THE COMPARISON OF RUNOFF PREDICTION ACCURACY AMONG THE VARIOUS STORAGE FUNCTION MODELS WITH LOSS MECHANISMS

AKIRA KAWAMURA, YOKO MORINAGA, KENJI JINNO
Institute of Environmental Systems, Kyushu University,
Hakozaki Higashi-ku, Fukuoka 812-8581, Japan

GRAEME C. DANDY
Department of Civil and Environmental Engineering, University of Adelaide,
Adelaide, South Australia 5005, Australia

The storage function model has been widely used for the rainfall-runoff analysis in Japan due to the ease of expressing the nonlinear relationship of rainfall-runoff events with simple equations and its ability to provide relatively easy computation. In this study, to examine and to compare the performance and the characteristics of the runoff prediction accuracy among the various storage function models with loss mechanisms, four types of storage function model, each of them is known as the Hoshi's model, the Prasad's model, the Kimura's model and the linear model, are applied to 6 flood events from the Koishiwara River basin located in Fukuoka, Japan. The shuffled complex evolution (SCE-UA) method as a relatively new global optimization strategy is applied to the optimal parameter identification for all four models.

INTRODUCTION

The flood forecasting is essential to prevent or deal with the damages caused by the flood events. For the proper flood forecasting, the model that simulates the complicated phenomenon of the non-linear relationship of the rainfall-runoff events is required. The storage function model, one of the process-based lumped models, has been widely used for the rainfall-runoff analysis in Japan due to the ease of expressing the nonlinear relationship of rainfall-runoff events with simple equations and their ability to provide relatively easy computation (Morinaga *et al.* [1]). However, the application of this storage function model to actual catchments usually requires an estimate of effective rainfall as an input in advance, which is computed by use of runoff coefficient or runoff-component separations. There are no deterministic methods for runoff component separations to obtain effective rainfall. Because of this, the selected method may change the effective rainfall values and the model parameters involved in the storage function model. To overcome this difficulty, the storage function model coupled with loss mechanisms has proposed by Baba *et al.* [2]. This model enables the use of observed rainfall directly as inputs and does not require effective rainfall to be determined. The storage function model with loss mechanisms has the advantage of the real-time flood forecasting, because the hydrologic data are directly processed.

In this study, to examine and to compare the performance and the characteristics of the runoff prediction accuracy among the various storage function models with loss mechanisms, four types of storage function model, each of them is known as the Hoshi's model, the Prasad's model, the Kimura's model and the linear model, are applied to 6 flood events from the Koishiwara River basin located in Fukuoka, Japan. In order to estimate the model parameters involved in four types of storage function model, the shuffled complex evolution (SCE-UA) that is proposed by Duan *et al.* [3] as a relatively new global optimization strategy method is applied.

STORAGE FUNCTION MODEL WITH LOSS MECHANISMS

The storage function model coupled with loss mechanisms, as proposed by Baba *et al.* [2], is given by the following equations Eqs. (1)-(3).

$$s(t) = k_1 q^{p_1}(t) + k_2 \frac{d}{dt} q^{p_2}(t) \tag{1}$$

$$\frac{ds(t)}{dt} = r(t) - q(t) - p(t) \tag{2}$$

$$p(t) = aq(t) \tag{3}$$

where s: storage (mm), q: observed runoff (mm/h), r: observed rainfall (mm/h), p: loss (mm/h), t: time (hours), k_1, k_2, p_1, p_2, a: model parameters. There are five unknown model parameters (k_1, k_2, p_1, p_2, a) involved in this model. Eq. (1) is empirical representation of the storage function known as Hoshi's model (Hoshi *et al.* [4]). Eq. (2) is the continuity equation and Eq. (3) is represents loss. By substituting Eqs. (2), (3) into Eq. (1), the following equation is obtained.

$$k_2 \frac{d^2 q^{p_2}(t)}{dt^2} = -k_1 p_1 q^{p_1 - 1}(t) \frac{dq(t)}{dt} + r(t) - (1+a)q(t) \tag{4}$$

With conducting transformation of variables by Eq. (5), Eq. (4) is transformed into the first order simultaneous differential equations Eq. (6).

