A CONVECTIVE-DISPERSION MODEL FOR REAL-TIME PREDICTION OF URBAN-SCALE RAINFALL

Kenji Jinno*, Akira Kawamura*, Magnus Larson**, Ronny Berndtsson**, and Janusz Niemczynowicz**

* Department of Civil Engineering (SUIKO), Faculty of Engineering, Kyushu University, Fukuoka, Japan

** Department of Water Resources Engineering, University of Lund, Lund, Sweden

ABSTRACT: This paper presents a model for real-time prediction in time and space of urban-scale rainfall under the assumption that the rainfall process is governed by the convective-dispersion equation. In contrast to other real-time models, the proposed model uses an a priori calculated storm speed and direction from a relationship given by Niemczynowicz and Dahlblom (1984), thus reducing the number of parameters to predict. The model is applied to a synthetically generated short-term, high-intensive rainstorm for a dense twelve-gage system in the city of Lund, Sweden. A recursive least-square technique is used for real-time estimation of the parameters in the model.

KEYWORDS: Real-Time Rainfall Prediction, Convective-Dispersion Model, Urban-Scale Rainfall

1. INTRODUCTION

Real-time prediction of urban runoff is of great interest to engineers providing information for online decision-making of how to optimize the operation of sewer systems, minimizing problems such as flooding and combined sewer overflow (e.g., Takasao and Shiiba, 1984). In this context, prediction of the dynamic development of the rainfall field in space and time is required to determine rainfall input to runoff simulation models. Urban runoff may be modeled with an accuracy sufficient for engineering applications if the studied catchment has been properly instrumented to yield enough data for calibration. The prediction of space-time characteristics of the rainfall field on real-time basis, however, has met only limited success so far primarily due to the scarce knowledge of the physical mechanisms which govern the small-scale rainfall development in time and space (Berndtsson and Niemczynowicz, 1988).

Methods for predicting macro-scale quantities associated with rainfall kinematics, mainly concerning the movement of individual rain cells, such as storm speed and direction, are well established (Hindi and Kelway, 1977; Niemczynowicz, 1987; Rodriguez-Iturbe and Eagleson, 1987). Information about expected velocity and direction of rainfall movement may be obtained from meteorological data (Shearman, 1977; Wickerts and Granath, 1979; Niemczynowicz and Jönsson, 1981). Incorporating these quantities into a schematized model of the rainfall field should give predictions of rainfall intensities in time and space reliable enough for engineering use. Urban-scale rainfall, appearing in northern Europe as short-term highly convective storms, develops rapidly and moves with little change in cell shape and intensity distribution over small urban catchments (Niemczynowicz, 1989). A first approach to schematize the rainfall field may be to assume the convective-dispersion equation to govern the space-time characteristics of the rainfall.

2. PHYSICAL CHARACTERISTICS OF SHORT-TERM URBAN-SCALE RAINFALL

When modeling rainfall for use in urban drainage simulation models the smallest space-time rainfall units are of interest, namely the individual convective rainfall cells. Urban catchments in Sweden as well as in the rest of northern Europe, where problems with flooding and combined sewer overflow are common, are usually small and with a dense building structure. The size of these catchments ranges from about 1-5 km^2. The smallest space-time rainfall units are individual cells with an areal extention of about 2-5 km^2. Considering the speed of individual raincells (5-25 m/s according to Niemczynowicz, 1987) it does not, as a first step, seem worthwhile to describe the development stages of each cell but instead to consider the shape of a rainfall cell as approximately constant during its passage over the small catchment. Changes in the cell shape are characterized through the dispersion coefficients and assumed to be sufficiently regular to allow stable estimates of these coefficients in a predictive situation.

Individual convective rainfall cells often display a more or less Gaussian-shaped areal intensity pattern, which has been confirmed both by use of radar and by gages. In a classical study, Zawadzki (1973) found using radar that normalized space autocorrelations of the rainfall intensity displayed elliptical Gaussian-shaped patterns. Similar features were also noted by Marshall (1980) and Sharon (1972) using raingage data. Hobbs and Locatelli (1978) followed the temporal evolution of convective cells embedded within elongated rainfall fields. The individual cells displayed approximately Gaussian-shaped spatial patterns.

