered over the basin as shown in Figure 1. Five target events were selected from the data, whose 60-minute maximum rainfall (R60) is greater than 30 mm and is capable of producing flash floods. Table 1 shows the characteristics of the five selected rainfall events. The inflow component $I$ in the continuity equation [3] was fixed at 0.0012 mm/min based on the annual report of the Bureau of Construction, TMG. The water intake $O$ from the basin and evapotranspiration $E$ were set at 0 as there is no intake from the target basin and the evapotranspiration during heavy rainfall is insignificant. The maximum storm drainage, $q_{R\,max}$ was estimated as 0.033 mm/min using Manning's equation.

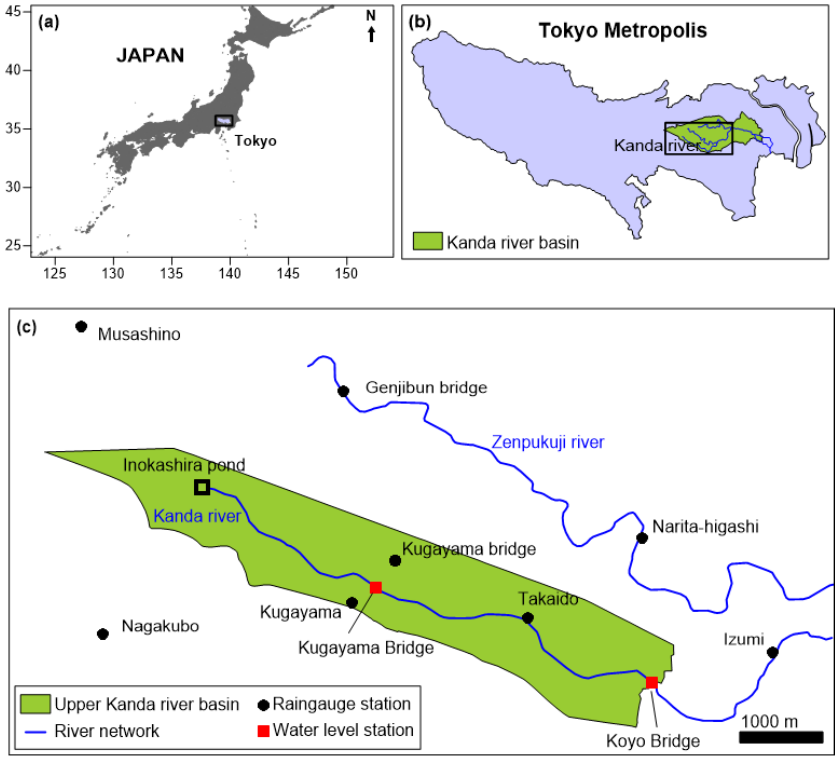


**Figure 1**. Index map of (a) japan, (b) Kanda river basin in Tokyo and (c) target watershed - upper Kanda river basin at Koyo Bridge.

**Table 1**. Characteristics of the five selected events.

| Event No. | Event date | $R_{60}$ (mm) | Total R | Meteorological factors |
|---|---|---|---|---|
| 1 | 13-10-2003 | 53.9 | 57.5 | Intensive localized storm |
| 2 | 25-06-2003 | 42.6 | 46.2 | Frontal rainfall |
| 3 | 8~10/10/2004 | 42.0 | 261.1 | Typhoon |
| 4 | 11-09-2006 | 32.7 | 37.9 | Frontal rainfall |
| 5 | 15-07-2006 | 31.5 | 31.5 | Frontal rainfall |

## 4 RESULTS AND DISCUSSION

### 4.1 Parameter estimation and model performance

The SCE-UA optimization method was applied for calibrating the USF model in the target watershed with RMSE as the objective function. The convergence of parameters was checked and the parameters were found to converge before the 50th generation in each SCE-UA application run. Thereafter, the best parameter set, $\hat{\theta}$ among the population at the 50th generation with a minimum RMSE value was used for the estimation of calibrated discharge series. Table 2 shows the calibrated parameters of the USF model along with their descriptions and search range (Takasaki et al., 2009) used in the SCE-UA parameter estimation method. It is clear from Table 2 that the parameter $z$ lies on the lower limit of search range whereas the rest of the parameters remained within the search range rather than lying on the search range boundaries. The calibrated value of zero for parameter $z$ indicates that there will be always a groundwater-related loss because $z$ represents the infiltration hole height in USF model (Takasaki et al., 2009).

