On the Application of Self-Tuning Controller to Reservoir Control Problems

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ABSTRACT

The self-tuning controller (STC) of Clark and Gawthrop (1975) is used to control the weir gates' openings to maintain a certain water level in the reservoir dammed by the weir gates for domestic, agricultural and industrial water supplies. The Kalman filter is used to estimate, in real-time, the parameters needed to compute the optimal control strategy. Using data of the actual operation of the weir gates during periods of high inflows at Onga River, Kyushu Island, Japan, the performance of STC is compared with that of the existing control strategy. Some interesting characteristics of the use of STC to the control of weir gates' openings are presented and discussed.

INTRODUCTION

In Japan, many weir gate structures are constructed in tidal inlets (river mouths) because feasible sites for reservoir construction in the upstreams are limited. The estuarial reservoirs dammed by weir gates are abstracted for domestic, agricultural and industrial water supplies. Since river flows into the reservoir, variation of the reservoir water surface levels is expected, and releases under the gates are made to maintain a certain water level. This water level is required to prevent sea water from entering the reservoir and to avoid drainage problems due to hydraulic connection between reservoir and aquifer in agricultural lands beside the reservoir. Automatic feedback control can greatly enhance the efficiency of operation of gates and increase the benefits associated with their use. With automatic controllers, gates' openings are optimally determined so that the energy consumed in operating the gates are minimized and unnecessary depletion of the reservoir is avoided.

Automatic controllers presently in operation employ readily available inexpensive computer systems and on-line observation systems using telemeters. An existing automatic controller uses data of upstream and downstream water levels, present gates' openings, and river discharge at a nearby upstream station from on-line observation system. These data are entered into the computer system where the gates' openings are decided according to a control strategy; computer commands are then carried out to operate the gates. However, the control strategy is not very efficient in treating the stochastic nature of the system. It requires that the inflow be accurately measured or forecasted and that the system be accurately modelled as the deviation of reservoir water surface level from the required one is not used to correct any modelling errors. Also, it is not adaptive, i.e., if there are changes in the system, the control strategy will be incorrect. Failure to take into account the stochastic and dynamic properties of the system in designing the control strategies has resulted in largely unsatisfactory and sometimes even unstable control. An alternate procedure for designing control strategies is therefore needed.

Adaptive control systems automatically adjust controller parameters on-line in response to changes in the system dynamics or noise characteristics or both. This study demonstrates the application of a stochastic adaptive controller, the self-tuning controller (STC) of Clarke and Gawthrop (1975), for the real-time control of weir gates' openings. STC consists of an on-line recursive system identification algorithm and a feedback control law whose parameters depend on the currently estimated system model. In this paper, the system to be controlled is represented by an ARMAX-type model. The system input, which is the control variable, is the total release under the gates, while the system output is the reservoir water surface level; the uncontrollable but observable inflow and abstractions are the exogeneous variables. The total release under the gates is determined by the feedback control law so that the resulting reservoir water surface level is equal to the required one.

The same gate opening control problem for lock and dam has been discussed by Duong, et al. (1978) using one type of self-tuning regulator (Astrom and Wittenmark, 1973) reported by Wieslander and Wittenmark (1971). Though the application of STC to the optimal control of groundwater abstraction for river flow augmentation has already been discussed by O'Connell (1980), as far as we know, its availability to other control problems in water resources engineering has not yet been investigated.

The Onga River weir gates is chosen to demonstrate the effectiveness of the technique. Specifically, a comparison is made between the existing control strategy and the STC-designed control strategy. Results indicate that the STC design technique can be more effective. The work described in this paper is a preliminary step to actual field demonstration of this optimal control concept.

SELF-TUNING CONTROLLER

This section outlines the derivation of an STC algorithm for the system described by an ARMAX-type model, following the derivation of O'Connell (1980) and Clark and Gawthrop (1975).

