EFFECTIVE AND EFFICIENT TECHNIQUE FOR NODAL DEMANDS PREDICTION IN WATER SUPPLY NETWORK

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Abstract: This paper presents a method for on-line prediction of nodal water demands in a supervisory water supply network. Based on Kalman filtering theory, the developed prediction algorithm had used two equations in the simulation of water supply network, the continuity equation and the head loss equation after being linearized using Taylor expansion series. The observable variables in Kalman filter algorithm are recorded from flow meters and pressure gauges which are connected to the pipes and nodes of the network. To investigate the performance and accuracy of this technique, this prediction method was applied to a certain block of Fukuoka City water supply network. As Kalman filter parameters are variable with time, results had shown the efficiency of this technique in the prediction of all nodal demands in the studied water supply network and thus decrease the uncertainty in determining the values of these quantities. On the other hand, predicted on-line values of water supply network variables as pipe discharges and hydraulic pressures are calculated in parallel with the computation of nodal demands. With the highly accurate results obtained from applying Kalman filter technique in the field of on-line prediction, a promising improvement of the existing water supply models is expected.

Keywords: Water Supply Network, Kalman filter, On-line prediction, Water demands.

1. INTRODUCTION

The purpose of a water supply network is to convey water to consumers in the required quantity at appropriate pressure, of acceptable quality, as economically as possible. According to the importance of the previous subjects, computer models for analyzing and designing water distribution systems have been available since the mid-1960's. Since then, however, many advances have been made with regard to the sophistication and application of this technology[1A]. For simulating the water distribution system by a computer model to obtain the different pipe flows and node pressures, data associated with the different components of the system should be provided. The essential data that should be provided are elevation, demand and demand pattern of junctions, diameter, length and roughness coefficient of pipes and elevation of water levels in the different reservoirs. In case of presence of pumps and valves, additional data should be available like the type of valves and its characteristics and the head-discharge curves of the used pumps[1B]. Among all previous properties, the two variables that possess the highest degree of uncertainty are the roughness coefficient of pipes and the different junction demands. To overcome the difficulty in determining those two previous quantities, a model calibration should be done. Available algorithms for pipe network calibration are based on the use of analytical equations, simulation models and optimization methods to adjust the assumed pipes roughness coefficients and nodal demands[1C]. The step of network model calibration is essential and always should be done before the analysis step.

Regarding the problem of estimating pipe-roughness coefficient values, an accurate estimate of these values could be obtained using average empirical values in the literature[1D] or from field measurements[1E], therefore, following any of these two methods will reduce the uncertainty in determining these quantities.
Determining an initial accurate estimate of the average nodal demands in any water distribution network could be done by a categorized and classification method of land use, type and number of dwellings, meter route and individual meter billing records[1F]. As nodal demands varies with time, it should multiplied by an adjustment hourly factor equal to the consumption of this hour divided by the average daily consumption[1G]. Recently, some demand forecast models had considered such factors as daily weather conditions, weather forecasts, seasons of the year and past trends in water use[1H].
However, review of the literature shows that the available techniques used in the calibration of the hydraulic network are very sensitive to the initial given values to the nodal demand and could not detect the real values of water demands in all the network nodes. In addition, the performances of these models are long-interval calibration, especially when an optimization technique is used like simulated annealing and genetic algorithm.
In this paper, a method for prediction on-line values of water demands in all nodes of a water supply network is presented. This method is based on the Kalman filtering theory[2]. The filter uses continuous sensor information on pipe discharges, hydraulic pressures and valve openings from some measurement points in the network. In addition, predicted on-line values of water supply network variables as pipe discharges and hydraulic pressures are calculated in parallel with the computation of nodal demands[2]. To demonstrate the accuracy of the method, it is applied to Block No. 12 of the water supply network in Fukuoka City.