$$x_1(t) = q^{p_2}(t) \qquad x_2(t) = \frac{dq^{p2}(t)}{dt} \tag{5}$$

$$\frac{dx_1(t)}{dt} = x_2(t) \qquad \frac{dx_2(t)}{dt} = -\frac{k_1}{k_2}\frac{p_1}{p_2} x_1^{\frac{p_1}{p_2}-1}(t) x_2(t) - \frac{1+a}{k_2} x_1^{\frac{1}{p_2}}(t) + \frac{1}{k_2} r(t) \tag{6}$$

Once unknown parameters (k_1, k_2, p_1, p_2, a) are given, the first order nonlinear simultaneous ordinary differential equations Eq. (6) are solved using various numerical methods such as Runge-Kutta-Gill method. Here in this study, however, Eq. (6) is furthermore linearized and transformed into difference equation in order to solve it with easy computation. The detailed derivation and solution for this transforming has been described by Kawamura [5].

OTHER STORAGE FUNCTION MODELS

In addition to the original version of the storage function model with loss mechanisms as above, three other versions are obtained as special cases of the storage function model with loss mechanisms. Some simplifications are implemented to the equation (1) as follows. Firstly, if we set p_2=1 in Eq. (1), we obtain the following equation which is known as Prasad's model [6].

$$s(t) = k_1 q^{p_1}(t) + k_2 \frac{d}{dt} q(t) \tag{7}$$

Secondly, if we set k_2=0 in Eq. (1), we obtain the Eq. equation (8) which is known as Kimura's model [7].

$$s(t) = k_1 q^{p_1}(t) \tag{8}$$

For a further simplification, Eq. (9) is obtained by setting p_1=1 in Eq. (8). This model is referred to as linear model hereafter.

$$s(t) = k_1 q(t) \tag{9}$$

Each of Eqs. (7) to (8) is adopted instead of Eq. (1) for the three special cases of the storage function model with loss mechanisms Eqs. (1) to (3).

STUDIED AREA AND DATA USED

Studied river basin is the Koishiwara River basin mainly located in Amagi City, Fukuoka Prefecture Western Japan, with a catchment area of 85.9km^2 and a mean annual rainfall of 2247.6mm. The Koishiwara River is the tributary of the Chikugo River as shown in Figure 1, which is the largest river in Kyushu Island of Japan. The Chikugo River had often been affected by both droughts and floods that inflicted large damage to the surrounding areas until the crop of dams were built at the upper reaches of the Chikugo River. Especially the very severe flood attacked the basin in 1953 which collapsed the levees along the river and, hence, extensive damage occurred including 147 toll of dead.