$$A(q^{-1})y(t) = B'(q^{-1})Q(t-k) + C'(q^{-1})I(t-k) + D'(q^{-1})L(t-k) + E(q^{-1})v(t) \quad (1)$$

where $y(t)$ is the system output (reservoir water surface level), $Q(t)$ is the system input or control variable (total gate release), $I(t)$ (inflow) and $L(t)$ (abstractions) are uncontrollable (but observable) inputs, $v(t)$ is the uncorrelated zero-mean random sequence, and t and k denote the sampling instant and the pure time delay between input and output respectively. The polynomials $A(q^{-1})$, $B'(q^{-1})$, $C'(q^{-1})$, $D'(q^{-1})$ and $E(q^{-1})$ are expressed in terms of the backward shift operator, q^{-1}

$$A(q^{-1}) = 1 + a_1q^{-1} + \cdots + a_nq^{-n}$$

$$B'(q^{-1}) = b_0 + b_1q^{-1} + \cdots + b_nq^{-n}$$

$$C'(q^{-1}) = c_0 + c_1q^{-1} + \cdots + c_nq^{-n}$$

$$D'(q^{-1}) = d_0 + d_1q^{-1} + \cdots + d_nq^{-n}$$

$$E(q^{-1}) = 1 + e_1q^{-1} + \cdots + e_nq^{-n}$$

where n is the order of the system.

Using the identity

$$G = (E - AF)q^k \quad (2)$$

where F and G are polynomials of order k-1 and n-1 respectively.

$$F(q^{-1}) = 1 + f_1q^{-1} + \cdots + f_{k-1}q^{1-k} \quad (3)$$

$$G(q^{-1}) = g_0 + g_1q^{-1} + \cdots + g_{n-1}q^{1-n} \quad (4)$$

The optimal k-step predictor for the system equation (1) is given by eq.5 which is derived following the minimum square error predictor of Astrom (1970).

$$\hat{y}(t+k|t) = \frac{G}{E}y(t) + \frac{B'F}{E}Q(t) + \frac{C'F}{E}I(t) + \frac{D'F}{E}L(t) \quad (5)$$

The prediction error is given by

$$\varepsilon(t+k) = Fv(t+k) \quad (6)$$

The objective of STC is to predict the input $Q(t)$ such that the system output $y(t+k)$ is equal to a target value or 'set point'. The cost function considered is of the form

$$J = \mathrm{E}\,\{[y(t+k) - y^*(t+k)]^2 + \lambda^*[Q(t) - Q(t-1)]^2\} \quad (7)$$

Here E is the expectation operator, $y^*(t+k)$ is the set point which is the target reservoir water surface level, and the factor λ^* is used to stabilize the controller by penalizing the control effort, defined as the change in the control variable from one time step to the next (O'Connell, 1980).

Unknown at time t, $y(t+k)$ must be set equal to $\hat{y}(t+k|t)$ (eq.5) with prediction error $\varepsilon(t+k)$ (eq.6), as

$$y(t+k) = \hat{y}(t+k|t) + \varepsilon(t+k) \quad (8)$$

and the cost function becomes

$$J = \mathrm{E}\,\{[\hat{y}(t+k|t) + \varepsilon(t+k) - y^*(t+k)]^2 + \lambda^*[Q(t) - Q(t-1)]^2\} \quad (9)$$

Taking the mathematical expectation and setting the derivative of J with respect to $Q(t)$ equal to zero to obtain the minimum cost:

$$\{\hat{y}(t+k|t) - y^*(t+k)\}\frac{\partial\hat{y}(t+k)}{\partial Q(t)} + \lambda^*\{Q(t) - Q(t-1)\} = 0 \quad (10)$$

Using $\partial\hat{y}(t+k|t)/\partial Q(t) = b_0$ (from eq.5) and $\lambda^* = b_0\lambda$, the control law is given by:

$$\hat{y}(t+k|t) - y^*(t+k) + \lambda\{Q(t) - Q(t-1)\} = 0 \quad (11)$$

The choice of a suitable value of λ is very important and is discussed later in the paper.

Defining a generalized output function ψ such that

$$\psi(t+k) = y(t+k) - y^*(t+k) + \lambda\{Q(t) - Q(t-1)\} \tag{12}$$

In order to predict $\psi(t+k)$, $y(t+k)$, unkown at time t, must be set equal to $\hat{y}(t+k|t)$ so that

$$\hat{\psi}(t+k|t) = \hat{y}(t+k|t) - y^*(t-k) + \lambda\{Q(t) - Q(t-1)\} \tag{13}$$

with output prediction error equal to $\varepsilon(t+k)$ (eq.6), i.e.

$$\hat{\psi}(t+k) = \hat{\psi}(t+k|t) + \varepsilon(t+k) \tag{14}$$

Therefore the minimization of J is the same as setting the prediction of ψ equal to zero at every time step so that the control law (eq.12) is satisfied. Now, eq.6 is substituted into eq.14 to obtain

$$E\hat{\psi}(t+k|t) = Gy(t) + BQ(t) + CI(t) + DL(t) - Ey^*(t+k) \tag{15}$$

where $B = B'F + \lambda E\{1 - q^{-1}\}$, $C = C'F$ and $D = D'F$.