Thereafter, the model performance on estimating discharge was analyzed using the statistical indicators of RMSE, Nash-Sutcliffe efficiency (NSE) (Nash and Sutcliffe, 1970), percentage error in peak (PEP), and percentage error in volume (PEV) (Padiyedath et al., 2017) and is tabulated in Table 3. It is apparent from Table 3 that the model received lower values of RMSE and higher values of NSE in all the events which further reveals better model performance in simulating individual flood events using the optimal parameters identified during the SCE-UA optimization process. It is evident from the table that the PEP values estimated by the model were

 

**Table 2**. The calibrated parameters of the USF model along with their descriptions and search range.

| Parameter | Definition | Search range | Calibrated Parameter, $\hat{\theta}$ |
|---|---|---|---|
| $k_1$ | Physical watershed characteristics (Sugiyama et al., 1997) | [10, 500] | 43.47 |
| $k_2$ | Loop relationship between the storage and discharge ( Prasad, 1967) | [100, 5000] | 619.90 |
| $k_3$ | Groundwater related loss | [0.001, 0.05] | 0.0052 |
| $p_1$ | Index of flow regime (Sugiyama et al., 1997) | [0.1, 1] | 0.41 |
| $p_2$ | Non-linear unsteady flow effects ( Hoshi and Yamaoka, 1982) | [0.1, 1] | 0.33 |
| $z$ | Infiltration hole height | [0, 50] | 0 |
| $\alpha$ | Effect of storm drainage diverted to the treatment plant | [0.1, 1] | 0.42 |

very low (close to zero) except for events 4 and 5. Likewise the PEP, USF model shows the best performance in PEV values as shown in Table 3 which is less than 10% except for certain events. The results revealed that the USF model was able to reproduce the peak discharge as well as the shape of the observed hydrograph with slight variations. Moreover, to have an idea about the amount of lower predictions and over predictions by the USF model, linear regression of the observed and calibrated discharge data are drawn as shown in Figure 2. For easy understanding, the x=y line is also plotted as a reference in blue color. From Figure 2, it can be observed that the regression line is almost overlapping with the x=y line even though there is a very slightly lower prediction of discharge values greater than 0.3 mm/min. In a similar fashion, the low flows exhibit almost close fit except for very few data points and they exhibit slight over prediction. Overall, the $R^2$ and NSE values were found to be 0.98 and 98.3% respectively which indicate a close fit between the observed and calibrated discharge series which makes the model parsimonious.

**Table 3**. Performance evaluation of USF model using different statistical indicators.

| Events | RMSE (mm/min) | NSE (%) | PEP (%) | PEV (%) |
|---|---|---|---|---|
| Event 1 | 0.013 | 99.16 | 0.13 | 11.68 |
| Event 2 | 0.010 | 98.76 | -6.18 | 1.67 |
| Event 3 | 0.010 | 98.41 | -3.95 | 4.39 |
| Event 4 | 0.012 | 92.91 | 17.63 | -9.90 |
| Event 5 | 0.014 | 91.14 | 16.72 | -18.02 |

