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MODELLING AND PREDICTION OF THE TIDAL RIVER WATER SURFACE LEVELS DURING TYPHOON PERIODS

by

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ABSTRACT

This paper describes techniques for the modelling and prediction of water levels when flood passes through the tidal reach of a river during typhoon periods. Two simple time series models, a periodic function model and an ARMAX model, are used to describe the water-level variations at different locations in the tidal reach. The Kalman Filter is used to automatically update the estimates of model coefficients or make forecasts recursively, based on the latest information. These techniques are applied to the problem of predicting the water levels in the tidal reach of the Sendai River, Kyushu, Japan. Also, evaluation of the rate at which the water level rises and specification of noise covariances are discussed.

INTRODUCTION

Japan is frequently visited by typhoons. During these typhoon periods, there are possibilities that storm surge running up the tidal reach of a river may occur simultaneously with the passage of flood, which should make the water levels in the reach higher than those due to either flood or surge alone. Many of these tidal reaches are inside the cities. Based on experience in past typhoon disasters, Hashino and Kanda (1985) have noted that the designs of levees along river channels still lack special consideration on this concurrence. In order to protect lives and properties against such disasters, vigilance should therefore be exercised. In this case, an accurate real-time prediction of the tidal river water level during typhoon period appears indispensable. In addition, the rate at which water level rises should be evaluated, since this could also be a crucial information for control of the river and for the civil defense people who have to decide when to evacuate people.

Subjects related to the modelling of tidal river water levels during typhoon periods have been investigated by a number of researchers (Okamoto, 1940, Unoki, 1969, Yano, 1969, and Kanda and Hamamura, 1986). Hashino and Kanda (1985) have treated the case of typhoon-induced concurrence of flood flow and storm surge in a river through numerical simulations, giving valuable insight into the characteristics of the water surface level distribution along the tidal reach during the concurrence. However, these researches are confined to tidal reach with simple configuration and constant water depth. The treatment usually requires solution of the partial differential form of momentum and continuity equations and assumption of simplified boundary conditions. Numerical model based on approximate solution of the two partial differential equations may be difficult to implement for real-time use. It may consume considerable computer time and may also entail a complicated initial procedure for the estimation of parameters (O'Connell, 1980).

River basin systems throughout Japan are equipped with river water level observation systems using telemeters. On-line information from these observation systems can be fully utilized to make real-time water level forecasts. In this paper, a periodic function model and an ARMAX model are coupled with the Kalman Filter algorithm to make forecasts of water levels at different locations in the tidal reach, based on the latest information. ARMAX is an autoregressive moving average (ARMA) time series model with exogeneous input. Applications of the Kalman Filter to periodic function and ARMAX models have been used in streamflow forecasting (see, for example, Sen, 1980, and Ngan and Russell, 1986). The modelling and prediction techniques are applied to the forecasting problem of water levels in the tidal reach of Sendai River, Kagoshima Prefecture, Kyushu, Japan.

BACKGROUND OF THE FORECASTING PROBLEM IN THE SENDAI RIVER BASIN

The Sendai River has a drainage area of 1409 sq. km. at the main gauging station at Sendai. Fig. 1 shows the main features of the river basin. The tidal reach extends up to Onobuti Gauging Station. The operation of the Tsuruta Dam during flood periods is a practically important problem, because with the absence of big tributaries, it alone influences the behaviour of the water levels in the tidal reach. Along this river, people live, with the biggest population center at Sendai, and rice fields exist. Hence, incorrect operation of the relatively small reservoir may cause severe damage. In fact, operation of the dam during flood period is constrained such that the rise of the water level in the tidal reach does not exceed 30 cm. per 30 min. This rate will provide enough time for people to be warned and evacuated. However, prediction of the water level fluctuations due to dam operation is generally made subjectively, and there are occassions when the water level rises more than 60 cm in one hour. Although much of the tidal reach is protected by dikes, high water levels cause worries of possible flooding. Accurate water level forecasts, even one hour ahead, are valuable during typhoon seasons.

