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SEQUENTIAL OPTIMAL CONTROL OF INLAND BASIN DRAINAGE FOR A ONE-BLOCK MODEL

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ABSTRACT: In this study, sequential optimal control (SOC), a method to sequentially optimize pump drainage from an inland basin, is formulated for a one-block model. The method is applied to synthetically generated data to demonstrate the effectiveness and to study the characteristics of the SOC for controlling the pump drainage volume. The results show that sequential optimal control is well suited for the optimal control of inland water drainage. The relationships between the control time period and cost function, and between control time period and computation time, are shown. Other characteristics of the SOC for controlling inland inundation are also presented and discussed.

INTRODUCTION

Recently, inland inundation, which may be viewed as a type of water disaster, has been frequently happening and has become a serious public problem. At present, pumping and/or sluice pipes are generally used to decrease the effect of inland inundation (Ministry of Construction,1995). The present pump control of inland water drainage is based on a predetermined operation scheme. However, in practical applications automatical and optimal real-time control of pumps and/or sluice pipes based on the measurements of inflow, rainfall and main river water level, is desired.

For the optimal control of inland water drainage, it is reasonable that pump or sluice pipe control should be updated in sequence, considering the difficulties of long span inflow forecast. If all data such as inflow, rainfall and main river water level are known beforehand, we do not have to execute non-linear programing, using depth of flooding water and pump drainage volume as decision variables, sequentially for the optimal control of inland water drainage. However, it is generally impossible to forecast all of those data. Even if that is possible, forecast precision decreases and computational load increases rapidly because of the increase of the number of control periods.

In this study, we apply sequential optimal control (SOC) (IAWPRC Task Group on Real Time Control of Urban Drainage Systems,1989; Nelen, A.J.M.,1992), a method to sequentially optimize pump drainage, to the problem of inland water drainage. First, an equation for the problem of inland water drainage for a one-block model, which is the simplest model of inland inundation, is introduced in order for SOC to be formulated. Second, SOC is applied to synthetically generated data to demonstrate the effectiveness and to study the characteristics of the SOC for controlling the pump drainage discharge. Other characteristics of the SOC for controlling inland water inundation are also presented and discussed.

FORMULATION OF SEQUENTIAL OPTIMAL CONTROL FOR A ONE-BLOCK MODEL

Fig.1 shows a schematic illustration of a one-block model. The equation of continuity for the model is expressed by

$$A\frac{d\{h_1(t)-h_d\}}{dt}=Q_{in}(t)-Q_c(t)-Q_p(t)+AR(t) \quad (1)$$

where

A : inland basin area (m^2)
t : time (sec)
h_1 : inland water level (m)
h_d : ground level (m)
Q_{in} : inflow (m^3/s)
Q_c : sluice pipe drainage discharge (m^3/s)
Q_p : pump drainage discharge (m^3/s)
R : rainfall (m/s)

Inland water depth $H=h_1-h_d$ and pump drainage discharge Q_p are taken as decision variables. The objective function (cost function) of inland water drainage control used in this study is defined by