If $\hat{\psi}(t+k|t)$ is forced to zero at every time step, then rearrangement of eq.15 yields the feedback control law that minimizes the variance of $\psi(t)$

$$Q(t) = -\frac{1}{B}\{Gy(t) + CI(t) + DL(t) - Ey^*(t+k)\} \tag{16}$$

Unlike the control law used by Duong, et al. (1978), the self-tuning controller (eq.16) considers a variable set-point which is incorporated directly in the cost function.

In order to implement the feedback control law, the coefficients of the polynomials B, C, D, E and G can be estimated by Kalman filter for the process

$$\theta(t+1) = \theta(t) \tag{17}$$

$$\psi(t) = H(t)\theta(t) + \varepsilon(t) \tag{18}$$

where the state and observation vectors are

$$\theta^T(t) = \{b_0, \cdots, b_{j-1}, c_0, \cdots, c_{l-1}, d_0, \cdots, d_{m-1}, -e_0, \cdots, -e_{p-1}, g_0, \cdots, g_{s-1}\} \quad (19)$$

$$H(t) = \{Q(t), \cdots, Q(t-j+1), I(t), \cdots, I(t-l+1), L(t), \cdots, L(t-m+1), y^*(t+k), \cdots, y^*(t+k-p-1), y(t), \cdots, y(t-s-1)\} \quad (20)$$

The parameters j, l, m, p and s are the number of terms in the respective polynomials B, C, D, E and G. The measurement equation (18) results from the substitution of eq.15 into eq.14 as

$$\psi(t+k) = H(t)\theta(t) - \{E - 1\}\hat{\psi}(t+k|t) + \varepsilon(t+k) \quad (21)$$

with $\hat{\psi}(t+k|t)$ assumed to be zero. At every time step t, $\psi(t)$ is computed as

$$\psi(t) = y(t) - y^*(t) + \lambda\{Q(t-k) - Q(t-k-1)\} \quad (22)$$

which is considered as the measured value of ψ at time t.

The control law (16) uses the estimated parameters $\hat{\theta}(t)$ at each sample instant t as if they had the true values.

EXAMPLE SYSTEM SIMULATION STUDY

The example discussed below illustrates the behaviour of STC and compares its performance with that of the existing control strategy. Both existing and STC-designed control strategies involve the estimation of a target gate release which is the basis for calculating the opening heights of the gates.

The Onga River is located in Fukuoka Prefecture, Kyushu Island, Japan. It has a drainage basin area of 938.6 sq. km. at the weir gate structure and a length of 61 km. The average annual rainfall in the basin is 1500 to 1700 mm in the plains and 2500 to 2700 mm in the mountains. The weir gate structure is located at 2.0 km from the Sea of Hibiki. The reservoir dammed by the weir has a total capacity of 11,140,000 cu. m. and 8,840,000 cu. m. of which is the effective volume. It has a surface area of 2.94 sq. km. and a length of 9.39 km. At 6.85 km from the weir, 172,800 cu. m. of water is being abstracted daily for municipal and industrial water supplies. The weir gate structure has a total length of 517 m and is composed of seven main gates and two auxilliary gates for small flows and fishway. The control of the weir gates' openings is done every 10 min. Considering the shortness of the period between adjustments, an automatic controller is justifiably important.

The existing control strategy deterministically estimates the target gate release as

$$Q(t) = g\,[y(t) - y^*(t)] + I(t) - L(t) \qquad (23)$$

where

$$g = [aI(t) + b]\left[\frac{B|I(t) - I(t-3)|}{I(t)} + 1\right] \qquad (24)$$

For $I(t) \leq 24$ cu. m/s, $a = 0$, $b = 200$ and $B = 0$; for 24 cu. m/s $\leq I(t) \leq 300$ cu. m/s, $a = 4.71$, $b = 87$, and $B = 3$; and for $I(t) > 300$ cu. m/s, $a = 3.57$, $b = 428.6$ and $B = 3$. The target reservoir water surface level $y^*(t)$ is at El. 1.45 m for $I(t) > 24$ cu.m/s and at El. 1.485 m for $I(t) \leq 24$ cu. m/s. If $I(t) \geq 1200$ cu. m/s, the control is terminated, and the gates are fully opened.