2. KALMAN FILTER

The Kalman filter[3] is a set of mathematical equations that provides an efficient computational solution of the least squares method. The Kalman filter algorithm is composed of three components: 1) System model; 2) Measurement model; and 3) Kalman filtering. This algorithm is based on observations using a measurement system, and has the ability to make on-line prediction without the need of long historical data by combining two well-known ideas; the state transition method of describing dynamic systems; and the linear filtering regard as orthogonal projection in Hilbert space[3]. All the required calculations of Kalman filter are made off-line, only the final step when updating the system model will require the recorded data from the measurement model. The following section will present the details of Kalman filtering algorithm in conjugation with water network model.

3. FORMULATION OF THE MODEL USING KALMAN FILTER

For a network has n_1 junction nodes, n_2 links, n_3 fixed-grade nodes, n_4 observation points, the two basic equations used in the water supply network are the continuity equation at each node, **Eq. 1** and the head loss equation for each pipe (Hazen-Williams relationship), **Eq. 2**.

$$\sum_j Q_{ij}(k) = -q_i(k) \tag{1}$$

$$H_i(k) - H_j(k) = r_{ij}^{-1/\alpha} \left|Q_{ij}(k)\right|^{(1/\alpha)-1} Q_{ij}(k) + \frac{8 f_{vij}}{g\pi^2 d_{ij}^4} \left|Q_{ij}(k)\right| Q_{ij}(k) \tag{2}$$

where

$$r_{ij} = 0.27853 \times C_{HW_{ij}} d_{ij}^{\beta} l_{ij}^{-\alpha} \tag{3}$$

and

$$f_{vij}(\theta) = \begin{cases} 165226 \times 10^{-0.18\theta} & (0 \le \theta < 13) \\ 3696 \times 10^{-0.06\theta} & (13 \le \theta < 40) \\ 221 \times 10^{-0.03\theta} & (40 \le \theta \le 100) \end{cases} \tag{4}$$

Here k is time step; Q_{ij} (m^3/hour) is pipe discharge from node i to j, and $Q_{ij} = - Q_{ji}$; $| \ |$ means absolute value, q_i (m^3/hour) is nodal demand at node i; H_i (m) and H_j (m) are hydraulic pressures at nodes i and j; $\alpha = 0.54$ and $\beta = 2.63$ are numerical constants; g is acceleration of gravity; r_{ij} defined by **Eq. 3** is constant for pipe ij and $r_{ij} = r_{ji}$; f_{vij} is valve loss coefficient determined by **Eq. 4** where θ *(%)* is the percentage of valve opening; C_{HWij} Hazen-Williams coefficient; d_{ij} (m) is diameter of the pipe, and l_{ij} (m) is pipe length. The second term of **Eq. 2** is required only if there is a valve connected to the pipe and **Eq. 4** is taken for the typical type of electrically motor valves used in Fukuoka City water distribution network[4].

Hazen-Williams equation, **Eq. 2** can be linearized using Taylor expansion series by considering the first-order term as follows:

$$\begin{aligned} \Delta H(t+\Delta t) &= \Delta H(t) + (\partial \Delta H(t)/\partial t)\Delta t \\ &= \Delta H(t) + (\partial \Delta H(t)/\partial Q)(\partial Q/\partial t)\Delta t \\ &= \Delta H(t) + (\partial \Delta H(t)/\partial Q)\Delta Q \end{aligned} \tag{5}$$

where

$$(\partial \Delta H(t)/\partial Q)\Delta Q = (1/\alpha) r_{ij}^{-1/\alpha} |Q_{ij}(t)|^{(1/\alpha)-1} + \frac{16 f_{vij}}{g\pi^2 d_{ij}^4} |Q_{ij}(t)| \equiv f_{ij}(t) \tag{6}$$

The following linearized equation for time step k is obtained by substituting **Eq. 6** into **Eq. 5** and rearranging.

$$H_i(k+1) - H_j(k+1) - f_{ij}(k)Q_{ij}(k+1) = H_i(k) - H_j(k) - f_{ij}(k)Q_{ij}(k) \tag{7}$$

