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# REAL–TIME PREDICTION OF URBAN–SCALE RAINFALL BY USE OF A TWO–DIMENSIONAL STOCHASTIC CONVECTION–DIFFUSION MODEL

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**ABSTRACT**: This paper presents a two-dimensional stochastic convection-diffusion model for use in real-time prediction of the space-time rainfall structure in urban areas. The model is used to parameterize the rainfall intensity from individual rain cells. Especially, we are interested to quantify the rainfall intensity resulting from different separable components of the rain cell. This is done by a Lagrangian approach and dividing the rain from one cell into rainfall resulting from apparent turbulent diffusion and development-/decay as a consequence of cell morphological changes. The results can be used as first choice of parameter values when modeling rain cell movement over ungaged areas and the presented methodology can be used to study the effects of different cell components on rainfall intensity.

## INTRODUCTION

There are distinct needs to develop real-time prediction models of the space-time rainfall structure for urban areas. The prediction time scale that these models should operate on is typically less than one hour and with a space scale of a few square kilometers (e.g., Berndtsson et al. [1992], Berndtsson and Niemczynowicz [1988]). Thus, the rainfall scale that is of main interest is the meso-$\gamma$ scale (2-20 km$^2$; see Orlanski [1975] for the definition of scales) and smaller. The rainfall variation on this scale generally corresponds to the behavior of individual rain cells.

Previously published rainfall prediction models (e.g., the model by Lee and Georgakakos [1990]) for common hydrologic scales like the meso-$\beta$ scale (20-200 km$^2$) can not be used for real-time prediction in typically-sized urban catchment areas. The requirements of the temporal and spatial resolution of the rainfall input to urban runoff models are usually 1-5 minute values for 1-2 km$^2$ areas (e.g., Schilling [1990]). These catchment areas are frequently instrumented with dense on-line rain gages. Hence, it is possible to make repeated short-term forecasts using the latest information on the spatial rainfall distribution. A common technique for this is nowcasting (Browning and Collier [1989]) where the present weather situation is assumed to travel without change for short periods of time. This method, however, is dependent on the time ahead for which linear extrapolation is valid. Thus, there are needs to further develop these methods by use of current understanding of rain cell behavior.

Jinno et al. [1992] introduced a model methodology which can be used for real-time prediction of the rainfall distribution on the aforementioned time and space scales. This methodology is based on a two-dimensional stochastic convection-diffusion equation in combination with a Fourier domain shape method and an extended Kalman filter algorithm. In their paper it was shown that the model can be used to forecast motion, shape, size, and intensity distribution of individual rain cells. Since a Fourier domain shape method is used to represent the rainfall field and the coupled Gaussian noise, the model domain, such as irregularly spaced rain gage networks, does not have to be discretized. Furthermore, the model is suitable for inclusion in an interactive urban management system because it uses a recursive estimation procedure for parameter identification.

There are many uses of a model like the one described above. Not only can such a model be used for practical real-time forecasting applications but also to parameterize the variability pattern of spatial rainfall.

This, however, relies on the inherent assumption that the governing equations are based on physical properties or at least that the parameter values in the model reflect some physically-based properties of the actual rainfall process. If so, the model parameters can be conveniently used to characterize the variability pattern of observed rainfall in terms of physically explainable characteristics. This, in turn, may be practically utilized, e.g., when choosing model parameters for ungaged areas or to study effects of varying parameter values on the rainfall intensity in artificial simulation studies.

In view of the above, the objective of the present paper is to use the aforementioned model methodology in order to study variability patterns of individual rain cells as observed in a dense rain gage network. Especially, we are interested in trying to separate the effects of convective velocity, apparent turbulent diffusion, and the development/decay of rainfall intensity resulting from cell morphological changes on the overall rain cell intensity rate.

## THEORETICAL CONSIDERATIONS

It is assumed that a two-dimensional convection-diffusion equation in the horizontal plane is able to describe the flux of rainfall at ground level. Consequently, the behind-lying assumption is that the actual three-dimensional Gaussian-shaped rainfall field in space is represented by a two-dimensional Gaussian-shaped rainfall intensity field at ground level (Jinno et al. [1992]). Under this assumption a two-dimensional convection-diffusion equation may be used to describe the flux of rainfall at ground level. Indications that this assumption is relevant are the often found Gaussian-shaped pattern of individual rain cells (Sharon [1972], Zawadzki [1973], Marshall [1980], Berndtsson et al. [1992]).