CHARACTERISTICS OF THE TIDAL REACH DURING TYPHOON PERIODS

The characteristics of the tidal reach are obtained by means of spectral analyses of the water level hydrographs at different locations in the reach. Fig. 2 shows the power spectra of the water level hydrographs at Kumisaki, Takaeomote, Sendai and Onobuti Gauging Stations during the typhoon in August of 1971. These power spectra are obtained by maximum entropy method (MEM). The power spectrum corresponding to a frequency of 12 hours is associated with the astronomical tides, which is the semidiurnal tidal frequency. This frequency component is dominant in Kumisaki and Takaeomote, which are located at the lower end of the tidal reach, and is drowned in Sendai and Onobuti at the upper end of the reach. The peaks in the power spectra at zero frequency represent the temporal variations in mean due to the passage of flood. These peaks are so dominant in Sendai and Onobuti that they drown the semidiurnal frequency. Moreover, the peaks at periods longer than 24 hours in the power spectra of the hydrographs in Kumisaki and Takaeomote are so close to zero that they also appear as temporal variations in mean. An approximately 6-hour oscillation is also detected in Kumisaki and Takaeomote.

The above results can be summarized as follows. During the passage of flood, the water level at the lower end of the tidal reach is very much influenced by tidal motion. At the upper end, the tidal oscillation is easily drowned, and the water level is dominated by the shape of flood hydrograph. In-between these ends, the water level is governed by either flood or tidal oscillation or their combination, depending on the magnitude of the flood. These results are used to decide which of the two time series models, the periodic function model or the ARMAX model, is appropriate to describe the variations of water levels at certain locations in the tidal reach.

MODELLING TECHNIQUE

The water level time series in Kumisaki and Takaeomote at the lower end of the tidal reach may be represented adequately by a periodic function model in the form:

$$y(t) = M_y(t) + \sum_{i=1}^{q}(A_i(t)\sin 2\pi f_i t + B_i(t)\cos 2\pi f_i t) + w(t) \tag{1}$$

where y(t) is tidal water level at time t, $M_y(t)$ is mean of the sequence, q is number of significant frequency components, f_i is frequency component, $A_i(t)$ and $B_i(t)$ are periodic coefficients, and w(t) is stochastic component which is assumed to be white Gaussian noise with zero mean and variance R. Here, two frequency components are identified, $f_1 = 1/12$ and $f_2 = 1/6$ in cycles per hour, which are obtained from the results of the spectral analyses.

A simple ARMAX model is used to describe the variations in water levels in Sendai at the upstream of the tidal reach. The model is given in the following form:

$$y_{SEN}(t)= a(t)y_{SEN}(t-1) + b(t)[y_{YUD}(t-T) - y_{YUD}(t-T-1)] + w(t) \tag{2}$$

where $y_{SEN}(t)$ is water level at Sendai in meters, $y_{YUD}(t-T)$ is water level at Yuda in meters, t is time in hour, T is time lag of flow between Sendai and Yuda in hours, a(t) and b(t) are ARMAX coefficients and w(t) is noise term as defined above. This is an autoregressive model of order 1, with the differenced upstream water level at Yuda as the exogeneous input. The time lag T is estimated from the water level data to be three hours, hence the differenced water level serves as a signal for the next three hours trend in the water level at Sendai. This ARMAX model is similar to the one reported by Ngan and Russell (1986).

PREDICTION TECHNIQUE

The Kalman Filter provides a method for recursively updating the coefficients of the models, thus tracking the behaviour of the system in real-time. Using the latest estimates of the model coefficients, the water level forecasts are computed. The filter is based on a linear state-space model:

$$x(t+1) = M(t)x(t) + u(t) \tag{3}$$

$$y(t) = H(t)x(t) + w(t) \tag{4}$$

where eq.3 is the state equation and eq.4 is the observation equation. Both the states and observations are affected by independent, zero mean, white Gaussian noises, u(t) and w(t), with covariance matrices Q and R respectively.