In the previous linearized hydraulic model there is n_1 equations of **Eq. 1** and n_2 equations of **Eq. 7**. For the nodal demand q_i described in **Eq. 1,** we assumed that it could be presented by a stochastic periodical model which take into consideration the change in water demand during the day. The general form of this equation is presented as follows:

$$q_i(k) = M_i + \sum_{m=1}^{s} (a_{im} \sin 2\pi\lambda_{im} k + b_{im} \cos 2\pi\lambda_{im} k) + \upsilon_i(k) \tag{8}$$

In **Eq. 8,** s presents the number of changes in the daily water demands, $\lambda_{i1}, \ldots, \lambda_{is}$ presents the frequency components, M_i (m^3/hour) mean fluctuation of nodal demand, $a_{i1}, \ldots, a_{is}$ and $b_{i1}, \ldots, b_{is}$ (m^3/hour) are the amplitudes of frequency components. υ_i independent value with zero-mean Gaussian white noise $N(0, \sigma_i^2)$.

In the determination of the unknown parameters, M_i, $a_{i1}, \ldots, a_{is}$ and $b_{i1}, \ldots, b_{is}$, we use Kalman filter algorithm to optimally estimate these variables. Kalman filter with the following system equation, **Eq. 9** and observation equation, **Eq. 10** are used in this study.

$$X(k+1|k) = \Phi(k+1|k)X(k|k) + u(k+1|k) \tag{9}$$

$$Y(k+1|k) = \Gamma(k+1|k)X(k+1|k) + F(k+1|k) + w(k+1|k) \tag{10}$$

Consider $n = (1+2s)\times(n_1+n_3)$; here k is time step; X is system state vector $(n\times1)$; $\Phi(k+1|k)$ state transition matrix $(n\times n)$ for time $(k+1)$ at time k; $u(k+1|k)$ system error vector $(n\times1)$; $Y(k+1|1)$ measurement vector $(n_4\times1)$; $\Gamma(k+1|k)$ measurement transition matrix $(n_4\times n)$ determined from **Eq. 11**; $F(k+1|k)$ measurement transition vector $(n_4\times1)$ calculated from **Eq. 12**; $w(k+1|k)$ measurement error vector $(n_4\times1)$. The error vectors $u(k+1|k)$ and $w(k+1|k)$ are assumed to be independent Gaussian processes.

$$\Gamma(k+1|k) = DA_\delta^{-1}(k|k)C_\delta E(k+1|k) \tag{11}$$

$$F(k+1|k) = DA_\delta^{-1}(k|k)B(k|k)Z(k|k) \tag{12}$$

The previous two equations are used in the determination of the measurement vector in **Eq. 10**. Where $A_\delta(k|k)$ *is* $(n_1+n_2)\times(n_1+n_2)$ matrix representing the LHS of **Eq. 1** and **Eq. 7** with elements $0, 1, -1, -f_{ij}$; $B(k|k)$ *is* $(n_1+n_2)\times(n_1+n_2+n_3)$ matrix representing the RHS of **Eq. 7** with elements $0, 1, -1, -f_{ij}$; C_δ is $(n_1+n_2)\times(n_1+n_3)$ matrix representing the RHS of **Eq. 1** with elements $0, 1, -1$; D is $n_4\times(n_1+n_2)$ matrix representing all the observation points with elements