In applying STC to Onga River weir gates operation, the system model considered is

$$y(t) = a_1 y(t-1) - b_0 Q(t-k) + c_0[I(t-k) - L(t-k)] + v(t) \qquad (25)$$

and the cost function to be minimized is

$$J = \mathrm{E}\,\{[y(t+k) - y^*(t+k)]^2 + \lambda^*[Q(t) - Q(t-1)]^2\} \qquad (26)$$

giving a generalized function

$$\psi(t) = y(t) - y^*(t) + \lambda\{Q(t) - Q(t-1)\} \qquad (27)$$

The optimal control law is

$$Q(t) = \frac{1}{b_0}\{g_0 y(t) + c_0[I(t) - L(t)] - e_0 y^*(t+k)\} \quad (28)$$

Here the number of parameters to be estimated is four, $\theta^T = \{b_0\ g_0\ c_0\ e_0\}$. In executing the Kalman filter algorithm, the initial estimates are $b_0 = c_0 = 0$, $e_0 = -1$, and $g_0 = 1$; the diagonal elements of the error covariance matrix $P(0|0)$ equal 0.1 and off-diagonal elements equal 0.05 (except for the elements of the row and column corresponding to e_0, which are zeros); and the measurement noise $W = 0.1$. In the observation vector, $H(t-k) = \{Q(t)\ y(t-k)\ [I(t-k) - L(t-k)]\ y^*(t)\}$, $Q(t)$ is the known gate discharge at time-step t and is to be adjusted to correct the present water level $y(t)$, and $I(t-k)$, $L(t-k)$ and $y(t-k)$ are the observed inflow, abstraction and reservoir water surface level at time-step $t-k$ respectively. In eq.28, $Q(t)$ is the "adjusted" gate discharge or the target gate release. The time delay $k = 1$, and the generalized function $\psi(t)$ is determined using eq.27. One time-step equals 10 min.

Since the parameters b_0 and g_0 will be estimated as numerically equal but opposite in sign to c_0 and e_0 respectively, the STC-target gate release is computed simply as

$$Q(t) = \frac{1}{b_0}\{y(t) - y^*(t+k)\} + I(t) - L(t) \quad (29)$$

where $I(t)$, $L(t)$ and $y(t)$ are the observed inflow, abstraction and reservoir water surface level at time-step t respectively.

To obtain a quantitative measure of the performance of the controllers, the sum of $\{y(t)-y^*(t)\}^2$ is calculated for a period beginning after simulation starting transients have died out. A decision on the quality (goodness) of the controller is related to the lowest value of this sum.

For this simulation study, we select the period with high inflows since it is the most critical stage in the control of gates' openings. Both existing and STC designed control strategies are applied to three floods on June 23-29, 1985 (Flood1), July 2-4, 1985 (Flood2) and July 4-6, 1983 (Flood3).

RESULTS AND DISCUSSION

This section presents the results of the simulation study; these results illustrate the properties of the application of STC algorithm to the control of weir gates' openings. In particular, the effect of the coefficient λ on the performance has been studied for the three different flood samples. Figures 1-3 contain the graphs of $Q(t)$ and $y(t)$ for each flood; observed values are in full line and values associated with the control strategy are in broken line.

Because of the nonstationarity of river flows, changes in the parameters can not be avoided. According to Clarke and Gawthrop (1975), a small error in the parameter vector is rapidly propagated into poor control, and this can further bias future parameter estimates. This excessive sensitivity may be reduced by appropriate choice of λ. In this study, a threshold of $I(t)$ = 300 cu. m/s is set to account for the nonstationarity of the river flows, and two values of λ are used, λ_1 for $I(t) \leqq$ 300 cu. m/s and λ_2 for $I(t) >$ 300 cu. m/s. However, λ can be varied on-line to affect the closed-loop performance without changing the estimated $\hat{\theta}(t)$, as suggested by Clarke and Gawthrop (1975). In this study, various combinations of λ_1 and λ_2 are considered. The effect of the choices of λ_1 and λ_2 on the sum of $\{y(t)-y^*(t)\}^2$ over the flood period is summarized in Table 1 for each flood.

It can be seen from Table 1 that the best compromise values of λ_1 and λ_2 for this system are 0.003 and 0.001 respectively and that the

Table 1 Effect of λ. Symbol * means diverge.

