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Examination of Possibility, Fuzziness and Chaos in Sunspot Time Series

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Abstract

Fuzzy sets and chaos theory are two effective tools to analyze uncertainty as well as stochastic theory. While application of chaotic theory is widely considered in recent years, by use of sunspot time series as an example, analyzing the interface of stochastic, fuzzy and chaotic theories, and discussing the relations among them in a view point of methodology may be interesting. Sunspots have been well studied since they indicate the relative activity of the Sun that in turn influences terrestrial weather properties. For a long time statistical analysis methods have been used in the research of sunspot numbers. During recent years many studies have shown chaotic properties of them. On the other hand, the fuzziness has already existed in the ancient description of sunspots. When the influence of the Sun on the earth is mentioned, fuzzy theory seems specially convenient for certain aspects. As a conclusion, the stochastic, fuzzy and chaotic characteristics of sunspots exist at the same time; each of these theories is effective to certain aspects; therefore, the research method that combines the stochastic, fuzzy and chaotic theories properly is suggested for not only sunspots but also other research fields.

Keywords: sunspot number, possibility, fuzziness, chaos

1. Introduction

A new successful theory usually provides not only a new method for applied research, but also a way of thinking. In the 60's of this century two important theories—fuzzy sets and chaos—are presented, and have been affecting the research of almost all fields of technology. They are effective tools for the analysis of uncertainty as well as stochastic theory. While chaotic theory are widely considered in recent years, by use of a practical example, analyzing the interface of stochastic, fuzzy and chaotic theories, and discussing the relations among them in a view point of methodology may be interesting.

What the most important object affects the earth is the Sun. For hundreds of years the behavior of sunspots has been observed and studied. Several papers discussed the chaotic characters of sunspots recently. Comparing stochastic, fuzzy and chaotic features of sunspots and discussing the relations among them is the duty of this paper. The study shows that each of statistical, fuzzy and chaotic theories is suitable for analyzing certain aspects of a problem; combination of all or some of these theories is necessary when the corresponding features can not be ignored, and this methodological idea can be expanded to other technology fields.

2. Sunspot Time Series

Sunspots have been widely studied since they indicate the relative activity of the Sun that in turn influences terrestrial weather properties. The data length of sunspot numbers used here is 244 year's monthly data(1753.1～1996.12, 2928 months). The data before 1957 are quoted from the work of Chernosky et. al.[1958] and the remaining data are obtained from the volumes of Journal of Geophysical Research. Figure 1.

shows monthly mean of the Wolf sunspot numbers. The 22 peaks and 23 valleys can be found clearly. Following the time order, the peaks are noted as cycle 1, cycle 2, and so on.

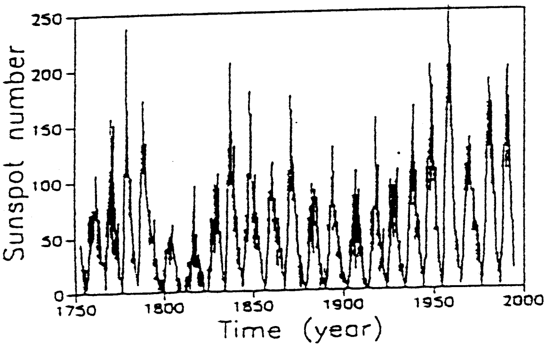

Fig.1 Monthly mean Wolf sunspot number time series

3. Stochastic Features of Sunspot Time Series

The study of sunspot numbers using the statistical methods has a long history. Many statistical coefficients of sunspot numbers, such as the mean value, the maximum, the minimum, the period, the regression prediction model and so on, are studied. The reason of the fact is concerned with the natural feature of the sunspots and the situation of human being's understanding natural world. As known, sunspots appear on the surface of the Sun. Not only the states of sunspots change continuously as time, but also the distribution, the size, the shape and the color of them change very much. Therefore, the possibility can only be a outline of uncertain sunspots. The observed data of sunspot numbers play a great role in understanding them. Figure 2(a), 2(b) and 2(c) show daily (1994.1--1994.12), monthly (1974.1--1994.12), and annual (1753--1994) sunspot number records.