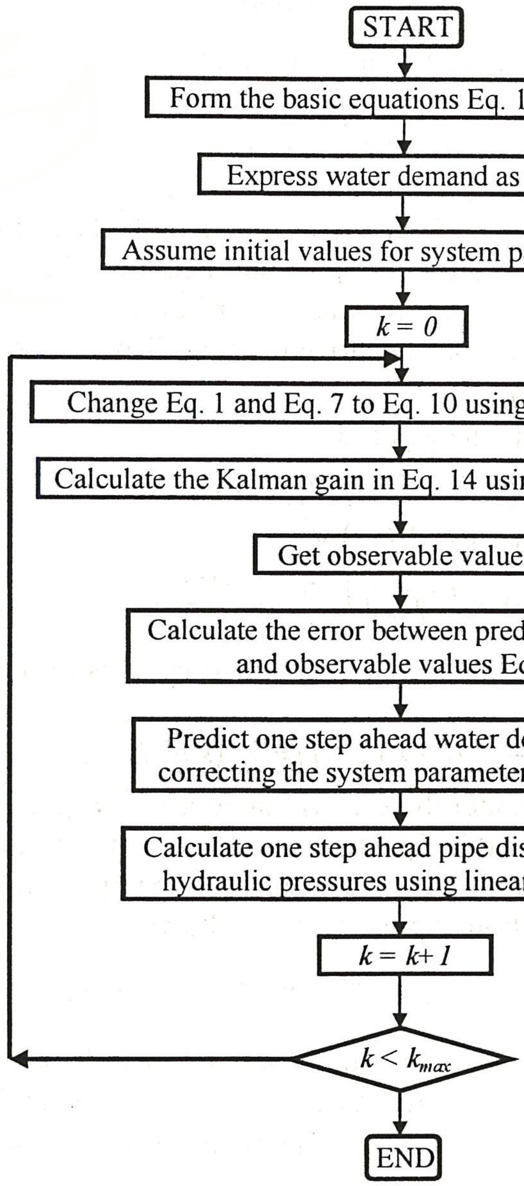

Fig. 1. Flowchart of the model using Kalman filter algorithm

0 and 1; $E(k+1|k)$ is $(n_1+n_3)\times n$ representing system state matrix with elements 0, 1, $\sin 2\pi\lambda_{im}(k)$, $\cos 2\pi\lambda_{im}(k)$ and $Z(k|k)$ is $(n_1+n_2+n_3)\times 1$ vector representing the predicted results of outflows, pipe discharges and nodal heads at time k.

Knowing the measurement transition vector $F(k+1|k)$ from **Eq. 10**, an estimated state vector $X(k+1|k+1)$ is obtained, **Eq. 13** by filtering the measurement error with use of Kalman gain $K(k+1|k)$, **Eq. 14** and measurement error vector $\Delta(k+1|k+1)$ **Eq. 15** as

$$X(k+1|k+1)=X(k+1|k)+K(k+1|k)\Delta(k+1|k+1) \tag{13}$$

$$K(k+1|k)=P(k+1|k)\Gamma^T(k+1|k)V^{-1}(k+1|k) \tag{14}$$

$$\Delta(k+1|k+1)=R(k+1|k+1)-Y(k+1|k) \tag{15}$$

$$V(k+1|k)=\Gamma(k+1|k)P(k+1|k)\Gamma^T(k+1|k)+W(k+1|k) \tag{16}$$

where $R(k+1|k+1)$observed vector $(n_4\times 1)$; $W(k+1|k)$ and $U(k+1|k)$ are symmetric nonnegative definite matrices with size $(n_4\times n_4)$ and $(n\times n)$, respectively; $P(k+1|k)$ is $(n\times n)$ covariance matrix of the state estimator for time $k+1$ at time k, **Eq. 17** and at time $k+1$, **Eq. 18**

$$P(k+1|k)=\Phi(k+1|k)P(k|k)\Phi^T(k+1|k)+U(k+1|k) \tag{17}$$

$$P(k+1|k+1)=[I-K(k+1|k)\Gamma(k+1|k)]P(k+1|k) \tag{18}$$

The algorithm of the proposed method of prediction is shown in **Fig. 1**.