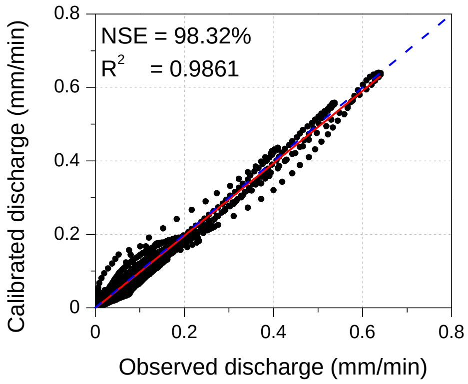


**Figure 2**. Linear regressions of observed and calibrated discharge for USF model.

4.2 Parameter uncertainty analysis

The computed model residual series (Eq. 1) was used to perform the jackknife approach by employing a block length of 50 and generated the associated jackknifed discharge series as described in section 2.1. These discharge series were calibrated to obtain the jackknifed parameter vector, $\hat{\theta}_d^*$ of USF model. Figure 3 shows the scatter plots of these parameter vectors with their uncertainty range in grey shading. The uncertainty range of these parameters was estimated by computing the 95% CI. It is apparent from Figure 3 that the jackknifed parameters are converged to a very narrow range with a slightly wide 95% CI. This wide uncertainty range can be attributed to the presence of several outlier values obtained after jackknifing. Most of the parameter values, such as $k_1, k_2, k_3$, and $z$, were lying close to the lower limit of the search range used during the calibration, whereas the parameters $p_1, p_2$, and $\alpha$ exhibited a widespread pattern within the search range. It was also noted that the parameter values were converged to a very narrow range after the uncertainty analysis compared with the wide search range (Table 2) used during the calibration process, indicating that the proposed jackknife approach is feasible for the parameter uncertainty analysis. The approach not only converges the parameter values to a narrow range but also provides a future reference of the search range for parameter calibration of the target model.

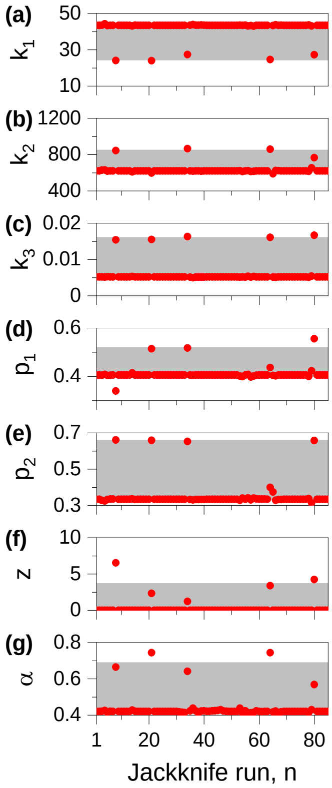


**Figure 3**. Scatter plots of parameter vector with block size 50 along with their 95% CI in grey shading.

The 95% CI gives the uncertainty range of each parameter from which it is difficult to identify the parameter with the highest and least uncertainty due to the different ranges of parameter values. Therefore, after obtaining the uncertainty range of each parameter, we computed different statistical estimators of the mean ($\bar{\theta}$), standard deviation ($\sigma_\theta$), and coefficient of variation (CV) along with the calibrated parameter vector ($\hat{\theta}$) and are shown in Table 4. It is clear from Table 4 that the jackknife estimate of mean for all the parameters are close to the calibrated parameter values in the selected block size. The standard deviation values obtained from the jackknife analysis were almost similar and relatively small. The highest CV value was observed for parameter $z$ which was around 455%. It was followed by the parameters $k_3$ and $p_2$ whose CV values were about 44% and 20% respectively. The remaining parameters exhibited CV values of less than 20% and the least CV value was noted for parameter $p_1$. Further, the parameters were ordered based on their CV values and the order of parameters is as follows: $z > k_3 > p_2 > \alpha > k_1 > k_2 > p_1$. The results revealed that the highest uncertainty was demonstrated by parameter $z$, which was 10 times higher in magnitude compared with the next uncertain parameter $k_3$, whereas the parameter $p_1$ was the one with the least uncertainty based on their CV values.

**Table 4**. The different statistical estimators along with the calibrated parameter vector for block size 50.

| Parameter | Calibrated Parameter, $\hat{\theta}$ | Mean | Standard deviation | Coefficient of variation |
|---|---|---|---|---|
| $k_1$ | 43.47 | 42.40 | 4.28 | 10.10 |
| $k_2$ | 619.90 | 629.69 | 46.80 | 7.43 |
| $k_3$ | 0.0052 | 0.0058 | 0.0026 | 43.99 |
| $p_1$ | 0.41 | 0.35 | 0.03 | 6.05 |
| $p_2$ | 0.33 | 0.21 | 0.07 | 19.76 |
| $z$ | 0 | 0.44 | 0.95 | 454.51 |
| $\alpha$ | 0.42 | 0.35 | 0.06 | 14.22 |