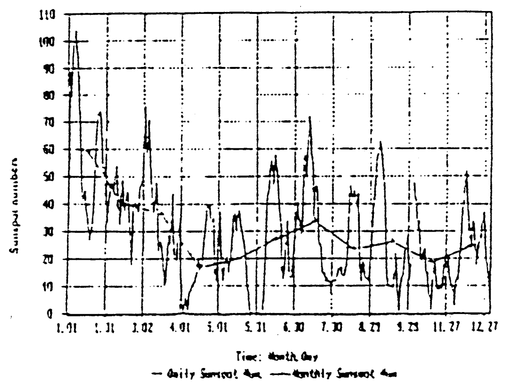

Fig.2(a) Daily sunspot number time series (1994.1.1～1994.12.31)

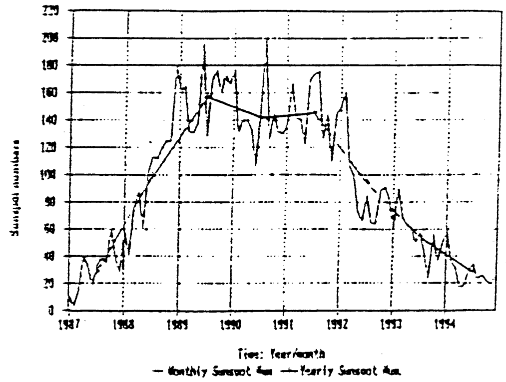

Fig.2(b) Monthly sunspot number time series (1974.1～1994.12)

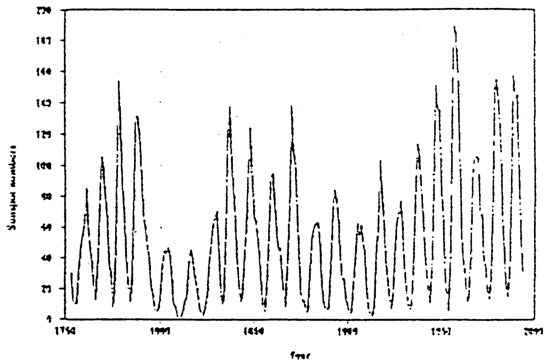

Fig.2(c) Annual sunspot number time series (1753～1996)

From these records, the first impression may be that sunspots time series might not be an exactly periodic one. Hence, in order to understand the solar activity in an average mean, the statistical methods are convenient. So far, the most popular statistical feature of sunspots is perhaps the 11 year periodicity that was discovered by an amateur astronomer, G. Schwabe [Ruzmaikin, 1981]. Even though the period of sunspot numbers changes from 9.5 to 14 years counted by minimums . and there are different periods for the same solar cycle counted by maximums or minimums. The basic 11 year cycle is modulated by a quasiharmonic (with period Ts) "secular" curve. This fact is sufficiently well found statistically, but the value of Ts is not well established [Ruzmaikin, 1981].

Statistical nature of the solar cycle was not only discussed in regarded to small-scale fluctuation, but also to the large scale. Ruzmaikin[1981] has shown that the extreme weakening of solar activity occur at random time, i.e. in a stochastic manner.

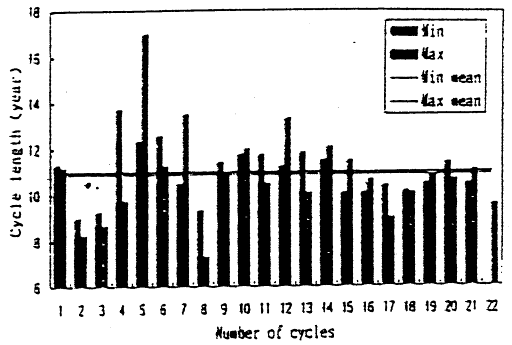

Fig.3 Cycle length of sunspot numbers

Before the dynamo model has been applied to predict sunspots, the current forecasting methods can be grouped in three broad categories[Withbore, 1989; Mundt et al. 1991]: (1) statistical period methods, which assume fundamental periods in the solar cycle, (2) statistical behavior methods, which, assuming a certain behavior of the Sun in a current cycle and together with the behavior in past cycles, will give the future of the current cycle, and (3) precursor techniques, which, assuming that the behavior of the solar magnetic field in the previous cycle, determine the behavior of the present cycle. This examination shows that the statistical methods play a great role in the research of sunspots.