Using a Lagrangian view of the rainfall intensity field (moving along with the same speed as the rain cell), the Lagrangian derivative of a two-dimensional stochastic convection-diffusion equation may be expressed as (assuming a homogeneous rainfall field)

$$\frac{dR(t)}{dt} = \frac{\partial R(x,y,t)}{\partial t} + u\frac{\partial R(x,y,t)}{\partial x} = D_x\frac{\partial^2 R(x,y,t)}{\partial x^2} + D_y\frac{\partial^2 R(x,y,t)}{\partial y^2} - \lambda\, R(x,y,t) + \varepsilon\,(x,y,t) \tag{1}$$

where $R(x, y, t)$ is the rainfall intensity in time and space (m/min), $u$ is the speed of the rainfall cell in the direction of movement (m/min), $D_x$, $D_y$ are the apparent diffusion coefficients in the $x$- and $y$-axis direction, respectively ($m^2$/min), $\gamma$ is the development/decay coefficient of the rainfall intensity ($min^{-1}$), and $\varepsilon(x, y, t)$ is a stochastic component with zero-mean Gaussian white noise in time and space (m/$min^2$).

The second-order partial differential equation (1) is transformed into a set of ordinary differential equations by applying a double Fourier series expansion. The rainfall field $R(x, y, t)$ and the coupled Gaussian white noise $\varepsilon(x, y, t)$ are expanded as

$$\begin{aligned} R(x,y,t) &= \frac{A_{0,0}(t)}{2} + \sum_{m=1}^{M}\sum_{n=0}^{N}[A_{m,n}(t)\cos F_1(x,y,m,n) + B_{m,n}(t)\sin F_1(x,y,m,n)] \\ &+ \sum_{m=0}^{M}\sum_{n=1}^{N}[C_{m,n}(t)\cos F_2(x,y,m,n) + D_{m,n}(t)\sin F_2(x,y,m,n)] \end{aligned} \tag{2}$$

where

$$\begin{aligned} \varepsilon(x,y,t) &= \sum_{m=1}^{M}\sum_{n=1}^{N}[E_{m,n}(t)\cos F_1(x,y,m,n) + F_{m,n}(t)\sin F_1(x,y,m,n) \\ &+ G_{m,n}(t)\cos F_2(x,y,m,n) + H_{m,n}(t)\sin F_2(x,y,m,n)] \end{aligned} \tag{3}$$

$$F_1(x, y, m, n) = \frac{2\pi mx}{\lambda_x} + \frac{2\pi ny}{\lambda_y} \tag{4}$$

$$F_2(x, y, m, n) = \frac{2\pi mx}{\lambda_x} - \frac{2\pi ny}{\lambda_y} \tag{5}$$

In (2)-(5), $A_{m,n}$, $B_{m,n}$, $C_{m,n}$, $D_{m,n}$, $E_{m,n}$, $F_{m,n}$, $G_{m,n}$, and $H_{m,n}$ are Fourier coefficients, $M$ and $N$ are the number of terms in the expansion procedure, and $\lambda_x$ and $\lambda_y$ are wave lengths for the $x$- and $y$-axis directions, respectively, that cover the entire area where rain gages are installed (25 $km^2$).

In the further analyses we assume a position in the center of the rain cell (peak rainfall intensity). Consequently, we may define the diffusion terms and the development/decay term of (1) (the non-random right-hand terms of (1)) at this point according to

$$D_{x,\,center} = D_x \frac{\partial^2 R(x, y, t)}{\partial x^2}\Bigg|_{\substack{x=X_L(t)\\ y=Y_L(t)}}$$

$$= D_x \sum_{m=1}^{M}\sum_{n=0}^{N}\left[-\left(\frac{2\pi m}{\lambda_x}\right)^2 A_{m,n}(t)\cos F_1(x,y,m,n) - \left(\frac{2\pi m}{\lambda_x}\right)^2 B_{m,n}(t)\sin F_1(x,y,m,n)\right]$$