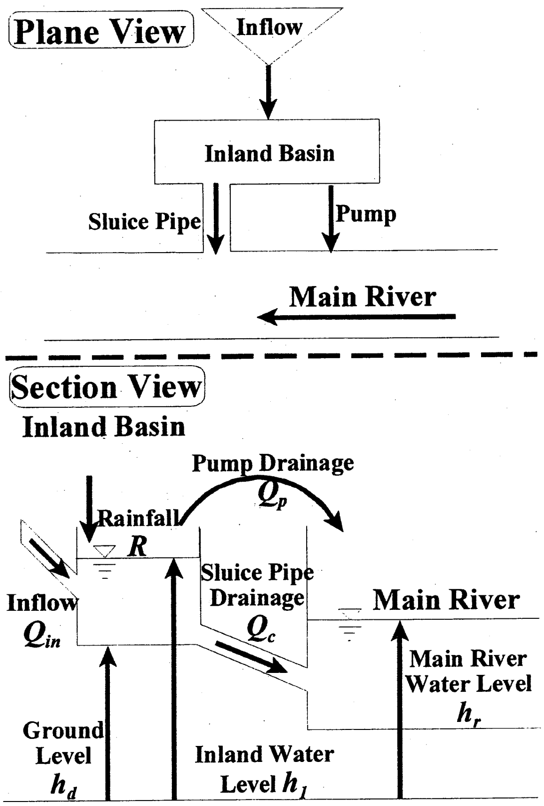

Fig. 1 Schematic illustration of a one-block model

$$Z(k)=\sum_{\tau=0}^{T}[\lambda_1 H(k+\tau+1)+\lambda_2 Q_p(k+\tau)] \quad (2)$$

where

k : time step

T : control time period at each time step

λ_1 : weighting factor for inland water depth (1/m)

λ_2 : weighting factor for pump drainage discharge (1/m^3/s)

The aim is to find the set of solutions for H's and Q_p's which minimizes the cost function (Eq. (2)). The equation of continuity (Eq. (1)) is discretized into Eq. (3) for $\tau=0\sim T$, as the constraint of equality for the optimization problem.

$$\begin{aligned}H(k+\tau+1)=&H(k+\tau)+R(k+\tau)\Delta t\\&+\{Q_{in}(k+\tau)-Q_c(k+\tau)-Q_p(k+\tau)\}\Delta t/A\end{aligned} \quad (3)$$

In this case, simple finite difference was conducted using a time increment Δt, and t and $t+\Delta t$ were replaced by k and $k+1$, respectively.

The constraint on pump drainage discharge Q_p is given by

$$Q_p(k+\tau)\leq P_{\max} \quad (4)$$

where

P_{max} : maximum pump drainage discharge (m^3/sec)

On the other hand, the sluice pipe drainage discharge Q_c in Eq. (3) is a function of the inland water depth H which can be expressed

$$\begin{aligned}Q_c(k+\tau)=&C_cA_c\sqrt{2g|H(k+\tau)+\Delta H(k+\tau)+h_d-h_r(k+\tau)|}\\&\times U[H(k+\tau)+\Delta H(k+\tau)+h_d-h_r(k+\tau)]\,U[h_r(k+\tau)-h_d]\\&+C_cA_c\sqrt{2g|H(k+\tau)+\Delta H(k+\tau)|}\,U[h_d-h_r(k+\tau)]\end{aligned} \quad (5)$$

where

C_c : discharge coefficient for sluice pipe

A_c : cross sectional area of sluice pipe (m^2)

g : acceleration of gravity (m/s^2)

h_r : main river water level (m)

ΔH : inflow compensation which considers inflow during calculation time increment Δt, which is calculated by

$$\Delta H(k+\tau)=Q_{in}(k+\tau)\times\Delta t/A \quad (6)$$

U : unit step function defined by

$$\begin{cases}U(x)=1 & \text{if } x>0\\ U(x)=1/2 & \text{if } x=0\\ U(x)=0 & \text{if } x<0\end{cases} \quad (7)$$

Eq. (5) is conveniently divided into two cases using the unit step function $\boldsymbol{U}$, *i.e.*; case 1: main river water level $\boldsymbol{h_r}$ is higher than ground level $\boldsymbol{h_d}$, and case 2: main water level $\boldsymbol{h_r}$ is lower than ground level $\boldsymbol{h_d}$. Furthermore, the following non-negative constraints exist for inland water depth $\boldsymbol{H}$ and pump drainage discharge $\boldsymbol{Q_p}$:

$$0 \leq H(k+\tau+1) \tag{8}$$

$$0 \leq Q_p(k+\tau) \tag{9}$$

Under the constraints Eq. (3)-Eq. (9), the values of $\boldsymbol{H}(\boldsymbol{k}+\tau+1)$ and $\boldsymbol{Q_p}(\boldsymbol{k}+\tau)$ which minimize $\boldsymbol{Z}(\boldsymbol{k})$ in the objective function (Eq. (2)) should be searched sequentially in this optimal control problem. This control problem becomes non-linear because Eq. (5) is expressed by non-linear functions such as the unit step function $\boldsymbol{U}$ and square roots and consequently non-linear programing must be used. In this study, we use Sequential Quadratic Programing (SQP) (Davidon, W.C., 1959; Fletcher, R., 1970; Gill, P.E *et al.*,1981). **Fig. 2** shows the flowchart for the SOC.