λ_1	λ_2	Flood1	Flood2	Flood3
0.003	0.0005	0.1268	6.4885	0.3135
0.003	0.001	0.1578	0.0163	0.0431
0.003	0.002	0.3986	0.1320	0.0573
0.003	0.003	0.5051	0.1891	0.0778
0.001	0.001	9.3036	3.5005	*
0.002	0.001	0.9705	*	0.4559
0.004	0.001	0.2364	0.0673	0.0493
Existing Control		0.3296	0.1366	0.1269

performance of STC with these values of λ is much better than that of the existing control strategy. Fig.1 shows the performance of STC for these values of λ, while Fig.2 demonstrates the performance of the existing control strategy. Comparison of the graphs of $y(t)$ (broken-line) for each flood in both figures indicates that better control can be achieved using STC than employing the existing control strategy. Fig.3 illustrates that, if the STC-target gate release is perfectly followed (i.e., $Q(t)$ in $H(t-k)$ equals the STC-target gate release at time-step $t-k$), $y(t)$ will be practically equal to $y^*(t)$. This suggests that the excessive STC-target gate release shown in Fig.1 can be avoided or atleast minimized.

Fig.4 shows the behaviour of the estimates of the controller parameters, where the variations of the parameter estimates (as shown in this figure) are typical of a self-turning algorithm.

The target gate discharge $Q(t)$ has to be aggregated; for a value of $Q_m(t)$, a corresponding value of $\phi_m(t)$ is calculated using the orifice equation, where $Q_m(t)$ is the discharge under gate m and $\phi_m(t)$ is the mth-gate opening height.

CONCLUSIONS

STC has been found to be well suited for the optimal control of weir gates' openings. The performance of STC has been found by simulation studies to be superior to that of the existing control strategy. The satisfactory results shown in this study could serve as a basis for more applications of STC to water flow and water level control problems arising in several water resources systems, water distribution networks and sewer control systems. Especially in the new application area of sewer network control (Papageorgiou and Messmer, 1985), the design of efficient and reliable water flow and water level control systems seems to be of great importance. In general, the simulation study has shown some interesting characteristics of the use of STC to the control of weir gates' openings.

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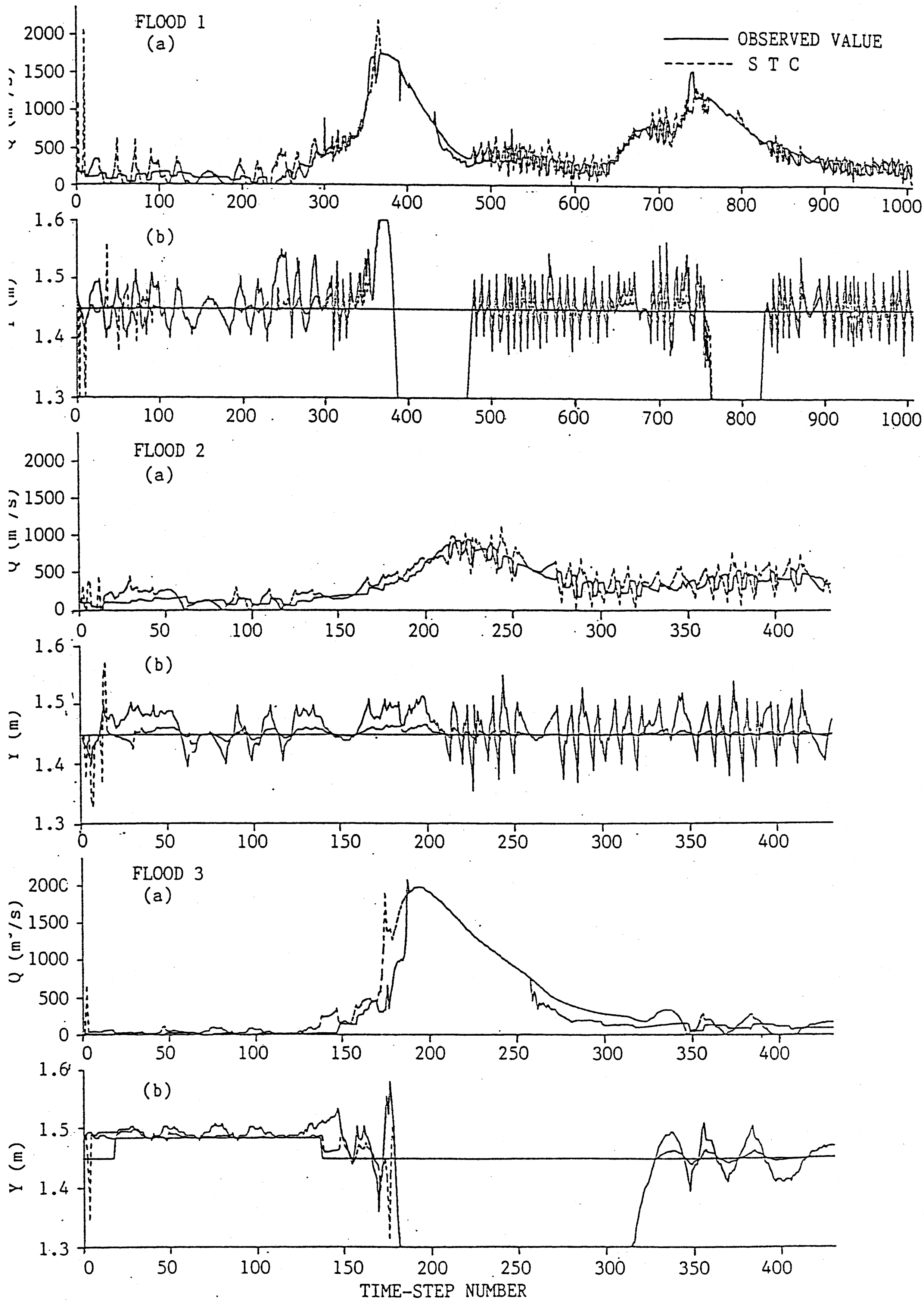