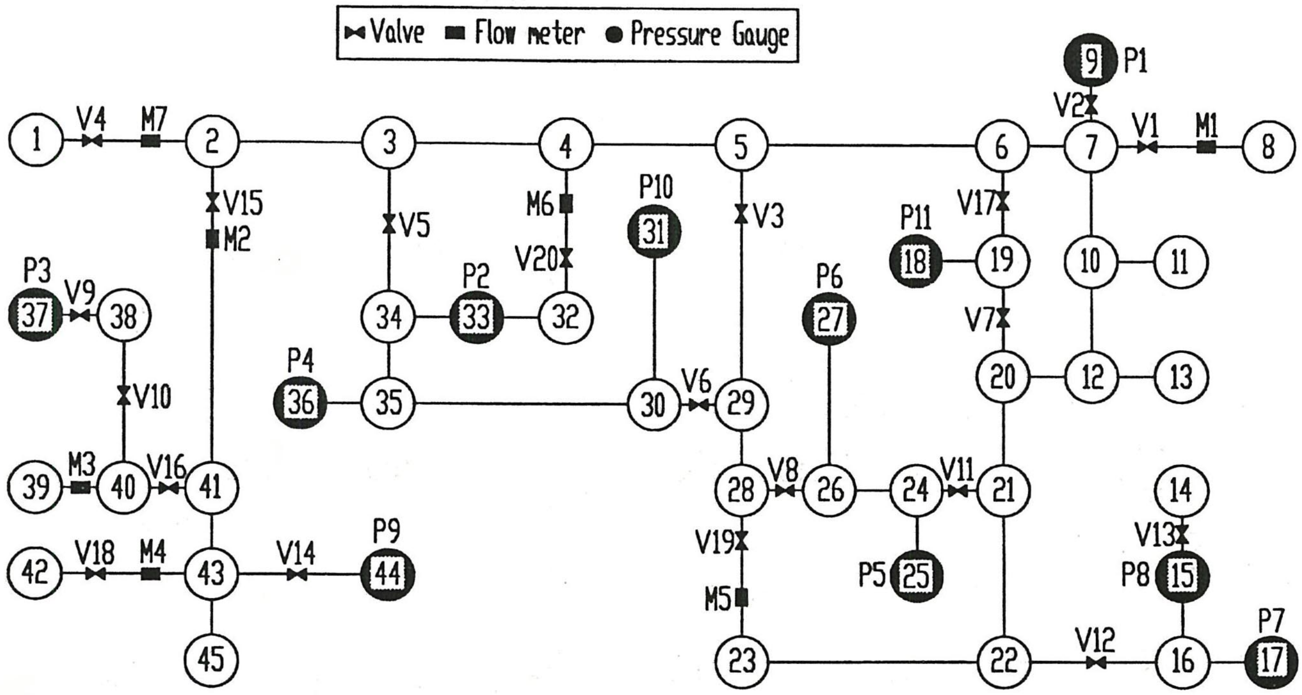

Fig. 2. Block 12 of the Fukuoka City water supply network

4. APPLICATION

In order to illustrate the capability of the proposed prediction method and its accuracy, the previous model was used to simulate Block 12 of the supervisory Fukuoka City water supply network. In this Block (see **Fig. 2**), there are 45 water demands, 45 hydraulic pressures, 49 pipe discharges, and 5 inflows from outside the network at nodes 1, 8, 39, 42 and 45. For the telemeters attached to the network, there are 7 flow meters *(M1, ..., M7)*, 11 pressure gauges *(P1, ..., P11)* and 20 electrically motor valves *(V1, ..., V20)*.

In the simulation of this block, we assume that the change of nodal demand are every 12 hours and 24 hours[4)], thus $s = 2$ in **Eq. 8** which describe the stochastic periodical model applied to synthetically generate the true nodal demand values. By using randomly chosen values which will be called "true values" to the parameters M_i, a_{i1}, a_{i2}, b_{i1}, b_{i2} and σ_i in **Eq. 8**, the initial state vector of parameters $X(0|0)$ are taken 50% of these true values. Then given the true values of parameters and known λ_{i1}, λ_{i2}, the 45 nodal demands are calculated. Here, calculations are done for 288 steps with one step equal to 10 minutes.

The number of available equations are 94, 45 of which are equations of continuity, **Eq. 1** and 49 equations of head loss, **Eq. 7**. Nevertheless, the number of unknown quantities excluding the 45 nodal demands is 99 which is broken down into 45 hydraulic pressures, 49 pipe discharges and 5 inflows. Unless five quantities are given, the network flow can not be solved uniquely. We choose 4 inflows at nodes 8, 39, 42 and 45 and one hydraulic pressure at node 1 as the five quantities.