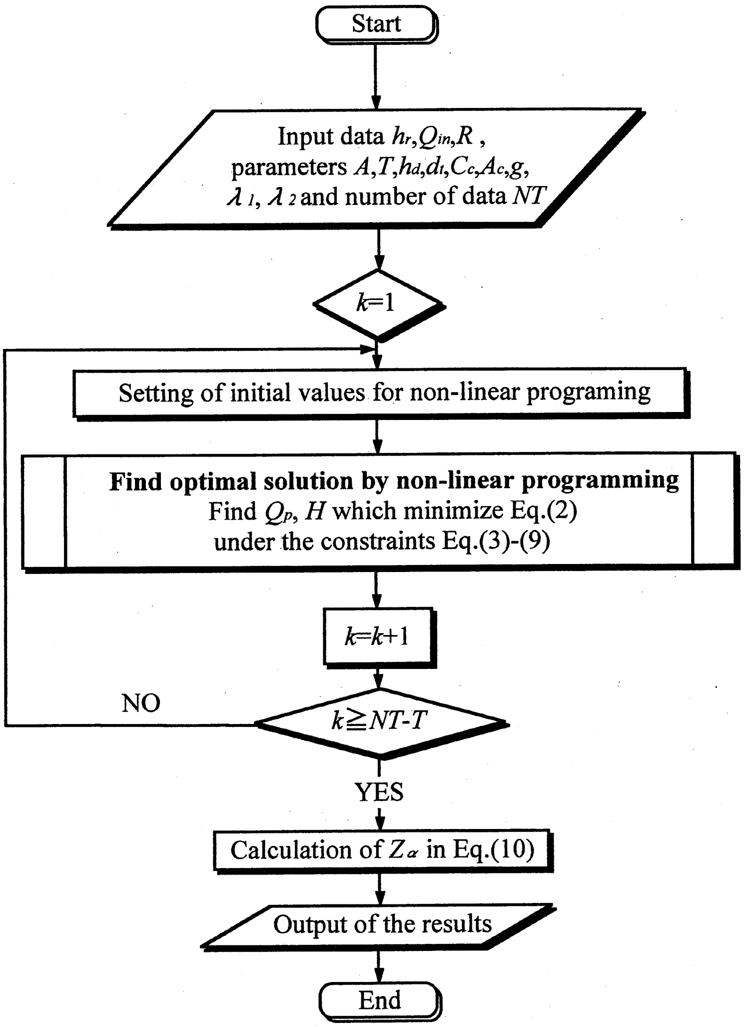

Fig. 2 Flowchart for the SOC

NUMERIC SIMULATION

Data and parameters

The SOC formulated in the former section is applied to synthetically generated data in order to demonstrate the effectiveness and to study the characteristics of this method. In this simulation, we synthetically generate values of inflow $\boldsymbol{Q_{in}}$ and main river water level $\boldsymbol{h_r}$ for 50 steps using $\boldsymbol{\Delta t}$ =600sec (10min) (which correspond to about 8 hours) shown in **Fig. 3(a)** and **(b)**. We set the inland basin area $\boldsymbol{A}$=2.0×10^5 (m^2), the ground level $\boldsymbol{h_d}$=1 (m), discharge coefficient for sluice pipe $\boldsymbol{C_c}$=0.6, the cross sectional area of sluice pipe $\boldsymbol{A_c}$=20 (m^3), the maximum pump drainage discharge $\boldsymbol{P_{max}}$=2.0 (m^3/s), and the initial inland water depth $\boldsymbol{H}$(0)=0 (m).