The parameter $z$ highly depends upon the height of river storage, rainfall intensity, etc. and will be highly varying from event to event with the highest uncertainty. Also, most of the time, the optimum value of $z$ estimated from the jackknife analysis by SCE-UA method is the lower limit of search range as shown in Figure 3. A slight deviation from zero to a higher value can cause high uncertainty of this parameter. The parameter with higher uncertainty after $z$ is $k_3$ which is associated with $z$ to depict the groundwater related loss as shown in Eq. [4] and there is a chance of high correlation between these two parameters which will lead to high uncertainty in $k_3$ values after $z$. The parameter $p_1$ received the least uncertainty which can be attributed to its characteristics that control the flow regime and is reasonably stable under the changing input data scenarios. Moreover, the simulated discharge is least affected by the parameters $p_1, k_2, k_1, \alpha$ and are highly sensitive with a small change in parameters $z, k_3, p_2$.

4.3 Model simulation uncertainty

Furthermore, the model simulation uncertainty range due to the parameter uncertainty was computed for each event by estimating the 95% CI of the simulated discharge series samples and is shown in Figure 4. It is desirable to have a narrow range for the model and can be seen from Figure 4(a) that the uncertainty range is wide at the peak flows compared to the range at the low flows of event 1 and it comprises most of the flows with a P-factor of 53%. However, the uncertainty range of event 2 was wide at the peak flows as well as at the low flows as demonstrated in Figure 4(b). It can be envisaged from the figure that the 95% CI was able to capture the observed values during the flood peak, whereas the same was unable to bracket the recession flows with a P-factor of 31%. Likewise event 2, the uncertainty band illustrated in Figure 4(c) was also not able to bracket the recession flows within the prediction band which further reduced the P-factor to 38%. However, the overall prediction band was narrow during event 3. During events 4 and 5, the flood peak values were falling inside the uncertainty band even though the low flows were not well captured as shown in Figure 4(d) and (e). The uncertainty range was wide during both the events with P-factor values of 51% and 40% in events 4 and 5 respectively. It is clear from the figure that the uncertainty range is narrow at the low flows and hence it can be envisaged that the model simulates peak discharge with high uncertainty compared with low flows. This can be attributed to the uncertainties involved in the rainfall during high flows. On average, the 95% CI was able to bracket 43% of observed discharge values with block size 50. This results implicated that the simulated discharge with block size 50 can capture around half of the observations within the uncertainty range with reasonable accuracy.