Fig.1 Performance of STC for λ_1=0.003 and λ_2=0.001.

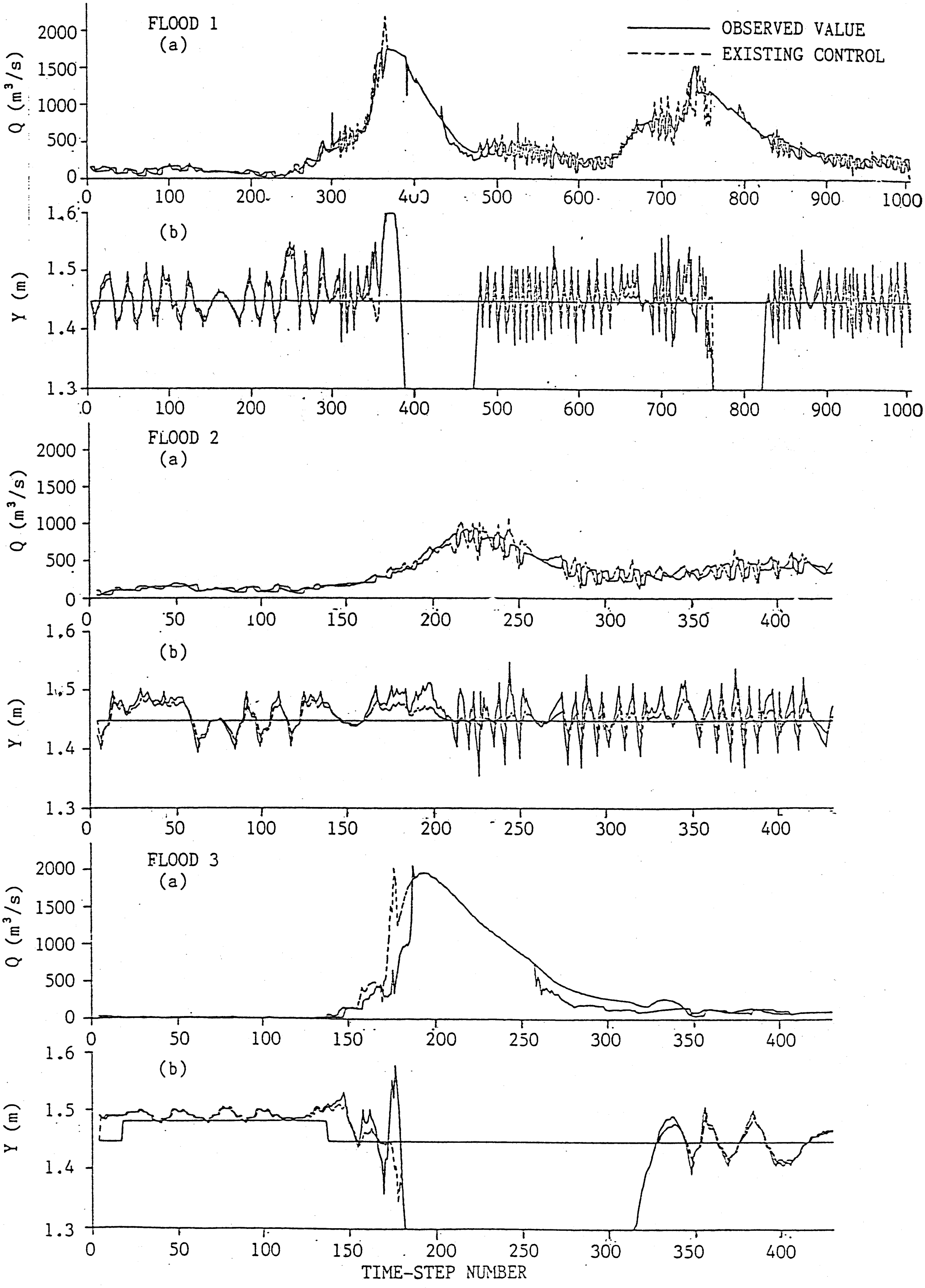

Fig.2. Performance of the existing control strategy.