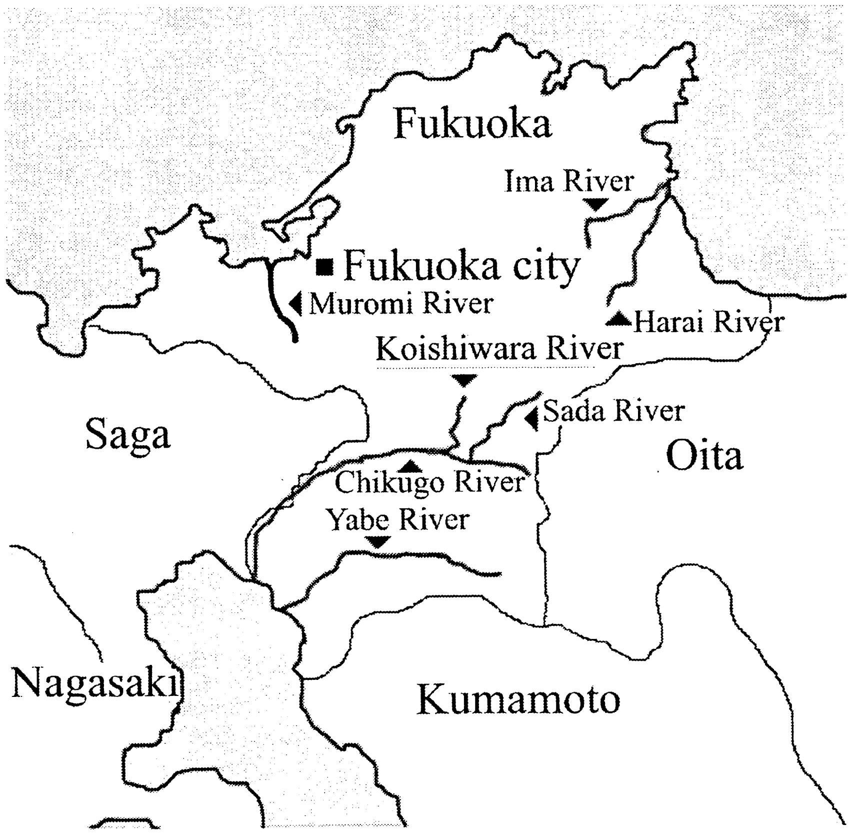

Figure 1. Location of the Koishiwara River basin.

In this study, six flood data sets (from flood event 1 to 6) that contain hourly rainfall and runoff data recorded at the Egawa dam during the period for 1993 to 1997 are used to examine and to compare the performance and the characteristics of the runoff prediction accuracy among the above mentioned four types of storage function model with loss mechanisms.

PARAMETER ESTIMATION

In this study, the model parameters of four types of the storage function model with loss mechanisms are estimated using the shuffled complex evolution (SCE-UA) method proposed by Duan *et al.* [3]. The SCE-UA method is a new global optimization strategy designed to be effective and efficient for a broad class of parameter estimation problems occurred in the calibration of nonlinear simulation models. The SCE-UA strategy combines the strength of the simplex procedure with the concepts of controlled random search, competitive evolution and the newly developed concept of complex shuffling. This method was originally applied to the conceptual rainfall runoff model optimization by Duen *et al.* [3]. Tanakamaru [8] applied SCE-UA method to the parameter estimation for the Tank model, and the authors [9] applied it to the searching of optimal valve openings for the pressure regulation of a water distribution networks. For the algorithmic parameters of the SCE-UA method such as m of the number of points in each complex and q of the number of points in each sub-complex, the values recommended by Duan *et al.* [3] of m=2n+1 and q=n+1 where n is the number of parameters to be optimized are used and the number of complexes of p is set equal to 10. In this study, the objective function to be minimized is selected as the root mean square error (RMSE) between the observed runoff and calculated one using the estimated parameters by SCE-UA method.

RESULTS OF PARAMETER ESTIMATION AND RUNOFF PREDICTION

The SCE-UA method is applied to the parameter estimation of the storage function model coupled with loss mechanisms for the each of the observed flood events (flood event 1 to 6) that contain hourly rainfall and runoff data during the period for 1993 to 1997. The SCE-UA method searches the combination of model parameters for each of the above mentioned four types of storage function model that makes the objective function minimum. Table 1 and 2 show the optimally estimated model parameters of four types of the model for flood event 1 and 2 out of the six flood events, respectively. In the tables and the following figures, Hoshi's model, Prasad's model, Kimura's model and linear model are referred to as 5, 4, 3 and 2-paremeter model, respectively, from the numbers of unknown parameters of those models.

Table 1. The estimated model parameter values for each model for

	5-parameter model	4-paraemter model	3-paraemter model	2-paraemter model
k_1	96.3487	290.6514	134.0332	19.4775
k_2	15.9492	6.6675		
p_1	0.2590	0.1000	0.1948	[1.0000]
p_2	0.3626	[1.0000]		
a	0.4416	0.4643	0.3981	0.4815

Table 2. The estimated model parameter values for each model

	5-parameter model	4-paraemter model	3-paraemter model	2-paraemter model
k_1	39.1903	49.9662	64.0975	17.3941
k_2	20.3647	6.2801		
p_1	0.5538	0.5707	0.3569	[1.0000]
p_2	0.2630	[1.0000]		
a	1.0898	1.9154	1.2908	1.4902

Hourly runoff of $q(t)$ is forecasted by each model with hourly rainfall of $r(t)$ as an input incorporating the estimated parameter values in Table 1 and 2. The runoff prediction results for flood event 1 and 2 are shown in Figure 2 and 3, respectively. The resulting values of the root mean square error (RMSE) computed between the observed and predicted runoff and the peak % error for each event are shown in Table 3.