$$+ D_x \sum_{m=1}^{M}\sum_{n=1}^{N}\left[-\left(\frac{2\pi m}{\lambda_x}\right)^2 C_{m,n}(t)\cos F_2(x,y,m,n) - \left(\frac{2\pi m}{\lambda_x}\right)^2 D_{m,n}(t)\sin F_2(x,y,m,n)\right] \tag{6}$$

$$D_{y,\,center} = D_y \frac{\partial^2 R(x, y, t)}{\partial y^2}\Bigg|_{\substack{x=X_L(t)\\ y=Y_L(t)}}$$

$$= D_y \sum_{m=1}^{M}\sum_{n=1}^{N}\left[-\left(\frac{2\pi n}{\lambda_y}\right)^2 A_{m,n}(t)\cos F_1(x,y,m,n) - \left(\frac{2\pi n}{\lambda_y}\right)^2 B_{m,n}(t)\sin F_1(x,y,m,n)\right]$$

$$+ D_y \sum_{m=1}^{M}\sum_{n=0}^{N}\left[-\left(\frac{2\pi n}{\lambda_y}\right)^2 C_{m,n}(t)\cos F_2(x,y,m,n) - \left(\frac{2\pi n}{\lambda_y}\right)^2 D_{m,n}(t)\sin F_2(x,y,m,n)\right] \tag{7}$$

$$\gamma_{center} = \gamma R(x, y, t)\Big|_{\substack{x=X_L(t)\\ y=Y_L(t)}} \tag{8}$$

where $X_L(t)$ and $Y_L(t)$ are the Lagrangian coordinates of the center of the cell with peak intensity. They are calculated as

$$X_L(t) = X_L(0) + \int_0^t u(\tau)\, d\tau \tag{9}$$

$$Y_L(t) = Y_L(0) + \int_0^t v(\tau)\, d\tau = Y_L(0) \tag{10}$$

where $u(\tau)$ and $v(\tau)$ are the mean velocities for the cell center in the $x$- and $y$-axis directions, respectively. The value of $v(\tau)$ is zero since this is taken in the direction of movement. Further details on the above modeling approach for real-time prediction of urban-scale rainfall is given by Jinno et al. [1992].

## PREDICTION OF INDIVIDUAL RAIN CELLS

Figure 1 shows an example of observed and predicted spatial rainfall intensities in real-time. The observations were done in a 12-gage 25 $km^2$ catchment described by Niemczynowicz [1987]. The observed rainfall intensities in the figure were smoothed by kriging procedures. It is seen that an individual cell advances in over the catchment area in a direction from NW to SE. The model parameters were continuously updated by extended Kalman filtering. Figure 2 shows the parameter identification for the physical parameters $u$ (speed of the rainfall cell in the direction of movement), $D_x$, $D_y$ (apparent diffusion coefficients in the $x$- and $y$-direction, respectively), $\gamma$ (development/decay coefficient of the rainfall intensity), and three of the Fourier series coefficients. Below we will use the rainfall event exemplified in Fig. 1 to in a more detailed way analyze properties of an individual rain cell.