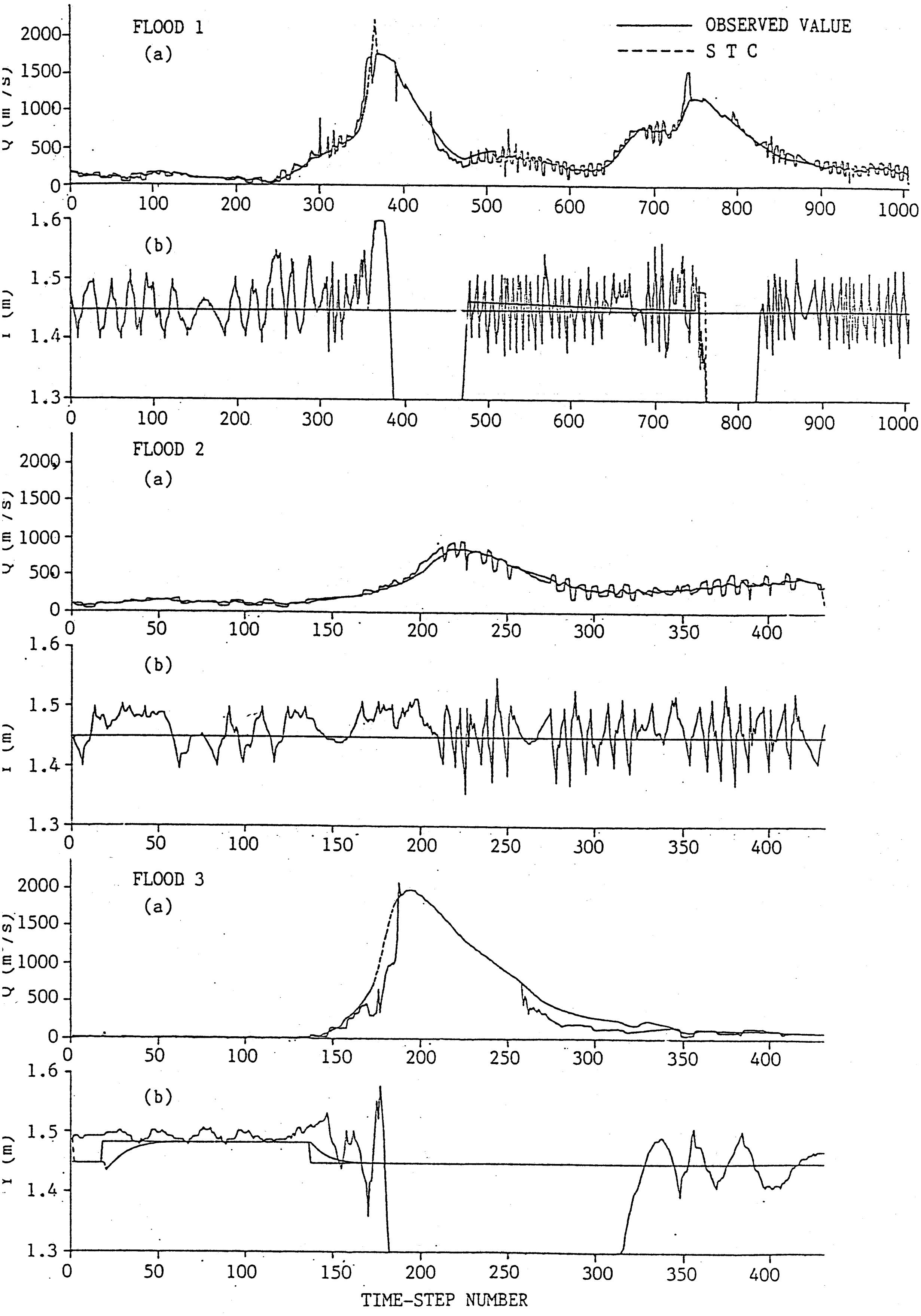

Fig.3 Performance of STC if the target gate release is perfectly followed.