As mentioned above, for a long time stochastic features of solar activity have been well investigated and many statistical methods to deal with sunspot time series have been developed. It is no doubt that statistical theory will also be an important theory for sunspot research before solar activity can be described in an exact way. On the other hand, some statistical results of sunspots, such as average period, maximum, minimum, mean value and so on, will have significant meaning for ever. Therefore the stochastic behavior and the statistical analysis of solar activity may not be ignored.

4. Fuzzy Feature of Sunspot Time Series

The theory of fuzzy sets has been presented in the 60's of this century. But the concept of fuzziness appeared much earlier. It can be said that human beings used a fuzzy way at the very beginning when they live against the natural world,. For example, they divided objects in the concepts of "big" or "small", not in kilogram or pound. They told temperature in the concepts of "hot", "warm" or "cold", not in Centigrade C or Fahrenheit F. Though modern science and technology provide us very exact ways to convey different physical objects, we are also used to employing fuzzy ways to explain or judge things. And for certain situations fuzzy ways have significant effects[Xu et al. 1991].

The Sun is so important for human beings and the earth that the different races of human beings had begun to notice the behavior of the Sun from very old time . The ubiquitous Robert Hooke, who lived, however, at the time of the Maunder minimum(he was born on 18 July, 1653 and died on 3 March, 1703),noted that the Sun's color resembles that of a flame. Hence, Hooke believed that the Sun can emit smoke and soot, and this could explain Sunspots and flares [Ruzmaikin, 1981]. Because the states of sunspots were described in natural language, the early records of the sunspots are somehow fuzzy.

Though Sun's activity today is observed and recorded in much better ways than that hundreds years ago, the fuzzy appearance of sunspots can not be disregarded either. It is known that sunspots are the areas on the surface of the Sun where nuclear reaction is relatively weaker than most other areas and the temperature is relatively lower. In fact, these areas are different from each other in many aspects, such as shape, size, color, location, existing time and so on, so that sunspots can be divided into several types. Zurich classification of sunspots has 9 types that are shown in Figure 4 [Japan astronomy station, Yearly physics report, 1995].

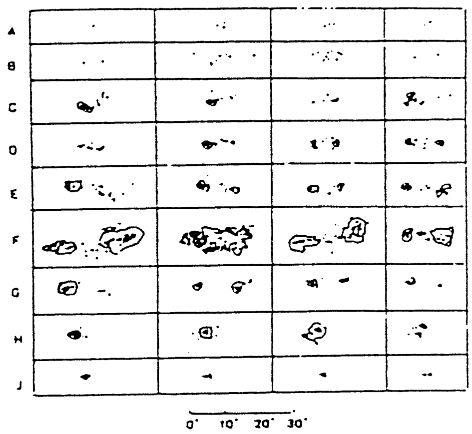

Fig.4 Zurich classification of sunspots

If describing these types of sunspots in words, they are like that:

A type: a single dark spot without dim parts, or a group of several points like that;

B type: a two pole group of dark spots without dim parts;

C type: a two pole group of dark spots with dim parts on one main spot;

D type: a two pole group of dark spots with dim parts, which width from east to west is less than 10°;

E type: a large two pole group of dark spots with dim parts, small dark spots separate between two poles, which width from east to west is larger than 10°;

F type: a huge two pole group of dark spots or a complicated group, which width from east to west is larger than 15°;

G type: a large two pole group of dark spots with dim parts, but there are no small dark spots between them, which width from east to west is larger than 10°;

H type: a single pole dark spot with dim parts, which diameter is larger than 2.5°;

J type: a single pole dark spot with dim parts, which diameter is less than 2.5°.

From the Figure 4 and the description of sunspots, it is seen that the states of sunspots is fuzzy. No matter how exact method is used, the fuzzy features of sunspots can not be neglected. Moreover, not to say a huge sunspot is 10 times as big as the earth, the state of sunspots changes frequently and continuously. It is the changes of sunspots in color and states that determine the fuzzy features of sunspots.

As mentioned above, sunspots are divided into 9 types. When their records are transformed to standard sunspot numbers, the following formula (Wolf sunspot numbers) is employed:

$$R = k\,(10g + f) \qquad (1)$$

Where R is Wolf sunspot numbers; g is the number of groups of sunspots; f is the sum number of sunspots observed; k is a coefficient that depends on the instruments used and observers. Apparently, the coefficient k gives a space for observers to deal with the same sunspots states in a personal way. Naturally, it may be regarded as a compensation for the fuzzy features of sunspots.