The 20 electrically motor valve openings are set to the mean values used in the normal daily operation of this Block according to the valve schedule operating system used by the operators of this block in Fukuoka City water supply network. The coefficient of valve loss, f_{vij} is calculated using **Eq. 4** by knowing the percentage of valve opening θ *(%)*.

With these assumptions, the simulation results in the 45 hydraulic pressures, 49 pipe discharges and 5 inflows are used as "observed" data.

The prediction method proposed in this paper will use 18 "observed" data of the previously mentioned data. These data are 11 hydraulic pressures and 7 pipe discharges from the locations in which pressure gauges and flow meters are attached to the nodes and pipes, respectively, of this block (see **Fig. 2**).

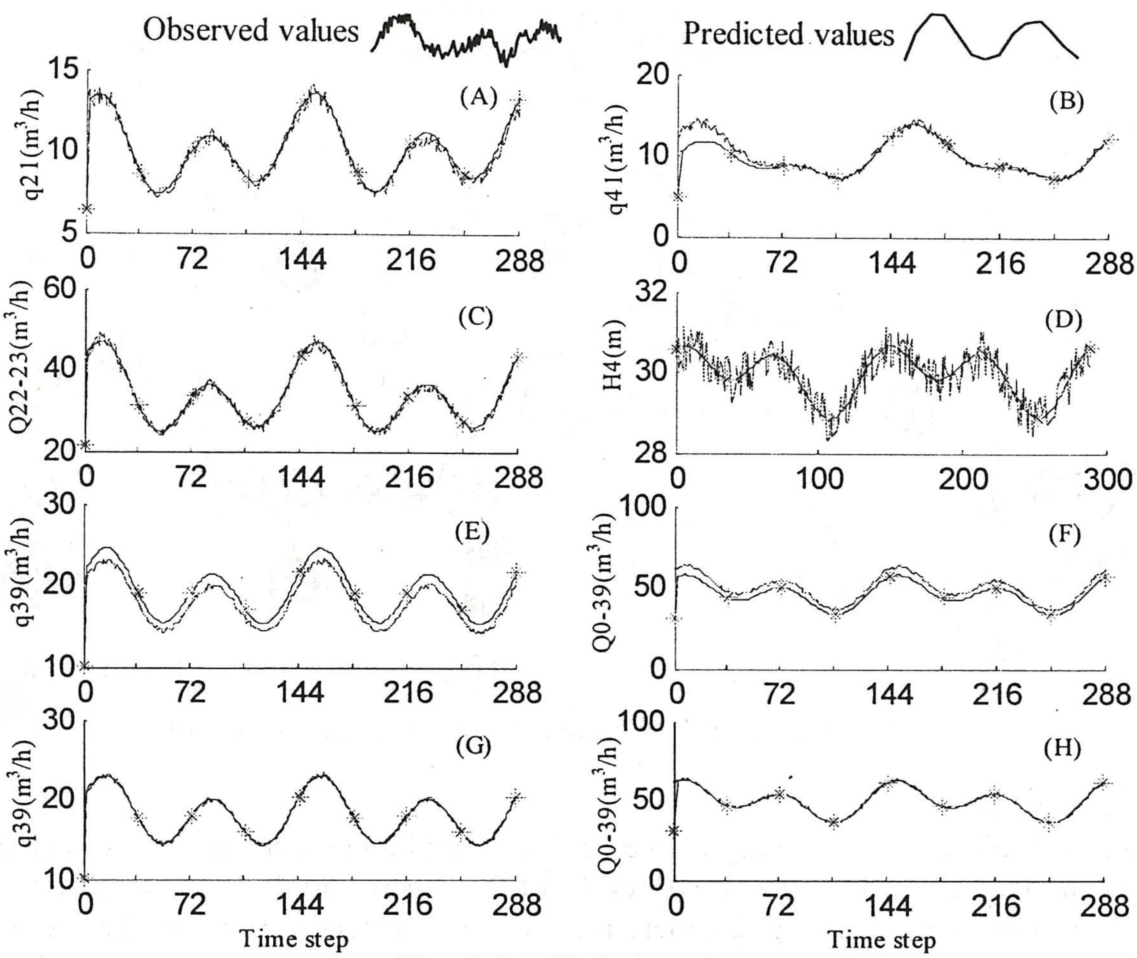

Fig. 3. Prediction results

5. RESULTS AND DISCUSSION

Analysis of results by using the linearized equations, **Eq. 1** and **Eq. 7**, shows that they are below 0.1% of the true values. The true values have been calculated using the method of extended linear graph theory for analyzing and simulating the water supply networks[5)]. It is a numerical method, based on the use of linear graph theory for the steady-state analysis of pipe network having different hydraulic components. Thus the results of the predictions can be discussed without any difficulty caused by ignoring the linearization errors.