We simply assume that rainfall in the inland basin $\boldsymbol{R}$=0. In this simulation, we consider inflow $\boldsymbol{Q_{in}}$ and main river water level $\boldsymbol{h_r}$ to be known.

In addition, we define the cost function $\boldsymbol{Z_\alpha}$ for the whole time period $\boldsymbol{N}$ as

$$Z_\alpha=\sum_{\tau=1}^{N}[\lambda_1 H(\tau+1)+\lambda_2 Q_p(\tau)] \qquad (10)$$

The cost function $\boldsymbol{Z_\alpha}$ is introduced to evaluate the total control operation performance for the simulation, whereas $\boldsymbol{Z(k)}$ in Eq. (2) is used to estimate the pump drainage which minimizes the cost during the control time period $\boldsymbol{T}$.

Results

One of the simulation results for the one-block model is shown in **Fig. 3**. In this case, we assume the control time period $\boldsymbol{T}$=5, the weighting factor for inland water depth $\boldsymbol{\lambda_1}$=1.0, and the weighting factor for pump drainage discharge $\boldsymbol{\lambda_2}$=0.015. SOC is carried out stepwise from $\boldsymbol{k}$=1 to $\boldsymbol{N}$-$\boldsymbol{T}$. Concerning the weighting factors, we set $\boldsymbol{\lambda_1}$ as 1.0 and $\boldsymbol{\lambda_2}$ as a relative value of $\boldsymbol{\lambda_1}$ (in this case $\boldsymbol{\lambda_2}$=0.015) which adequately represents pump operation. **Fig. 3(a)** shows the pump drainage discharge