From Figure 2, there are broadly three peaks for the flood event 1. The 5-parameter and the 3-parameter model provide approximately the same prediction with slight differences and they generally give good prediction for the first and second peaks. On the other hand, the 4-parameter and 2-parameter model overestimate the first peak. Furthermore 2-paremeter model underestimate the second peak. From Table 3 for flood

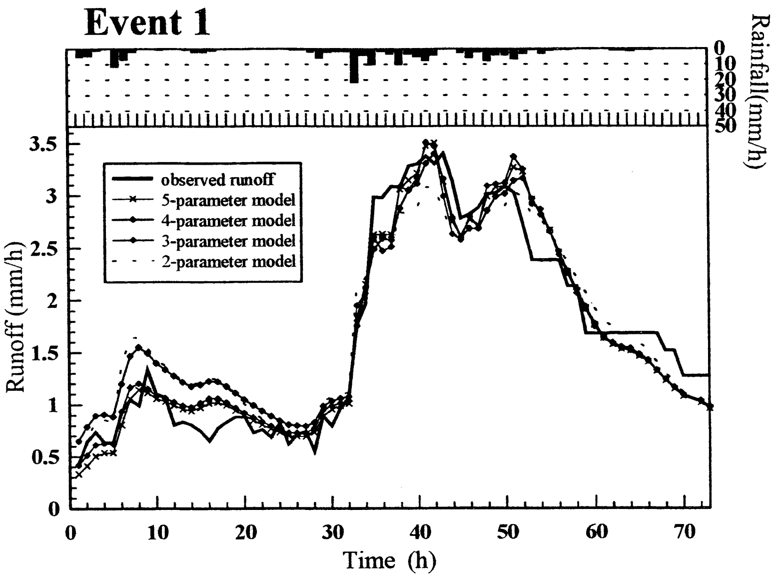

Figure 2. The runoff prediction by each storage function model for Event 1.

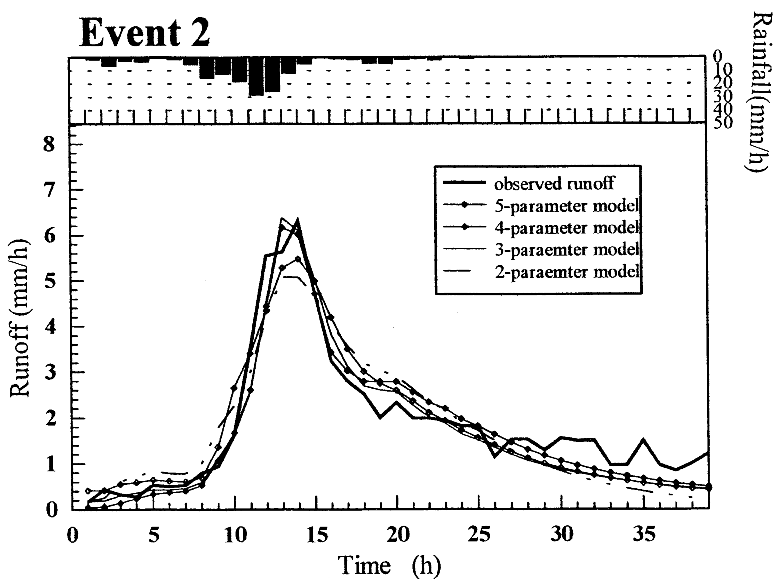

Figure 3. The runoff prediction by each storage function model for Event 2.

event 1, the smallest RMSE is given by the 5-parameter model followed by 3-paremeter model. The smallest peak % error is provided by the 4-parameter model. However, the RMSE provided by the 4-parameter model is slightly larger than that by the three-parameter model, which implies that the SCE-UA method failed to search the global minimum values of objective function.

From Figure 3, the 5-parameter model and the 3-parameter model provide good correspondence for the peak prediction for flood event 2. On the other hand, slight underestimation is found in the predictions by the 4-parameter model and the 2-parameter model. From Table 3 for flood event 2, the smallest RMSE value is provided by the 5-parameter model, and the smallest peak % error is given by the 3-parameter model. Again, the RMSE provided by the 4-parameter model is larger than that by the 3-parameter model.