M is known state transition matrix and H is known observation matrix. At each time step, the Kalman Filter algorithm gives estimates of the states x(t) based on new observation y(t). The Kalman Filter algorithm consists of

$$\hat{x}(t+1/t) = M(t)\hat{x}(t/t) \tag{5}$$

$$P(t+1/t) = M(t)P(t/t)M^T(t) + Q(t) \tag{6}$$

at an observation

$$\hat{x}(t+1/t+1) = \hat{x}(t+1/t) + K(t+1)[y(t+1) - H(t+1)\hat{x}(t+1/t)] \tag{7}$$

$$P(t+1/t+1) = [I - K(t+1)H(t+1)]P(t+1/t) \tag{8}$$

where

$$K(t+1) = P(t+1/t)H^T(t+1)[H(t+1)P(t+1/t)H^T(t+1) + R(t+1)]^{-1} \tag{9}$$

P is error covariance matrix of x and I is identity matrix.

Both ARMAX and periodic function models are formulated as the observation equation (4), with the model coefficients as states, $x(t) = [a(t)\ b(t)]^T$ for the ARMAX model and $x(t) = [M_y(t)\ A_1(t)\ B_1(t)\ A_2(t)\ B_2(t)]^T$ for the periodic function model. The ARMAX observation matrix is $H(t) = [y_{SEN}(t-1)\ \ (y_{YUD}(t-T) - y_{YUD}(t-T-1))]$ and periodic function observation matrix is $H(t) = [1\ \sin 2\pi f_1 t\ \cos 2\pi f_1 t\ \sin 2\pi f_2 t\ \cos 2\pi f_2 t]$. The transition matrix M(t) is an identity matrix.

The recursive application of the Kalman Filter requires initial estimates of states $\hat{x}(0/0)$ and error covariance P(0/0) and estimates of the system noise Q and observation noise R. Here, the values of the model coefficients estimated by least-square method are used as initial estimates of the states $\hat{x}(0/0)$. The diagonal elements of P(0/0) are all taken as ten and off-diagonal elements as five. On the other hand, both the diagonal matrix Q and scalar R are assumed to be time-invariant, but the model coefficients are allowed to be time-variant, as modelled explicitly in the state equation. To allow time-variant model coefficients, the diagonal elements of Q are taken as the square of the elements of $\hat{x}(0/0)$, which are the least-square estimates of the model coefficients, for convenience sake. However, this may mean that the true values of states x equal zero, which is definitely not assumed in this study. The scalar observation noise R is assumed equal to the variance of the residuals which resulted from the least-square fit of the model to the given observed water levels.

RESULTS AND DISCUSSION

The modelling and prediction techniques are applied to two sets of flood data on August 3-5, 1971 and September 10-13, 1976 at Kumisaki, Takaeomote and Sendai Gauging Stations. Fig. 3 shows the forecasting performance of the Kalman Filter when the noise variance of the ARMAX and periodic coefficients are zero. For Kumisaki and Takaeomote, the misfits between predicted and observed water levels confirm that the mean M_y and amplitudes and phases of the 12-hour and 6-hour periodicities are not time-invariant. Hence, as modelled explicitly in the state equation, M_y, A_i and B_i should be allowed to vary with time by assuming appropriate noise variance to these model coefficients. This assumption of time-invariant system is also true during nontyphoon periods. Comparing the prediction results for Sendai as shown in this figure with those with time-variant coefficients as shown in Figs. 4 and

5, it can be concluded that better forecasts can be obtained if the ARMAX coefficients are time-variant.