Figure 1 shows an example of the kinematics of a raincell observed in the Lund catchment. A typical cell-shaped intensity structure is moving over the catchment area from south to north, readily described by a Gaussian shape. The time resolution between the figures is one minute.

Several researchers have used a Gaussian function to describe the rainfall attenuation in space for a cell (Valdes et al., 1985; Sivapalan and Wood, 1987). Similar observations have been made in Lund (Niemczynowicz and Jönsson, 1981). The model presented in this paper thus employs a Gaussian-shaped raincell which constitutes a solution of the convective-dispersion equation.

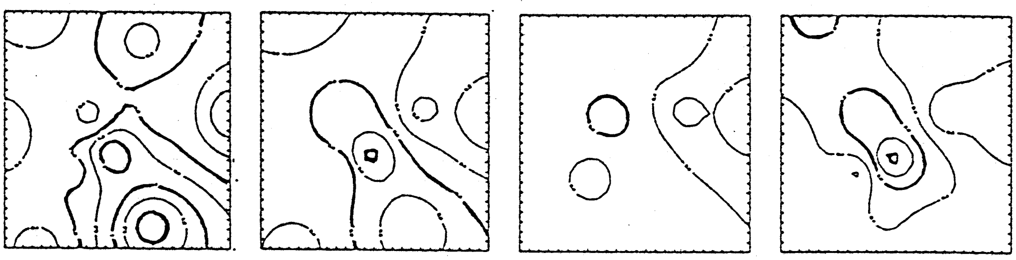

Figure 1. The temporal evolution of a high-intensive rainfall cell over the catchment in Lund (four consecutive observations during four minutes).

3. METHODOLOGY AND MODEL FORMULATION

The evolution of a raincell in time and space is assumed to be governed by the two-dimensional convective-dispersion equation in space. If the x-axis is taken along the direction of movement of the convective rainfall cell, the convective-dispersion equation may be expressed as

$$\frac{\partial R}{\partial t} + u\frac{\partial R}{\partial x} = D_x\frac{\partial^2 R}{\partial x^2} + D_y\frac{\partial^2 R}{\partial y^2} - \sigma R \tag{1}$$

where

- R = rainfall intensity in time and space (m/sec)
- u = velocity of the rainfall cell in direction of movement (m/sec)
- D_x, D_y = dispersion coefficients in the x- and y-direction (m^2/sec)
- σ = decay coefficient of the rainfall intensity (sec^{-1})

For sake of generality (1) also includes a source term which could describe the time evolution of a specific cell. However, the coefficient σ is set to zero in the following simulations.

The solution of (1) is

$$R(x,y,t) = \frac{I}{4\pi(D_xD_y)^{1/2}(t-t_o)} \exp\left[-\frac{\{x-x_o-u(t-t_o)\}^2}{4D_x(t-t_o)} - \frac{(y-y_o)^2}{4D_y(t-t_o)} - \sigma(t-t_o)\right] \tag{2}$$

where

- I = strength of the cell (m^3/sec)
- t_o = birth time of the cell (sec)
- x_o, y_o = initial location of the cell (m)

The rainfall intensity may be predicted at any time and location once the parameters I, u, D_x, D_y, σ, x_o, y_o, and t_o have been identified. Kalman filtering technique is used to identify these parameters. In this case the solution of (2) is non-linear and the extended Kalman filter needs to be applied (e.g., Jinno et al., 1989). The system equation and the observation equation in the Kalman filtering are

$$X(k+1) = \Phi(k)\, X(k) + S(k) + v(k) \tag{3}$$

$$Y(k+1) = H(k+1)\, X(k+1) + F(k+1) + w(k+1) \tag{4}$$

where

- X = system vector to be estimated (matrix size: (n*1))
- Φ = known state transition matrix (n*n)
- S = known constant vector (n*1)
- v = white Gaussian system noise (n*1)
- Y = observation vector (m*1)
- H = known observation matrix (m*n)