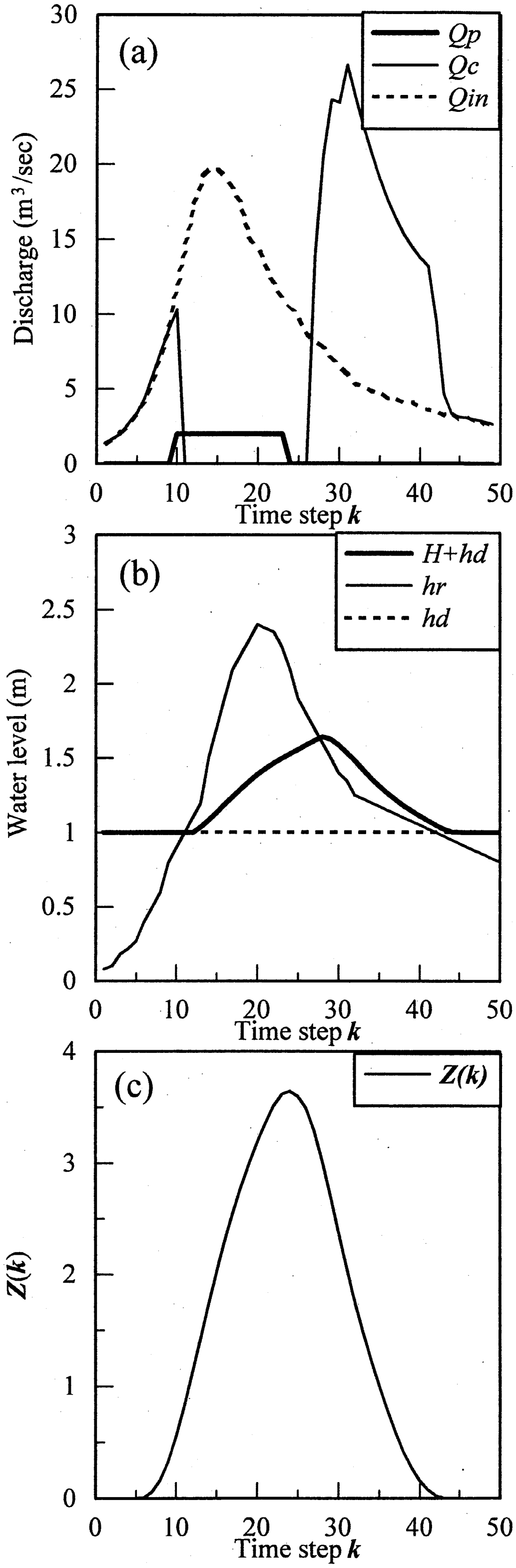

Fig. 3 Simulation results for $\boldsymbol{T}$=5, $\boldsymbol{\lambda_2}$=0.015

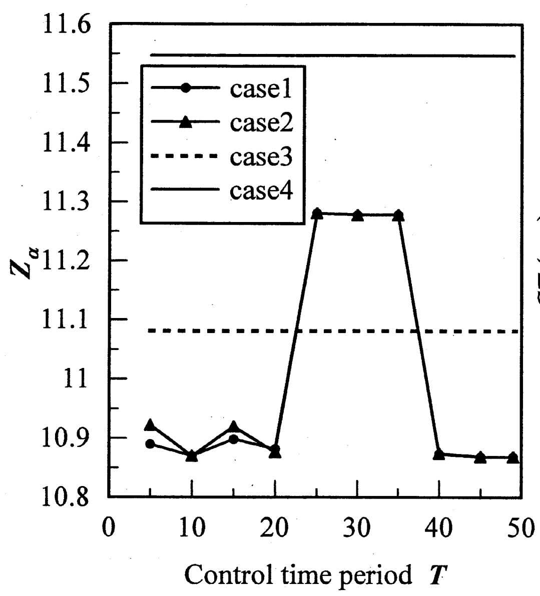

Fig. 4 Cost function Z_α as a function of control time period (λ_2=0.015)

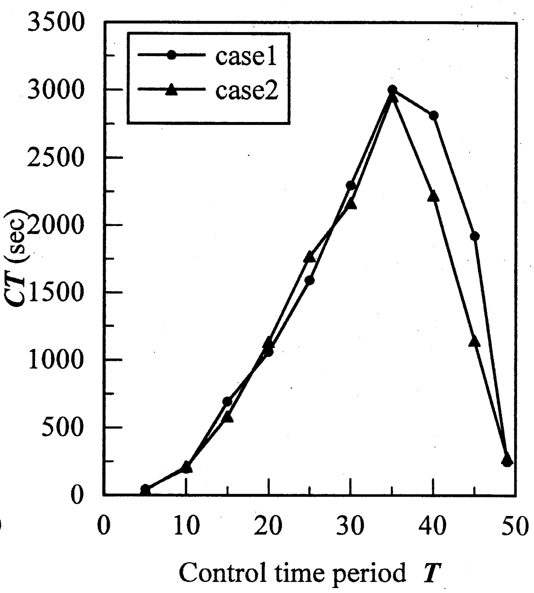

Fig. 5 Computation time CT as a function of control time period (λ_2=0.015)

$Q_p(k)$ and the sluice pipe drainage $Q_c(k)$ obtained by the SOC (the inflow $Q_{in}(k)$ is also included). **Fig. 3(b)** shows the time series of the inland water level H+h_d controlled by the SOC(main river water level $h_r(k)$ and ground level h_d are also included). **Fig. 3(c)** shows the objective function expressed by Eq. (2). In this simulation, we set $H(k+\tau+1)$ and $Q_p(k+\tau)$, τ=0～T to zero, as initial values of the non-linear programing at time step k=1. After time step k=2, however, the optimal solutions obtained by the SOC at the former time step are used as initial values. At each time step, optimal solutions up to T time steps ahead were calculated by the SOC. However, the solution at a certain time step k is only used for the actual real-time control by the SOC.

Values of Z_α and computation time CT obtained by the SOC as a function of the control time period T are shown as case 1 in **Fig. 4** and **Fig. 5,** respectively. The SOC calculation was carried out on a Pentium computer, 166 MHZ CPU, whose operating system is Windows95, and MATLAB was used for the programing.