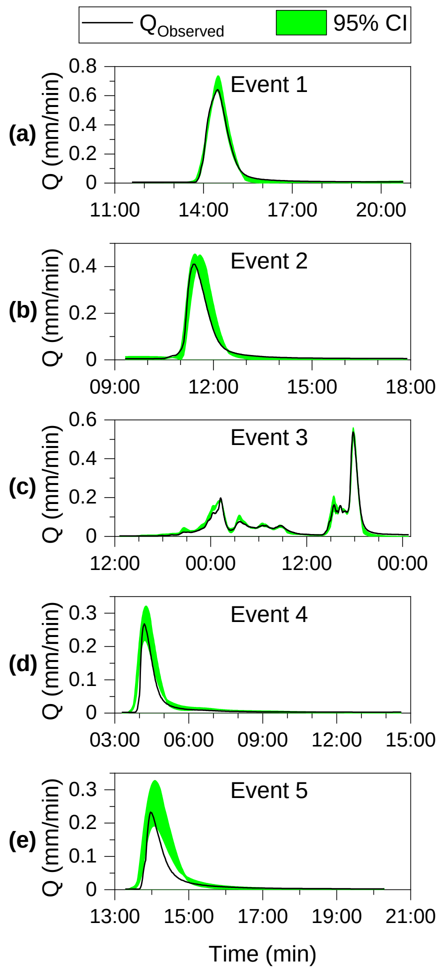


**Figure 4**. The simulation uncertainty of USF model with block size 50 for the selected events.

## 5 CONCLUSIONS

The proposed jackknife approach with a block length of 50 was implemented to analyze the parameter uncertainty and its subsequent effect on discharge simulation of USF model in the upper Kanda River basin, an urban watershed in Tokyo. The 95% CI of all the parameters was relatively wide and parameter z was identified with the highest uncertainty by comparing the jackknife estimate of CV. Further, the parameters from the highest to the lowest uncertainties were derived based on their order of CV values and hence the proposed jackknife approach will be useful in the future studies in order to derive the parameters with highest uncertainty. Additionally, the effect of parameter uncertainty on the model simulation uncertainty was investigated by computing the 95% CI of simulated discharge series. The results revealed that the model was able to bracket only 43% of the observations with block size 50, on average, within the confidence band. This further disclosed that the parameter uncertainty has a great impact on the simulation uncertainty of the USF model. Also, the uncertainty range was wide at the peak flows and the model simulated peak discharge with high uncertainty compared with low flows. As a conclusion, the parameter uncertainty and its effect on model simulation uncertainty were successfully evaluated using the proposed jackknife approach for the USF model.

The primary objective of this study was to propose a jackknife approach for the parameter uncertainty analysis and its subsequent effect on model simulation uncertainty in order to cope with the block length selection. We applied only one block length in the jackknife approach, and further research is required to determine whether the same or better results could be obtained when other block lengths are used.

## ACKNOWLEDGEMENTS

This study was carried out as a part of the research project entitled “Study on guerrilla rainstorm, flood, and water pollution in megacity urban watersheds - Countermeasures against megacity urban water-related disasters bipolarized by climate change” supported by Tokyo Metropolitan Government, Japan (Represented by Prof. Akira Kawamura).

## REFERENCES

Abbaspour, K.C. (2015). SWAT-CUP: SWAT Calibration and Uncertainty Programs–A User Manual. Swiss Federal Institute of Aquatic Science and Technology, Duebendorf, Switzerland.

Abbott, M.B., Bathurst, J.C., Cunge, J.A., O’Connell, P.E. and Ramussen, J. (1986). An introduction to the European Hydrological System – Systeme Hydrologique Europpen, ‘‘SHE”. 2: Structure of a physically based, distributed modeling system. *J. Hydrol*. 87, 61–77.

Bates, B.C. and Townley, L.R. (1988). Nonlinear, discrete flood event models, 3. Analysis of prediction uncertainty. *J. Hydrol*. 99, 91–101. doi:10.1016/0022-1694(88)90080-7.

Brocca, L., Melone, F. and Moramarco, T. (2011). Distributed rainfall-runoff modelling for flood frequency estimation and flood forecasting. *Hydrol. Process*. 25, 2801–2813. doi:10.1002/hyp.8042.

Cover, K.A. and Unny, T.E. (1986). Application of computer intensive statistics to parameter uncertainty in streamflow synthesis. *JAWRA J. Am. Water Resour. Assoc*. 22, 495–507. doi:10.1111/j.1752-1688.1986.tb01905.x.

Donnelly-Makowecki, L.M., Moore, R.D., 1999. Hierarchical testing of three rainfall-runoff models in small forested catchments. J. Hydrol. 219, 136–152. doi:10.1016/S0022-1694(99)00056-6.

Duan, Q.Y., Gupta, V.K. and Sorooshian, S. (1993). Shuffled complex evolution approach for effective and efficient global minimization. *J. Optim. Theory Appl*. 76, 501–521. doi:10.1007/BF00939380.

Duan, Q.Y., Sorooshian, S. and Gupta, V.K. (1992). Effective and efficient global optimization for conceptual rainfall-runoff models. *Water Resour. Res*. 28, 1015–1031. doi:10.1029/91WR02985.

Duchesne, C. and MacGregor, J.F. (2001). Jackknife and bootstrap methods in the identification of dynamic models. *J. Process Control*. 11, 553–564. doi:10.1016/S0959-1524(00)00025-1.

Efron, B. (1982). The Jackknife, the Bootstrap and Other Resampling Plans. *CBMS-NSF regional conference series in applied mathematics*. Philadelphia. doi:10.1137/1.9781611970319.

Freer, J., Beven, K. and Ambroise, B. (1996). Bayesian estimation of uncertainty in runoff prediction and the value of data: An application of the GLUE approach. *Water Resour. Res*. doi:10.1029/95WR03723.