F	-	known constant vector (m*1)
w	-	white Gaussian observation noise (m*1)
k	-	time step number

The estimated system vector X is formed as $(I, u, D_x, D_y, \sigma, x_o, y_o, t_o)^T$, where T denotes transposed. Y corresponds to the measured rainfall intensity R, Φ is a unit matrix, $S = 0$, $H = J(X^*)$, where $J(X^*)$ is the Jacobian matrix for R, $F = R(X^*) - J(X^*)X^*$, and $R(X^*)$ is the rainfall intensity represented by (2) with the estimated parameter X^*.

By use of Kalman filtering technique the unknown parameters I, u, D_x, D_y, σ, x_o, y_o, and t_o are estimated and updated at each time step.

The above methodology was employed for a synthetically generated rainfall cell with elliptic shape moving over the existing observation network in the city of Lund. These simulations constituted a first test of the applicability of the convective-dispersion model for rainfall prediction.

4. RESULTS

Short-term high-intensive rainstorms have been observed during many years in a dense twelve-gage system for rainfall observation in the city of Lund (Niemczynowicz and Jönsson, 1981). A first approach to real-time modeling of spatial and temporal variations in rainfall intensity during these short-term events is to use synthetic data. This was done by simulating a convective rainfall cell moving over the existing network in Lund. Figure 2 shows the cell as it moves over the catchment area. The parameters values used in (2) were: $I = 10^4$ (m^3/min), $u = 200$ (m/min), $D_x = 30\ 000$ (m^2/min), $D_y = 10\ 000$ (m^2/min), $\sigma = 0$ (min^{-1}), $x_o = -5\ 000$ (m), $y_o = 2\ 000$ (m), and $t_o = -15$ (min).

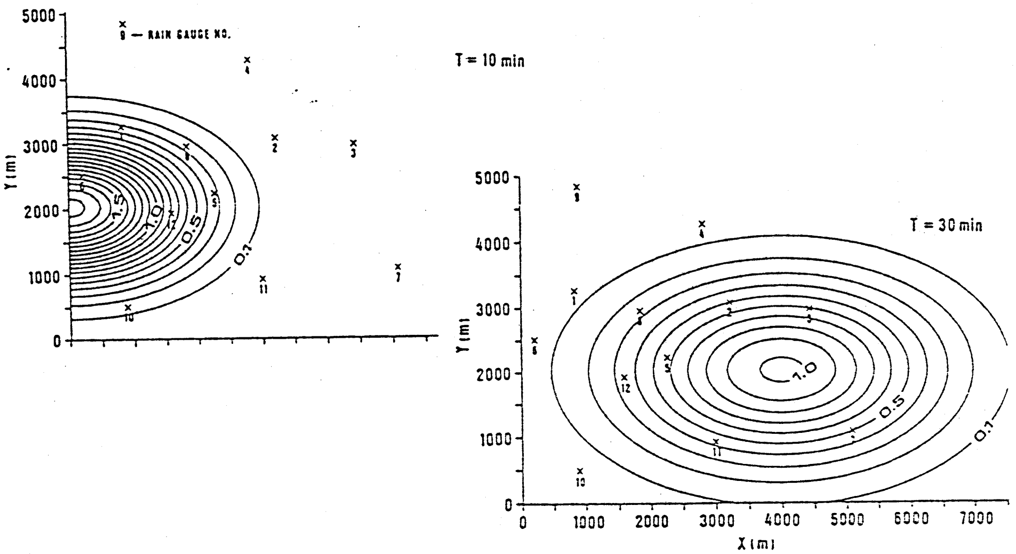

Figure 2. Synthetic rainfall cell used for model calibration moving over the studied catchment.

The methodology previously discussed was applied using both 1-minute and 5-minute ahead real-time prediction of the rainfall intensity and for the estimation of the eight unknown parameters. Figure 3 shows the results of the prediction for three different gages (1, 3, and 5). An observation noise of 0.05 mm/min was added to the synthetically generated rainfall intensities.