As a comparison, the results in which we set zero values for time step k=1, and after time step k=2 use also zero values as initial values of the non-linear programing are shown as case 2. Furthermore, the pump control performance obtained by using the pump maximally without consideration of the pump cost is shown as case 3 in **Fig. 4**. The pump control performance obtained without operating the pump at all is also shown as case 4 in **Fig. 4**.

In the next simulation, the weighting factor for pump drainage discharge λ_2 is set to be 0.010 which is a reduction by 0.005 from the former simulation. The results for a control time period of T=5 are shown in **Fig. 6**. **Fig. 7** shows Z_α obtained by the SOC as a function of the control time period T. In this figure, case 2 is omitted because the results for case 2, in which the initial values for the non-linear programing are zero are almost same as those for case 1. The resulting computation time CT for various control time period T were almost the same as in **Fig. 5**.

DISCUSSION

The values of the sluice pipe drainage discharge Q_c in **Fig. 3(a)** is zero from time step k=12 to 27, when the main river water level h_r in **Fig. 3(b)** is higher than the inland water level H+h_d.

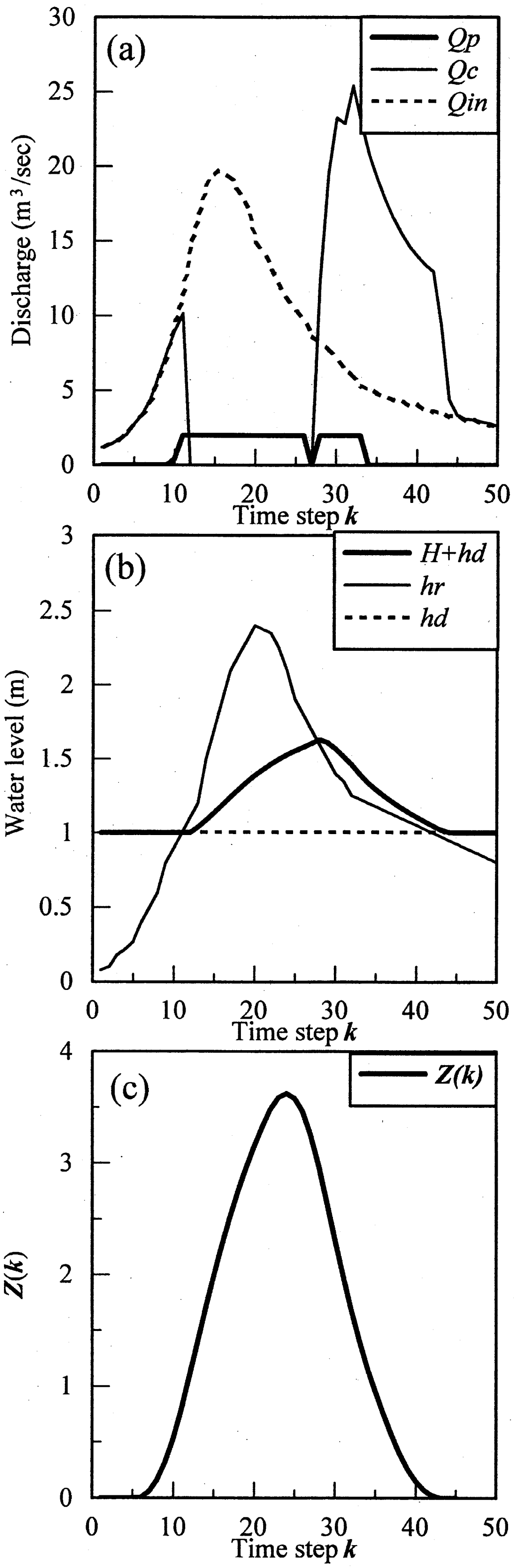

Fig.6 Simulation results for T=5, λ_2=0.010

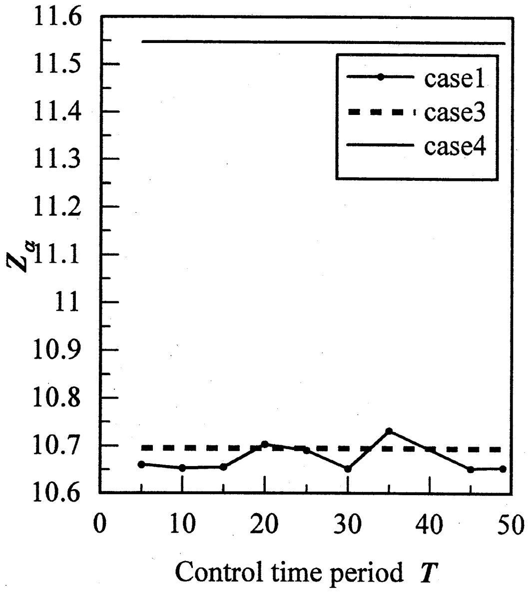

Fig. 7 Computationst time CT as a function of control time period T (λ_2=0.010)

After time step k=27, however, Q_c increases rapidly because the main river water level h_r becomes lower than the inland water level $H+h_d$. During the period in which the sluice pipe can not be used, only pumping is carried out at the maximum capacity P_{max}=2.0 (m^3/s). On the other hand, during the period in which sluice pipe drainage is carried out, the pump is not activated. In this simulation, the sluice pipe drainage is changing very smoothly compared with the results from a former simulation by Koga *et al.* (1998). This is because of the inflow compensation of ΔH in Eq. (6).

The cost function Z_α for the whole time period in Eq. (10) is expected to decrease with increasing of control time period T. However, Z_α's corresponding to T=25~35 are typically larger than for other T values. There are two reasons for this phenomenon. First, sequential optimization does not always give the optimal solution for the whole period. Second, the non-linear programing cannot always find the global minimum but only a local one.