Gallagher, M. and Doherty, J. (2007). Parameter estimation and uncertainty analysis for a watershed model. *Environ. Model. Softw*. 22, 1000–1020. doi:10.1016/J.ENVSOFT.2006.06.007.

Gaume, E., Mouhous, N. and Andrieu, H. (2007). Rainfall stochastic disaggregation models: Calibration and validation of a multiplicative cascade model. *Adv. Water Resour*. 30, 1301–1319. doi:10.1016/j.advwatres.2006.11.007.

Gomes, M.I., Hall, A. and Miranda, M.C. (2008). Subsampling techniques and the Jackknife methodology in the estimation of the extremal index. *Comput. Stat. Data Anal*. 52, 2022–2041. doi:10.1016/J.CSDA.2007.06.023.

Hollis, G.E. (1975). The effect of urbanization on floods of different recurrence interval. *Water Resour. Res*. 11, 431–435. doi:10.1029/WR011i003p00431.

Hoshi, K. and Yamaoka, H. (1982). A relationship between kinematic wave and storage routing models. *Proc. 26th Japanese Conf. on Hydraul. JSCE*. 273-278. doi:10.2208/prohe1975.26.273.

Jones, D.A. and Kay, A.L. (2007). Uncertainty analysis for estimating flood frequencies for ungauged catchments using rainfall-runoff models. *Adv. Water Resour*. 30, 1190–1204. doi:10.1016/j.advwatres.2006.10.009.
Kuczera, G. (1988). On the validity of first-order prediction limits for conceptual hydrologic models. *J. Hydrol*. 103, 229–247.
Kuczera, G. and Parent, E. (1998). Monte Carlo assessment of parameter uncertainty in conceptual catchment models: The Metropolis algorithm. *J. Hydrol*. 211, 69–85. doi:10.1016/S0022-1694(98)00198-X.
Li, Z., Shao, Q., Xu, Z. and Cai, X. (2010). Analysis of parameter uncertainty in semi-distributed hydrological models using bootstrap method: A case study of SWAT model applied to Yingluoxia watershed in northwest China. *J. Hydrol*. 385, 76–83. doi:10.1016/j.jhydrol.2010.01.025.
Lilly, A., Nemes, A., Rawls, W.J. and Pachepsky, Y.A. (2008). Probabilistic Approach to the Identification of Input Variables to Estimate Hydraulic Conductivity. *Soil Sci. Soc. Am. J*. 72, 16. doi:10.2136/sssaj2006.0391.
Mason, D.C., Davenport, I.J., Neal, J.C., Schumann, G.J.-P. and Bates, P.D. (2012). Near Real-Time Flood Detection in Urban and Rural Areas Using High-Resolution Synthetic Aperture Radar Images. *IEEE Trans. Geosci. Remote Sens*. 50, 3041–3052. doi:10.1109/TGRS.2011.2178030.
Mockler, E.M., Chun, K.P., Sapriza-Azuri, G., Bruen, M. and Wheater, H.S. (2016). Assessing the relative importance of parameter and forcing uncertainty and their interactions in conceptual hydrological model simulations. *Adv. Water Resour*. 97, 299–313. doi:10.1016/j.advwatres.2016.10.008.
Mousavi, S.M., Tavakkoli-Moghaddam, R., Azaron, A., Mojtahedi, S.M.H. and Hashemi, H. (2011). Risk assessment for highway projects using jackknife technique. *Expert Syst. Appl*. 38, 5514–5524. doi:10.1016/j.eswa.2010.10.085.
Nandakumar, N. and Mein, R.G. (1997). Uncertainty in rainfall-runoff model simulations and the implications for predicting the hydrologic effects of land-use change. *J. Hydrol*. 192, 211–232.
Nash, J.E. and Sutcliffe, J.V. (1970). River flow forecasting through conceptual models part I – A discussion of principles. *J. Hydrol*. 10, 282–290. http://dx.doi.org/10.1016/0022-1694 (70)90255-6.
Padiyedath, S.G., Kawamura, A., Sahoo, B. and Amaguchi, H. (2017). Effect of evapotranspiration on the discharge estimation in Baitarani watershed, India, in the context of climate change. In: *World Environmental and Water Resources Congress 2017. ASCE*, Reston, VA, pp. 577–588. http://dx.doi.org/10.1061/9780784480595.052.