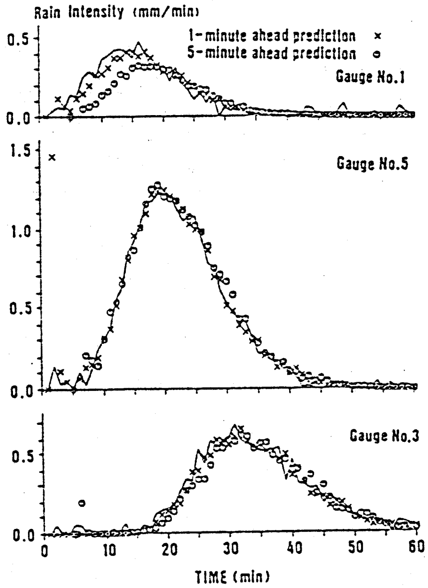

Figure 3. Resulting 1- and 5-minute ahead real-time prediction of rainfall intensity for three gages (the solid line shows the synthetic rainfall with 0.05 mm/min observation noise).

Figure 4 shows the corresponding 5-minute ahead prediction error for the same three gages.

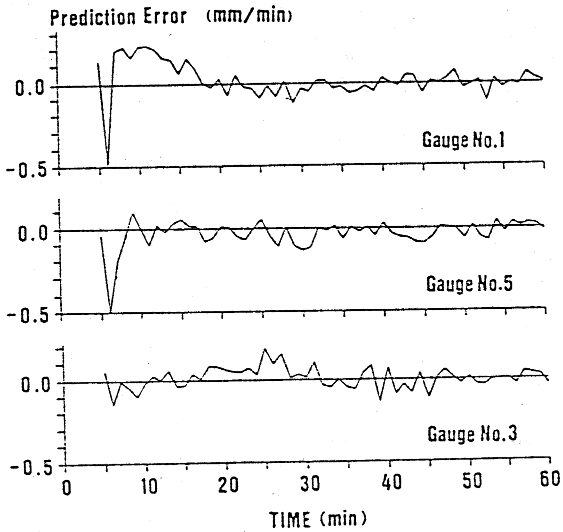

Figure 4. 5-minute ahead prediction error for the studied three gages.

Figure 5 displays the result of the parameter estimation procedure. As initial values of the eight unknown model parameters, I, u, D_x, D_y, σ, x_o, and y_o, were set to 70 % and t_o to 90 % of the true value. The identified parameter values at the end of the simulation are very close to the true values as seen in Figure 5.

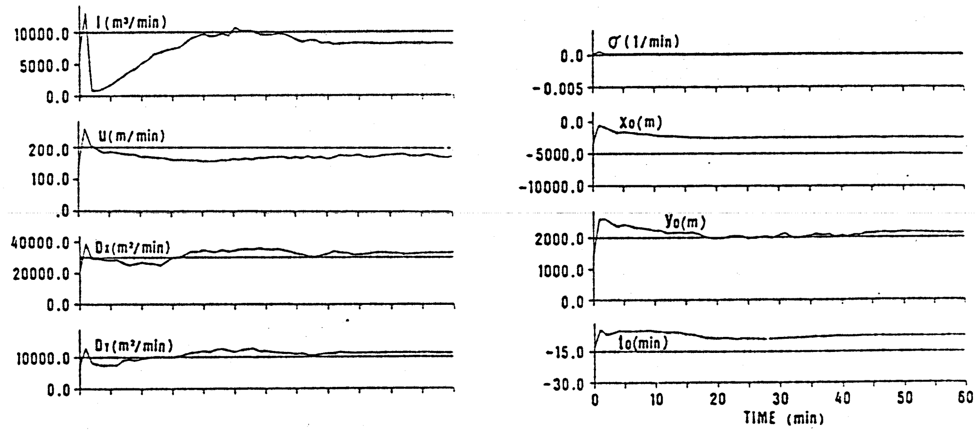

Figure 5. Result of the identification procedure for estimating values of the unknown model parameters.