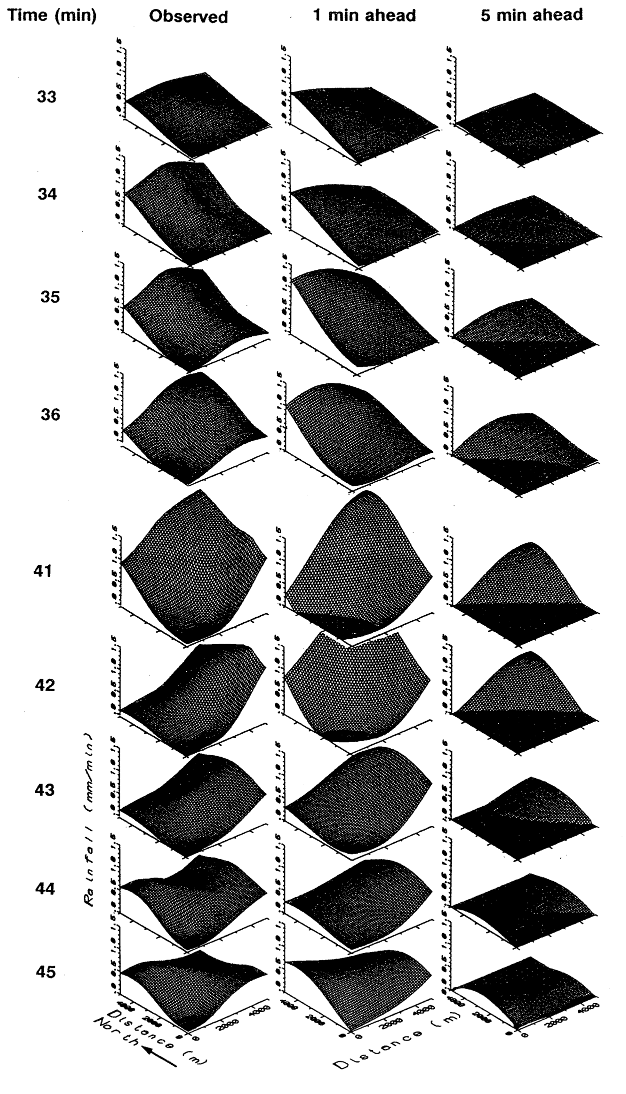


Figure 1 Observed and predicted time series of spatial rainfall distribution for event 1980 08 23 (Jinno et al. [1992]).

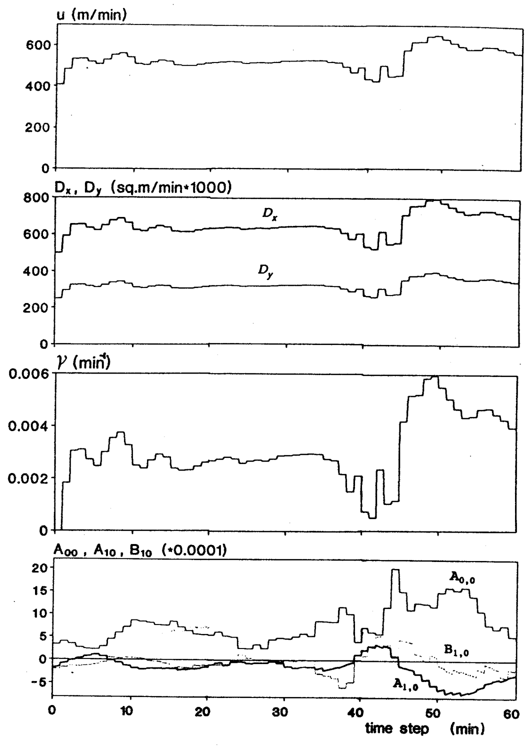


Figure 2 Parameter identification for the 1-hour rainfall event shown in Fig. 1 (Jinno et al. [1992]).

## PROPERTIES OF AN INDIVIDUAL RAIN CELL

Physical interpretation of observed individual rain cells is important in order to model small-scale rainfall variability. However, also for larger-scale modeling, the parameterization and description of individual cells are important for a realistic simulation approach. In this study we use the non-random right-hand side terms of (1) to infer physical properties of an individual rain cell. Consequently, we use a Lagrangian approach for tracking the diffusion terms and the development/decay term, (6)-(10). For the description of the rain cell shown in Fig. 1 we use the identified parameter values shown in Fig. 2. Figure 3 shows an example of comparison of the interpolated rainfall intensities by using these identified parameters. As a comparison it can be mentioned that the root mean-square error was 0.12 mm/min for the interpolated values (Fig. 3) and 0.17 and 0.29 mm/min for 1- and 5-min ahead predictions (Fig. 1), respectively.