The main advantage of Kalman filter technique in the field of predicting different variables in water supply network that the updating of system parameters $X(k+1|k+1)$ in **Eq. 13** doesn't need long historical data and it is based on the computation of Kalman gain $K(k+1|k)$ which could be estimated before reaching the required time step of prediction $(k+1)$.

Fig. 3 shows some of the results obtained from the predictions of the 45 water demands, 45 hydraulic pressures, 49 pipe discharges and 5 inflows. In this figure, the observed values are plotted with the predicted ones for the different 288 time steps used in the simulation. It is clear in this figure that the predicted water demand q_{21} in **Fig. 3A** and q_{41} in **Fig. 3B**, the pipe discharge $Q_{22\text{-}23}$, **Fig 3C** and the node pressure H_4, **Fig. 3D** are predicted with high accuracy. The predicted values of the nodal demands, pipe discharges and the hydraulic pressures follow the fluctuations of the observed values. Regarding the prediction results of nodal demand at node 39, q_{39}, **Fig. 3E** with the prediction of inflow $Q_{0\text{-}39}$, **Fig. 3F** (the suffix 0 means outside the network; $Q_{0\text{-}39}$ means inflow at node 39) are predicted with some bias. This error is due to the failure of sensor information from flow meters and pressure gauges in distinguishing between water demand and inflow at a certain node. Except for these unobservable quantities, other quantities (both observable and unobservable) not shown in **Fig. 3** converged to their true values.

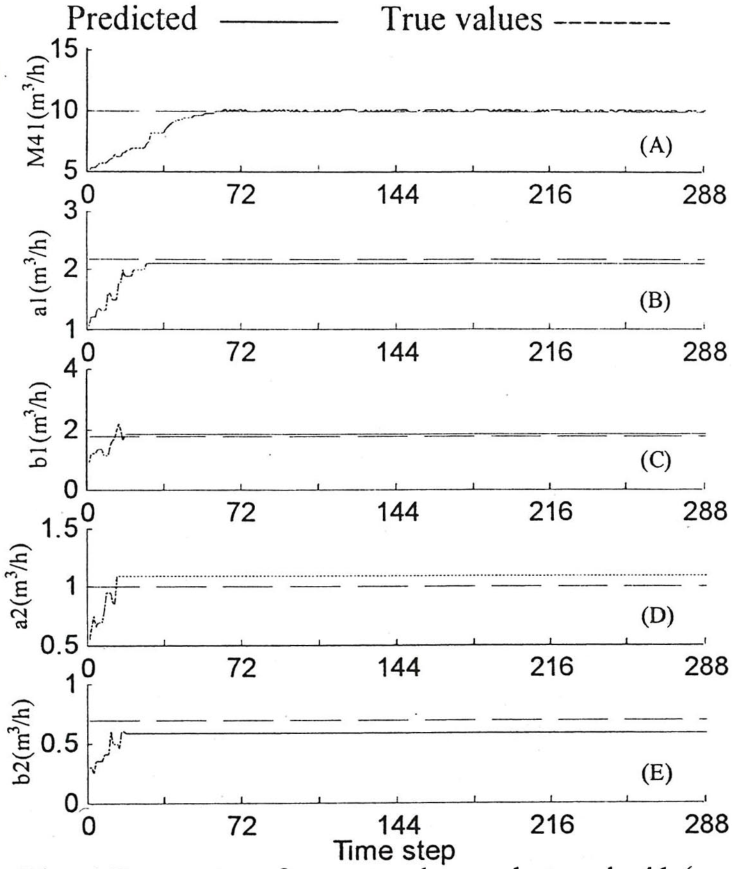

Fig. 4 Parameters for water demand at node 41 (q_{41})