5. SUMMARY AND DISCUSSION

A general methodology and a model have been presented for real-time prediction of rainfall intensity based on the convective-dispersion equation. Model parameters were identified using Kalman filtering technique. In this paper a synthetically generated rainfall cell moving over an actual catchment area was studied as a first step. The methodology seems to work properly for rainfall prediction. The next natural step is to apply the model to measured data in the Lund catchment.

Some general conclusions may be drawn from the present study:

1) The convective-dispersion equation seems appropriate to use for convective rainfall cell modeling in time and space

2) By using a priori calculated storm speed and direction from empirical relationships it is possible to reduce the number of unknown model parameters which has to be estimated in a predictive mode

3) If actual rainfall data with a sufficiently detailed time resolution (1-or 5-minute values) are available with detailed areal coverage it seems possible to model the raincell characteristics using the convective-dispersion model together with Kalman filtering technique

REFERENCES

Berndtsson, R. and J. Niemczynowicz (1988) Spatial and Temporal Scales in Rainfall Analyses - Some Aspects and Future Perspectives, J. Hydrol., 100, 293-314.

Hindi, W. N. A. and P. S. Kelway (1977) Determination of Storm Velocities as an Aid to the Quality Control of Recording Raingauge Data, J. Hydrol., 32, 115-137.

Hobbs, P. V. and J. D. Locatelli (1978) Rainbands, Precipitation Cores and Generating Cells in a Cyclonic Storm, J. Atmos. Sci., 35, 230-241.

Jinno, K., A. Kawamura, T. Ueda, (1989) Prediction of the Concentration Distribution of Groundwater Pollutants, Groundwater Managem.; Quantity and Quality, Proc. of the Benidorm Symp., Oct., 1989.

Marshall, R. J. (1980) The Estimation and Distribution of Storm Movement and Storm Structure, using a Correlation Analysis Technique and Raingauge Data, J. Hydrol., 48, 19-39.

Niemczynowicz, J. (1987) Storm Tracking using Raingauge Data, J. Hydrol., 93, 135-152.

Niemczynowicz, J. (1989) Rainfall data for urban hydrological applications, Inter. Workshop on Prec. Measurem., WMO, St. Moritz, 4-7 Dec. 1989.

Niemczynowicz, J. and P. Dahlblom (1984) Dynamic properties of rainfall in Lund, Nord. Hydrol., 15, 9-24.

Niemczynowicz, J. and O. Jönsson (1981) Extreme Rainfall Events in Lund 1979-1980, Nord. Hydrol., 12, 129-142.

Rodriguez-Iturbe, I. and P. S. Eagleson (1987) Mathematical Models of Rainstorm Events in Time and Space, Water Resour. Res., 23, 181-190.

Sharon, D. (1972) Spatial analysis of rainfall data from dense networks, Hydrol. Sci. Bull., 17, 291-300.

Shearman, R. J. (1977) The Speed and Direction of Movement of Storm Rainfall Patterns, Meteorol. Office, Rep. No. UDC 551.513.43, Bracknell, Berkshire.

Sivapalan, M. and E. F. Wood (1987) A Multidimensional Model of Nonstationary Space-Time Rainfall at the Catchment Scale, Water Resour. Res., 23, 1289-1299.

Takasao, T., and M. Shiiba (1984) Development techniques for on-line forecasting of rainfall and flood runoff, Natural Disaster Science, 6, 83-112.

Valdes, J. B., I. Rodriguez-Iturbe and V. K. Gupta (1985) Approximations of Temporal Rainfall from a Multidimensional Model, Water Resour. Res., 21, 1259-1270.

Wickerts, S. and S. Granath (1979) Measurements of Rainfall Intensity, National Defence Res. Inst., Rep. C 30157-E2, Stockholm.

Zawadzki, I. I. (1973) Statistical Properties of Precipitation Patterns, J. Applied Meteorol., 12, 459-472.