From old time to now the aim to study sunspots may be almost same, that is, to explain the influence of the Sun to the earth. No doubt, the Sun affects the earth so much as we can say. Considering that the Sun is 3.3×10^5 times as big as the earth, the Sun is 1.5×10^8 Km far from the earth, and about 55% of the light from the Sun is reflected or absorbed by the air before it reaches the surface of earth, the relation between the behavior of the Sun and a certain phenomena on the earth will be very confusion and fuzzy. So far findings show that the ice periods of the earth is concerned with the extreme weakening periods of the Sun, the disorder weather on the earth is related to the peaks of sunspots, the transport storage of the sand in the yellow river of china is affected by sunspot's change, and even the tempers of human beings have some relation with the situation of sunspots. Most of them can only be shown as a tendency or a likely relation. The theory of fuzzy sets may be an useful tool for them.

5. chaotic features of sunspot time series

As one of nonlinear phenomena, chaos is concerned with a kind of dynamical behaviors that future developments are sensitive to initial states. However, even though chaotic behavior changes very sharply at a certain part, it does not diverge or converge in a meaningful range. So that it can be considered that chaotic phenomena belong to such systems that are affected by many factors which react each other, compensate in values and keep dynamical balance of the system. As a result, chaotic phenomena do not have a fixed period, except a pseudoperiod or a likely similarity among some parts. When sunspot numbers are attended upon, the dynamical state of nuclear reaction on the surface of the Sun determines the dynamical characteristics of sunspot number time series.

Many studies during recent years have indicated chaotic characteristics for sunspot time series and solar activity in general [Ruzmaikin, 1981; Kurths and Herzel, 1987; Weiss, 1988; Feynman and Gabriel, 1990; Mundt et al., 1991; Berndtsson et al., 1994]. This paper will discuss the chaotic characteristics of sunspot time series in three aspects: strange attractor, fractal dimension and prediction by means of a method based on chaotic theory.

Studies have shown that nonlinear analysis methods are extremely noise sensitive [e.g., Grassberger et al., 1991; 1993]. Therefore, observed time series should firstly be cleaned by a noise reduction scheme. Figure 5 shows monthly noise-reduced sunspot numbers using the noise reduction algorithm of Schreiber [1993]. This algorithm was especially developed for dimension estimations.

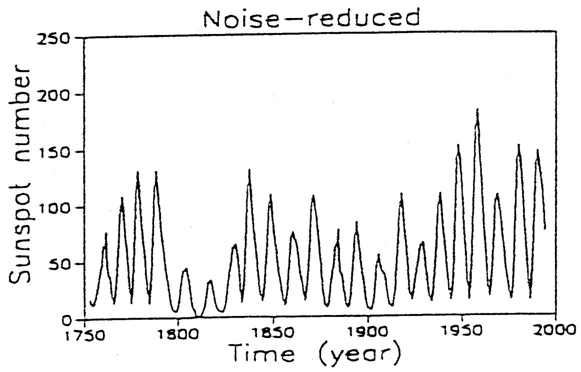

Fig.5 Noise-reduced monthly sunspot time series (1753.1—1994.12)

The results of noise reduction show the removal of high frequencies from the time series and remainder of low frequencies more or less unaffected as seen in the power spectrum diagram (Fig.6). Consequently, general properties of the time series are kept while small-scale variations are evened out.

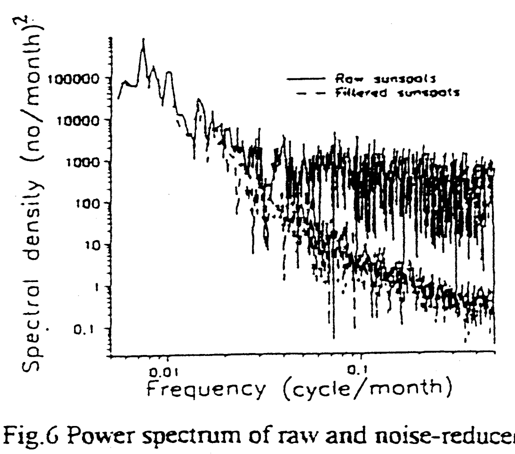

Fig.6 Power spectrum of raw and noise-reduced monthly sunspot time series

5.1 Strange Attractor

Fig.7 shows the phase space portrait, a strange attractor, of noise-reduced sunspot time series for a time lag τ = 10 months. From the figure, it is seen that the order of the cycles of sunspot time series does not correspond to the largeness of the cycles. This may be a general feature of chaotic time series. A remarkable impression is that the different sunspot cycles appear to follow about five preferential development or attraction lines. And the styles of these preferential development are somehow corresponding to the peak values of sunspot numbers.