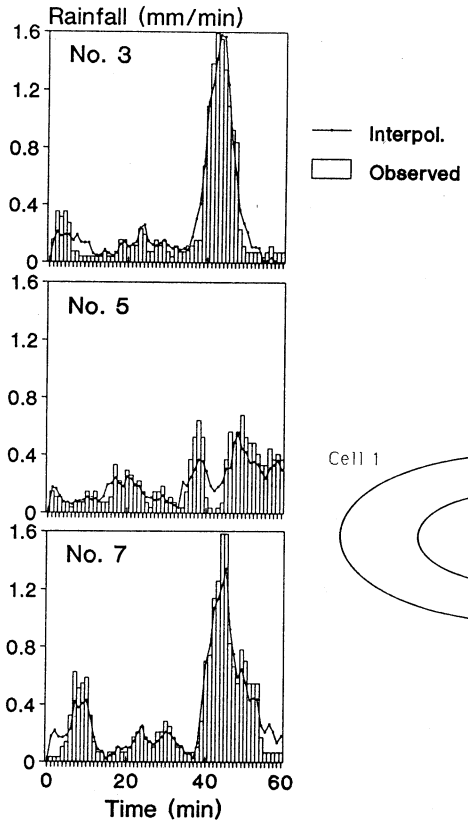


Figure 3 Comparison between observed and interpolated rainfall intensities for the event in Fig. 1 using the parameter values in Fig. 2.

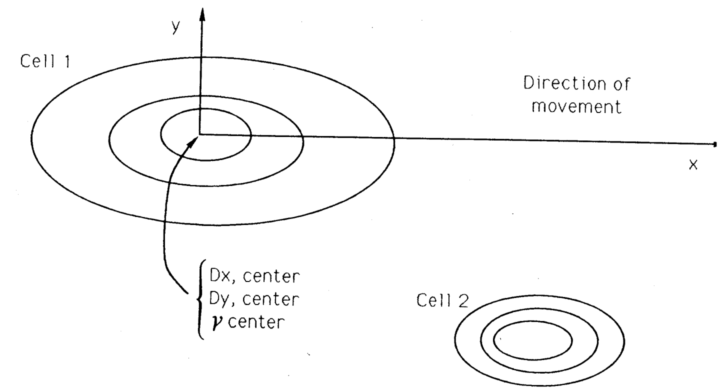


Figure 4 Schematic definition of $D_{x,\ center}$, $D_{y,\ center}$, and $\gamma_{center}$ adopted in the paper.

The use of (6)-(10) is convenient to obtain a physical description of individual cell behavior. Using the Lagrangian approach and an assumed position in the center of the cell we may analyze the turbulent flow field as indicated by the two-dimensional flux of rainfall at ground level. Figure 4 shows a schematic of the definition of $D_{x,\ center}$, $D_{y,\ center}$, and $\gamma_{center}$ which we adopt herein. The values of $D_{x,\ center}$ and $D_{y,\ center}$ should be interpreted as the change in the shape of the cell center with time in the *x*- and *y*-axis directions, respectively. Negative values mean that this shape is convex (peak in rainfall intensity). Similarly, positive values indicate a concave shape (depression in the rainfall intensity field) and that the coordinate center no longer is located in the center of the cell. The term $\gamma_{center}$, similarly, indicates the development/decay (sink/source) of the cell center. Positive values of $\gamma_{center}$ mean a decaying rainfall intensity behavior for the cell. Similarly, negative values mean that the cell is growing or developing in rainfall intensity.

Figure 5 shows calculated values of $D_{x,\ center}$, $D_{y,\ center}$, and $\gamma_{center}$ for the 1-hour rainfall event shown in Fig. 1 and 3. It is seen that the identified cell from the beginning has large negative values of $D_{x,\ center}$ and $D_{y,\ center}$. These values increase in a nonlinear way approaching zero after about 20 min. This means that the convex shape of the rainfall intensity field at the center of the cell is markedly peaked just after the cell birth but after about 20 min approaches constant near-zero values (a flat intensity field). The value of $\gamma_{center}$, on the other hand, is almost zero during the entire cell life. This means that the contribution of the development/decay term is negligible and that only the diffusion terms contribute to the total rainfall intensity as observed at ground level.