From Table 3 for all flood events, the 5-parameter model give the smallest RMSE for all of six events and the smallest peak % error for three out of six flood events. The 2-parameter model provided the worst results for the RMSE in all six predictions and the peak % error for five events out of six. Moreover, the 4-parameter model gave the inferior results to that by the 3-parameter model for flood event 1, 2, and 6. This indicates that the SCE-UA method failed to properly optimize the parameters in the 4-parameter model for those flood events.

Table 3. The RMSE and peak % error by each storage function model for six flood events.

	flood event 1		flood event 2		flood event 3	
	RMSE	peak % error	RMSE	peak % error	RMSE	peak % error
5-parameter model	0.2153	7.8506	0.4303	4.5696	0.5255	7.4759
4 parameter-model	0.2657	4.7059	0.5193	13.0302	0.6862	21.3647
3 parameter-model	0.2329	11.9344	0.4561	3.2092	0.8353	14.4269
2 parameter-model	0.3027	16.2150	0.6335	19.4960	1.0679	30.7432
	flood event 4		flood event 5		flood event 6	
	RMSE	peak % error	RMSE	peak % error	RMSE	peak % error
5-parameter model	0.3808	32.0652	0.1964	13.7490	0.2471	19.0668
4 parameter-model	0.4053	33.0192	0.2593	22.6764	0.2931	22.3836
3 parameter-model	0.4075	35.8780	0.2596	22.6009	0.2609	17.0872
2 parameter-model	0.4933	37.9287	0.2610	25.2829	0.3154	32.0528

CONCLUSION

In this study, four types of storage function models with loss mechanisms are applied to six flood events to compare the performance of the runoff prediction accuracy. SCE-UA method is used to estimate the model parameters of those storage function models. The following conclusions are obtained. The 5-parameter model (Hoshi's model) naturally showed the highest accuracy of all four models for the runoff prediction of the Koishiwara River basin. The 3-paremeter model (Kimura's model) can provide quite accurate runoff prediction. The 2-parameter model (linear model) could not provide the reasonable results in terms of both of the RMSE and the peak % error compared with other storage function models. However, even the 2-paremeter model can predict overall tendency of runoff. It was implied that the SCE-UA method failed to properly estimate the model parameters in the 4-parameter model for some flood events.

REFERENCES

[1] Morinaga, Y., Kawamura, A. and Jinno, K., "The global search method of the SCE-UA for the parameter optimization for the storage function model with loss mechanisms", *Proc. Annual Meeting of JSCE Western Branch*, (2001), pp B198-B199.

[2] Baba, H., Hoshi, K. and Hashimoto, N., "Synthetic storage routing model coupled with loss mechanisms", *Annual Journal of Hydraulic Engineering*, JSCE, Vol.43, (1999), pp 1085-1090.

[3] Duan, Q., Sorooshian, S. and Gupta, V., "Effective and efficient global optimization for conceptual rainfall-runoff models", *Water Resources Research*, Vol.28, No.4, (1992), pp 1015- 1031.

[4] Hoshi, K. and Yamaoka, I., "A relationship between kinematic wave and storage routing models", *Proc. the 26th Japanese Conference on Hydraulics*, (1982), pp 273- 278.

[5] Kawamura, A., "Real-time prediction of flood runoff by Kalman filter using storage function model", *Hydraulics Formulae: Hydraulics Worked Examples with CD-ROM,* JSCE, (2002), Example No. 1-11.

[6] Prasad, R., "A nonlinear hydrologic system response model", *Proc. ASCE*, Vol. 93, No. HY-4, (1967), pp 202-221.

[7] Kimura, T., "*The storage function model*", Publishing Company Kawanabe, Tokyo, (1975).

[8] Tanakamaru, H. and Burges, S.J., "Application of global optimization to parameter estimation of the tank model", *Proc. Int. Conf. on Water Resour. & Environ. Res.*, Vol.II, (1996), pp 39-46.

[9] Haytham, A., Kawamura, A., Jinno, K. and Kuno Y., "Evolutionary computing techniques for optimal pressure regulation in water distribution networks", *Annual Journal of Hydraulic Engineering*, JSCE, Vol.47, (2003), pp 865-870.