In order to improve the accuracy of prediction at nodes containing demands and inflows, the prediction method are repeated by increasing the observation points to 23. These additional five observations are the outflows at nodes 8, 39, 42 and 45 with the pressure head at node 1. With this new setting, **Fig 3G** and **3H** shows an increase of the prediction accuracy comparing with the case of unobserved outflows. The same accurate prediction was detected for the other four nodes of fixed grade when the outflows were observed. Considering these results, it can be proposed that the water supply network should be divided into blocks where the observable pipe discharges will become inflows to the next block.

After trying different numbers of total observation points, it is concluded that the accuracy of prediction for both observed and unobserved points is directly proportional with the total number of observation points.

By studying the effect of the observation point type, it was found that the installation of flow meters is more effective than installing pressure gauges in terms of getting high prediction accuracy. This conclusion conflict with the current situation of the majority of supervisory water supply networks; for example, there are eleven pressure gauges and seven flow meters in the studied example of Block 12. Only information from pressure gauges is used to control the distribution of hydraulic pressures in the network, and information from flow meters is used for the detection of extraordinary pipe discharges.

Fig. 4 shows the identification of parameters in the case of q_{41} in **Eq. 8**. In this figure, broken lines are the true parameter values. It is obvious that each parameter converges to the true value after around 60 steps. It is supposed that the small bias in the identification of parameters causes the small prediction error in **Fig. 3A**. Moreover, it is confirmed that the prediction could be done more accurately if the initial values of the parameters used in the prediction are closer to their true values.

The results of other calculations confirm that more accurate predictions of water demands can be obtained at nodes located downstreams of the flow meters comparing to the nodes located in the upstreams sites.

To generalize the application of the stochastic periodical function used in this study to any water supply network, it is suggested to increase the number of parameters and to expand this model to include an autoregressive model to simulate real fluctuations in water demand variations which varies according to the daily weather conditions, seasons of the year and past trends in water use. A function of the following form is suggested

$$q_i(k) = M_i + \sum_{m_1=1}^{s_1}\left(a_{im_1}\sin 2\pi\lambda_{im_1}k + b_{im_1}\cos 2\pi\lambda_{im_1}k\right) + \sum_{m_2=1}^{s_2} c_{im_2}\{q_i(k-m_2)\}_i + \upsilon_i(k) \tag{19}$$

where s_2 the number of factors affecting the nodal demand, c_{i1}, ..., c_{im2} additional parameters and the rest of variables are as previously mentioned in **Eq. 8**.

6. CONCLUSIONS

Regarding the problem of pipe network calibration which aims to find an accurate estimation of water consumption at nodes and pipe roughness coefficients, in this paper we have applied Kalman filter theory for the on-line prediction of nodal demands, pipe discharges and nodal heads of a water supply network by using sensor information from flow meters and pressure gauges. Application of this method to Block 12 of the Fukuoka city water supply network shows the effectiveness of the proposed method. The advantages of this prediction method are that once we compute the estimated system parameters we need only a little amount of data to be stored and that the computation of these parameter values are made on-line.

The following conclusions can be drawn from this study:

1) In spite of the initial estimated errors, predictions of nodal water demands, pipe discharges, nodal pressures and outflows were found to be in good agreement with observations.
2) When observed points are added, the accuracy of the predictions is improved for both observable and non-observable variables.
3) Accuracy of predictions of the observable variables is better using flow meters as compared to pressure gauges.
4) Results from simulations reveal that the accuracy of predicting pipe network variables increase when the outflows to the network are used as observed values.
5) The accuracy of the final predictions results could be increased if the assumed initial system parameters are near its real values.
6) More accurate prediction results can be obtained at nodes located downstreams of the flow meters.

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