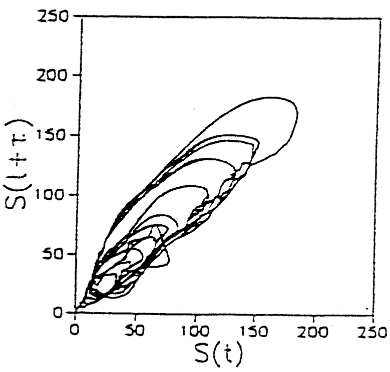

Fig.7 Strange attractor of noise-reduced sunspot time series (lag time τ = 10 months; monthly data during 241 years)

From the phase space portrait of sunspot numbers, another chaotic property, which future behavior is sensitive to initial values, may be assumed as that there exist five possible styles of future preferential development and each style include some possible attraction lines. Anyway, once such an attraction line, which the present cycle is following, is identified, it may be possible to make accurate short-term predictions of future states in the cycle. In the light of this idea, an updated prediction approach has been presented in the previous work of authors[Xu et al. 1993; Jinno et al. 1995].

5.2 Fractal Dimension

A real number dimension is a common feature of chaotic phenomena. Berndtsson et al.[1994] calculated the fractal dimension d of sunspot time series by the algorithm given by Grassberger [1990]. d is The ratio of $log C(r)$ to $log r$ according to:

$$log C(r) = d | log r | \quad (2)$$

where C(r) is the correlation integral defining the density of points around a specific coordinate x_i within a distance r related as (for small r):

$$C(r) \sim r^d \quad (3)$$

The correlation integral C(r) is calculated as:

$$C(r)=\frac{1}{N^2}\sum_{\substack{i,j=1\\ i\neq j}}^{N}\theta\,(r-|xi-xj|) \qquad (4)$$

where θ is the Heaviside function defined by $\theta\,(x) = 0$ if $x < 0$ and $\theta\,(x) = 1$ if $x > 0$. As shown in Fig. 8, there is a clear scaling region for $-2.0 < log r < 0$ ($dlog C(r)/dlog r$ does not change for various values of $log\ r$. According to the figure, noise-reduced sunspots display saturation at a correlation dimension $d < 2$. Mundt et al. [1991] found $d \fallingdotseq 2.3$ for the same sunspot data. However, they used a second-order, digital Butterworth filter with a cutoff frequency of 1/6 yr^{-1}. When comparing their power spectrum for the filtered data with the power spectrum in Fig.6, it appears that the algorithm by Schreiber [1993] preserves more of the high-frequency variation compared to the Butterworth filter. The reason for this is that the method of Schreiber [1993] is a nonlinear technique, while the Butterworth filter works as a linear procedure.

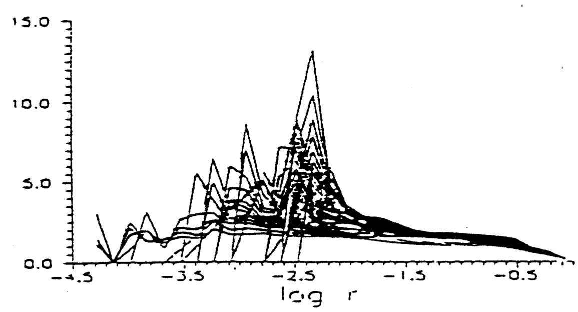

Fig.8 Slopes $dlog C(r)/dlog r$ vs. $log r$ for noise reduced sunspots(Embedding dimensions m = 2, 40).

5.3 Prediction Based on Chaotic Theory

So far no universal method exists to discriminate between colored noise with power-law spectra and underlying dynamical processes in data [Theiler et al., 1992]. However, observed time series may at least tentatively be viewed as deterministically chaotic if prediction methods based on underlying deterministic properties are significantly better as compared to autoregressive linear models based on stochastic theory [Farmer and Sidorowich, 1987; 1988; Sugihara and May, 1990]. As noted by Mundt et al. [1991], one reason why models based on periodic behavior fail to predict sunspot time series accurately, may be the nonlinear behavior.