Padiyedath, S.G., Kawamura, A., Takasaki, T., Amaguchi, H. and Azhikodan, G. (2018). An effective storage function model for an urban watershed in terms of hydrograph reproducibility and Akaike information criterion. *J. Hydrol*. 563, 657–668. doi:10.1016/j.jhydrol.2018.06.035.
Prasad, R. (1967). A nonlinear hydrologic system response model, *Proc. ASCE*, 93, HY4, 201-221.
Quenouille, M.H. (1949). Approximate tests of correlation in time-series. *Journal of the Royal Statistical Society. Series B*, 11, 68-84.
Refsgaard, J.C., Seth, S.M., Bathurst, J.C., Erlich, M., Storm, B., Jorgensen, G.H. and Chandra, S. (1992). Application of the SHE to catchments in India. Part 1. General results. *J. Hydrol*. 140, 1–23.
Selle, B. and Hannah, M. (2010). A bootstrap approach to assess parameter uncertainty in simple catchment models. *Environ. Model. Softw*. 25, 919–926. doi:10.1016/j.envsoft.2010.03.005.
Shao, J. and Tu, D. (1995). *The Jackknife and Bootstrap*, Springer Series in Statistics. Springer New York, New York, NY. doi:10.1007/978-1-4612-0795-5.
Shao, J. and Wu, C.F.J. (1989). A General Theory for Jackknife Variance Estimation. *Ann. Stat*. 17, 1176–1197. doi:10.1214/aos/1176347263.
Shrestha, D.L. (2009). Uncertainty analysis in rainfall-runoff modelling: application of machine learning techniques. *PhD Thesis*, CRC PRESS.
Sivakumar, B. (2017). *Chaos in hydrology: bridging determinism and stochasticity*, Chapter 3, Dordrecht: Springer Science+Business Media, 63-110. doi:10.1007/978-90-481-2552-4.
Sivakumar, B. and Berndtsson, R. (2010). *Advances in data-based approaches for hydrologic modeling and forecasting*. Chapter 3, World Scientific Publishing Co. Pte. Ltd., Singapore. doi:10.1142/9789814307987.
Sugiyama, H., Kadoya, M., Nagai, A. and Lansey, K. (1997). Evaluation of the storage function model parameter characteristics. *J. Hydrol*. 191, 332–348. doi:10.1016/S0022-1694(96)03026-0.
Sun, J., Liu, Y., Wang, Y., Bao, G. and Sun, B. (2013). Tree-ring based runoff reconstruction of the upper Fenhe River basin, North China, since 1799 AD. *Quat. Int*. 283, 117–124. doi:10.1016/j.quaint.2012.03.044.
Swain, J.B. and Patra, K.C. (2017). Streamflow estimation in ungauged catchments using regionalization techniques. *J. Hydrol*. 554, 420–433. doi:10.1016/j.jhydrol.2017.08.054.
Takara, K. (2009). Frequency analysis of hydrological extreme events and how to consider climate change. *The Nineteenth IHP training course (International Hydrological Program)*, 29. 11. 2009 – 12. 12. 2009. Nagayama and Kyoto, Japan.
Takasaki, T., Kawamura, A., Amaguchi, H. and Araki, K. (2009). New storage function model considering urban runoff process, *J. JSCE, B*, 65(3), 217-230. doi:10.2208/jscejb.65.217.
Tukey, J.W. (1958). Bias and Confidence in Not-Quite Large Sample. *Annals of Mathematical Statistics*, 29, 614.

Uhlenbrook, S., Seibert, J., Leibundgut, C. and Rodhe, A. (1999). Prediction uncertainty of conceptual rainfall-runoff models caused by problems in identifying model parameters and structure. *Hydrol. Sci. J*. 44, 779–797. doi:10.1080/02626669909492273.

Yang, J., Reichert, P., Abbaspour, K.C., Xia, J. and Yang, H. (2008). Comparing uncertainty analysis techniques for a SWAT application to the Chaohe Basin in China. *J. Hydrol*. 358, 1–23.

 