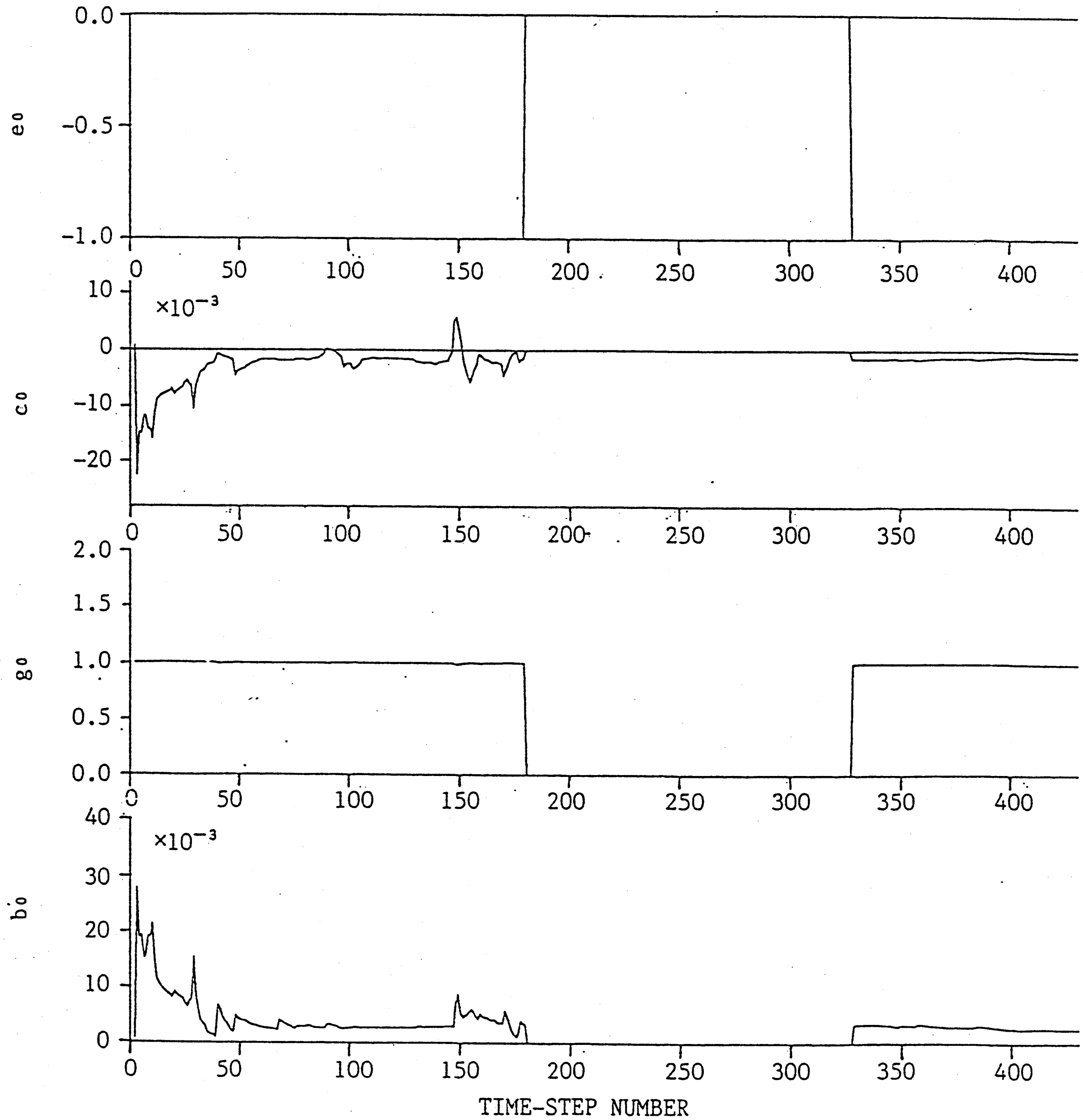

Fig.4 Variations of STC parameters (Flood 3).