After studying chaotic behavior of sunspot time series in detail, Jinno et al.[1995] presented a prediction procedure based on chaotic theory and got successful prediction results. The procedure was: (1) based on the behavior of observed time series and dimension estimation of the strange attractor, find reference equations that show similar basic feature as the time series(e.g., general appearance of the attractor, the amplitude, the pseudoperiod, etc.), (2)assume a general system equation by applying Taylor series expansion, (3) use the reference system equation as initial state for the general system equation and use it in an update procedure. Then the prediction of future state of the observed variable can be made by means of the updated and identified system equation.

Applied this approach to 241 year's monthly sunspot time series, the prediction result for 3-month ahead prediction is shown in Figure 9. Figure 10 is correlation coefficients between observed and predicted sunspot time series versus lead time. It is seen that, on average for the entire observation period, the correlation coefficients for prediction up to eight months ahead remain above 0.9. After this lead time, the prediction accuracy reduce remarkably. This kind of decrease in the prediction accuracy is known as a typical feature of chaotic systems as pointed out by Sugihara and May[1990]. Therefore, the prediction results indicate that the approach based on chaotic theory is effective to predict chaotic sunspot time series.

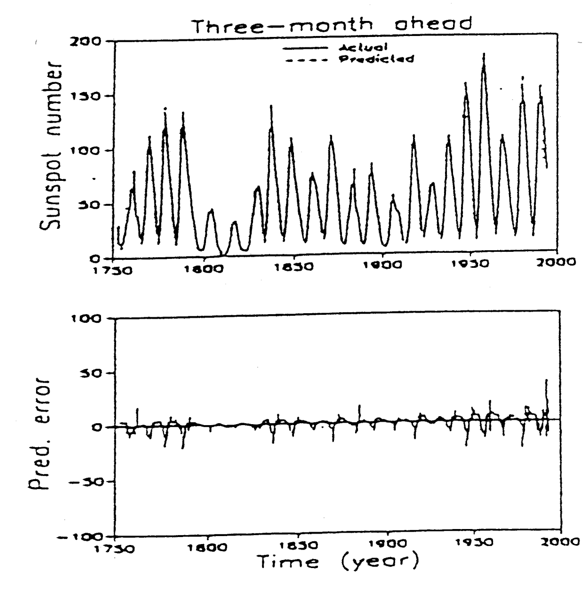

Fig.9 Three-month ahead prediction results

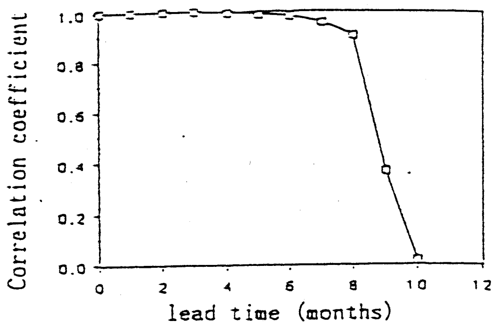

Fig.10 Correlation coefficients between observed and predicted sunspot time series versus lead time

6. Conclusion and Discussion

According to the basic concepts of stochastic, fuzzy and chaotic theories, it is illustrated that sunspot time series possesses stochastic, fuzzy and chaotic characteristics. Stochastic characteristic is concerned with nonperiod feature of sunspot numbers and the effectiveness of the results analyzed by stochastic methods. Fuzzy characteristic is born in describing the state of sunspots, quantifying sunspot numbers, and considering the influence of the Sun on the earth. Chaotic characteristic, as a nonlinear feature, appears in long term behaviors. Analysis shows that each of these theories are effective for analyzing corresponding features of sunspot time series. This can go to a summary that, as tools to comprehend the natural world, the each of these theories describes a different aspect of thing's characteristics. So that though they are theoretical independent each other, it is natural that they can be applied in a same example.

Therefore, conclusions come as (1) stochastic, fuzzy and chaotic theories correspond to different features of things. They are parallel theories for describe natural things. (2) combination of these theories is necessary if all or two of these features can not be ignored. (3) Sunspot time series is a successful example to show the existence of stochastic, fuzzy and chaotic phenomena in a same time series. The effect of these theories on sunspot time series is demonstrated true. (4) The combination of these theories has a general meaning for the application research.

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