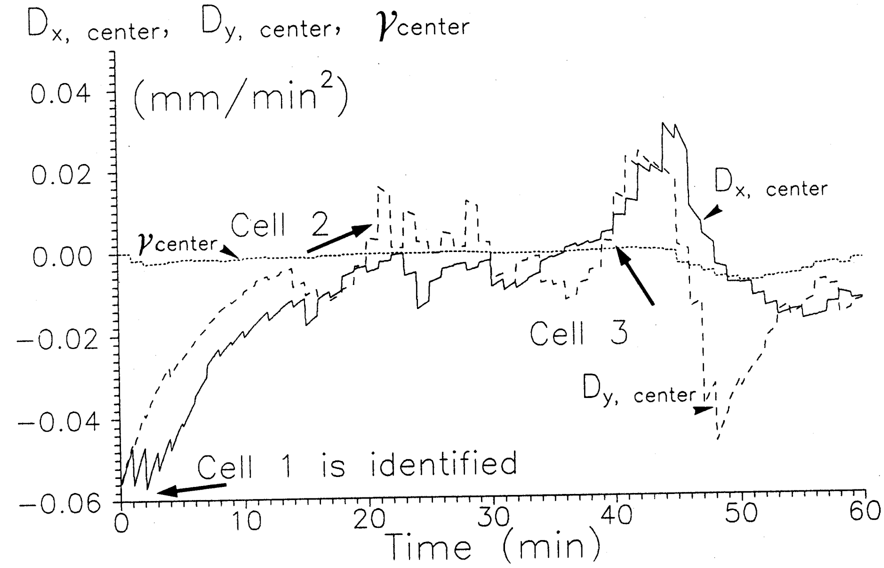


Figure 5 Values of $D_{x,\ center}$, $D_{y,\ center}$, and $\gamma_{center}$ for the 1-hour rainfall event shown in Fig. 1.

After about 35 min after the cell birth, the diffusion terms suddenly turn into large positive values. This indicates that the coordinate center not any longer is located in the center of the cell (large positive values indicating a concave shape). A physical interpretation of this behavior is the existence of more than one cell during the analyzed time period. Consequently, in the near vicinity of the first cell there rather suddenly develops a second cell with a higher rainfall intensity as compared to the firstly identified cell (therefore the concave intensity field). This is seen in Fig. 7 which shows time series of intensity contour lines for the period 13 - 42 min. The contour lines were interpolated by kriging routines. In Fig. 7 a new incoming cell is noticed at time 34 min. This is noticed in Fig. 5 as values of $D_{x,\ center}$ and $D_{y,\ center}$ which rapidly turns into large positive values. Also at time 20 min a new smaller cell is indicated in Fig. 7 (in the SW corner) which also gives rise to temporary positive values of $D_{y,\ center}$.

The occurrence of multiple cells also influences the parameter values of the model. This is seen in Fig. 2 at a time of about 35 min when an incoming major cell affects the updating system.

Figure 6 shows the same values of $D_{x,\ center}$, $D_{y,\ center}$, and $\gamma_{center}$ as in Fig. 5, but after that the coordinate center has been modified to automatically change to the point with highest rainfall intensity. In this case

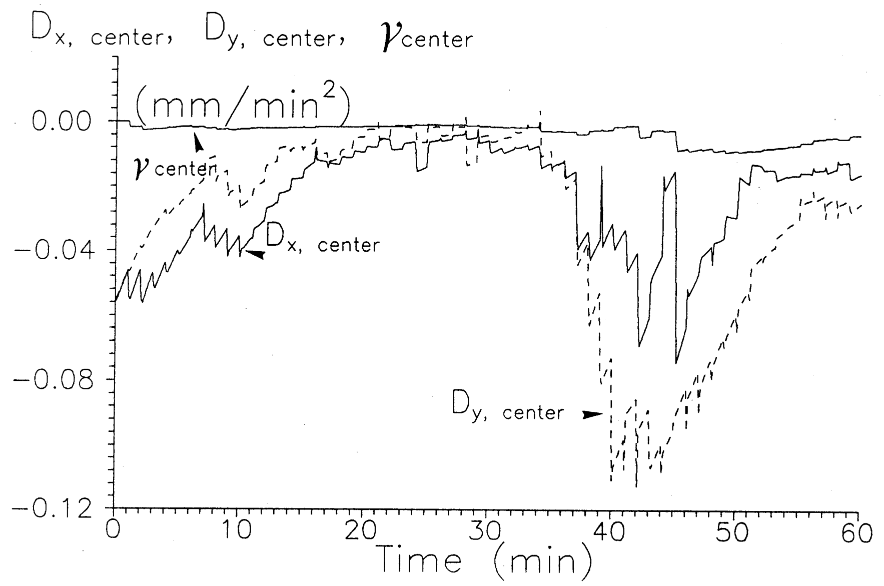


Figure 6 The same values as in Fig. 5, but after automatic change of the coordinate center to the points with highest rainfall intensity.