When the control time period T becomes larger, the required number of time steps for the sequential optimal control calculation decreases (*e.g.*, when T equals its maximum number 49, only one optimal calculation is required).

However, the number of decision variables for each step which must be calculated by non-linear programing increases with increasing T. As the result, the computation time CT increases exponentially with increasing of control time period T until T=35, but after T=35 it decreases drastically as shown in **Fig. 5**.

In **Fig.4** and **Fig.5**, the results for case 1 and case 2 are almost the same, which means in this simulation the initial values used in the non-linear programing do not affect the result very much.

In these cases, for which the weighting factor for the pump drainage discharge λ_2 is set to be 0.010 which is a reduction by 0.005 compared with **Fig. 3**, the pump is running even when the sluice pipe drainage is carried out during time step k=28 to 33 as shown in **Fig. 6(a)**. However, the inland water level $H+h_d$ in **Fig. 6(b)** is almost the same as in **Fig .3(b)**. The resulting cost function $Z(k)$ (**Fig. 6(c)**) is also almost the same as in **Fig. 3(c)**, although $Z(k)$ cannot be compared because of the weighting factors used.

In **Fig. 7**, the level of the cost function Z_a for case 4, in which the pump was not used at all, is much higher than that for case 1 and case 3, which demonstrates the effectiveness of pump operation. The cost function Z_a for case 1 which was obtained by the SOC becomes higher for some T's compared with that of case 3 in which the pump was used maximally without consideration of the pump cost, but on the whole, case 1 is better than case 3. From the results of **Fig. 4** and **Fig. 7**, the preferable range of T is from about 5 to 10 for this simulation.

CONCLUSIONS

In this study, sequential optimal control (SOC) of inland basin drainage was formulated for a one-block model and is applied to synthetically generated data. In this simulation, the initial values used in the non-linear programing did not affect the control performance very much. The importance of inflow compensation ΔH for smooth sluice pipe drainage was shown. Concerning the selection of control period T, between 5 and 10 seemed to be reasonable under the consideration of computation time and predictability of inflow and main river water level.

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