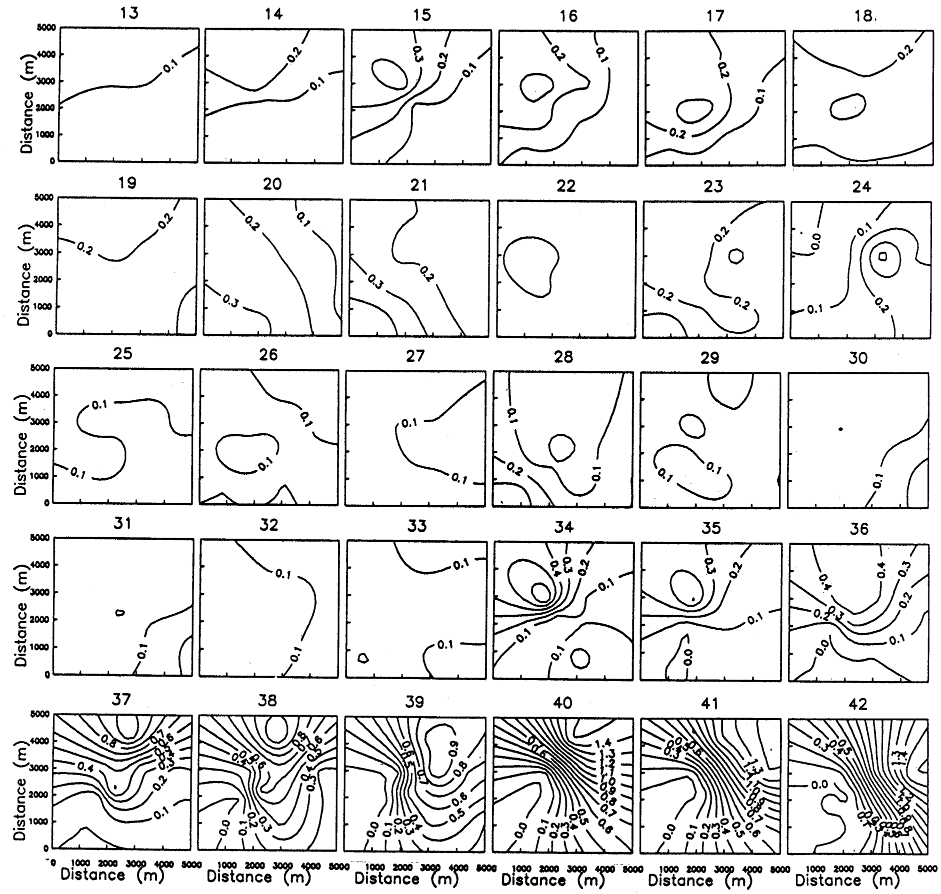


Figure 7 Time series of contour lines for rainfall intensity showing the two cells.

$D_{x,\ center}$ and $D_{y,\ center}$ remain positive. The incoming cell 3 results in large convex values for both diffusion terms. Note that as in Fig. 5, the development/decay term remains almost zero during the entire event.

## SUMMARY AND DISCUSSION

In the present paper we have developed a methodology to separate and to quantify different components of individual rain cells such as apparent diffusion and development/decay resulting from morphological changes. Using a Lagrangian approach the methodology was applied to a 1-hour rainfall event containing evidence of three major cells. It was shown that the major contribution of total rainfall could be explained by the diffusion terms. The development/decay term contributed only marginally to the overall rainfall intensity. It therefore appears that during short periods of time (less than 1 hour) the rain cell behavior may be simulated by using only the diffusion terms. This may be physically explained as a dominant effect of turbulent dissipation also affecting the rain drops. Analyses of more rainfall events are needed to confirm these preliminary results.

The results of this study may be practically utilized when choosing initial parameter values of the presented model. The overall methodology can be used to further study and to parameterize small-scale rainfall variability.

## ACKNOWLEDGEMENT

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