Figs. 4 and 5 show the results of one-hour ahead predictions of water levels. It is shown that the prediction performance is satisfactory, with the predicted behaviour of the water levels following the observed one. This means that the periodic function models for Kumisaki and Takaoeomote data and an ARMAX model for Sendai data could adequately describe the behaviour of water levels in the tidal reach of Sendai River. Note that the addition of the autoregressive error terms in the periodic function model could somewhat improve the one-hour ahead forecasts (results not shown in this report). Moreover, poor forecasts are obtained for the two-hour and three-hour ahead forecasting exercises (results not shown in this report). This is because the model coefficients identified at time t by Kalman Filter do not represent the system at time t+2 and at time t+3, as they are time-variant.

Fig. 6 shows the results of the prediction of the rate (dy/dt) at which the water level rises at Sendai Gauging Station. It is found that there is a pronounced effect of the errors between predicted and observed water levels on the prediction of the rate of water-level rise. This means that if the ARMAX model overestimates or underestimates the water level, the rate will be overestimated or underestimated as well. An accurate prediction of the rate of rise of water level based on the predicted water level is rather difficult to achieve, because the interval of interest, which is the rising limb of hydrograph, is where the coefficients of the linear model are unstable, as they are adapting themselves to the flood situation. However, it is recommended that appropriate confidence limits should be incorporated in the forecasted water levels, which should also be reflected as confidence limits for the forecasted rates of water-level increases. For this example, $\hat{y} + 2\sigma$ and $\hat{y} - 2\sigma$ are adapted as the limits (95% confidence limits), where σ is the standard deviation of the residuals, i.e., the difference between predicted and observed water levels. These limits are shown as the lower and upper broken curves in Fig. 6.

The identification of M_y, A_i and B_i are presented in Fig. 7 for the flood data in August, 1971 at Takaeomote. The temporal variations in the model coefficients suggest that they are not constants but functions of time. The passage of flood in the lower end of the tidal reach is satisfactorily described by M_y and A_1 and B_1 of the semidiurnal frequency component, with small contribution from A_2 and B_2 of the 6-hour oscillation. It is shown that the variation of M_y is the most dominant, which is what one would expect during the passage of flood. Since A_2 and B_2 are relatively very small, it is possible to exclude the 6-hour frequency component.

Fig. 8 shows the results of the identification of the coefficients of the ARMAX model, a and b. It is shown that the variation of the autoregressive coefficient is bigger than that of the upstream input, which is what one would expect from the physical interpretation. At Yuda, the water level is largely influenced by the releases from the Tsuruta Dam so the upstream term is relatively stable, compared to the autoregressive term which has to account for the more variable tidal oscillations (see Fig. 4).

The prediction of tidal water levels by combined periodic function model and Kalman Filter is applicable to the operation of weir gates constructed near the mouth of the river. The weir gates, which can be found in many rivers in Japan, control the inflow of seawater into the reservoir dammed by them. With very accurate predictions (usually as short as 10-minute ahead) of tidal water levels downstream of the weir gates, optimal operation of gate openings could be realized, which would maintain the required water level upstream of weir gates. One existing procedure uses a deterministic periodic function model involving 25 frequency components, with no allowance for the occurrence of storm surge and passage of flood.

CONCLUSIONS

It has been shown that simple time series models can be applied to predict the variations of tidal water levels during typhoon periods when there are possibilities of concurrence of storm surge and flood in the tidal reach. It has also been shown that a periodic function model with one or two frequency components can satisfactorily represent the behaviour of water level at locations where the influence of tidal motion is dominant, and AR(1) model with exogeneous term can be applied at locations where the shape of flood hydrograph predominates. MEM spectral analysis has been used to decide which model is to be used at certain locations in the tidal reach. Moreover, this study has shown that mean M_y, periodic coefficients A_i and B_i, and ARMAX coefficients a and b should be allowed to vary with time by assuming proper noise variances to these parameters, when implementing Kalman Filter. It has been concluded that accurate prediction of the rate at which the water level rises is rather a difficult task using the present ARMAX linear model, and it has been recommended that confidence limits should be incorporated in the forecasted rate of water-level rise. Lastly, to demonstrate the full potential of the present modelling and prediction techniques, more case studies should be conducted when surge as well as nonsurge data are available.

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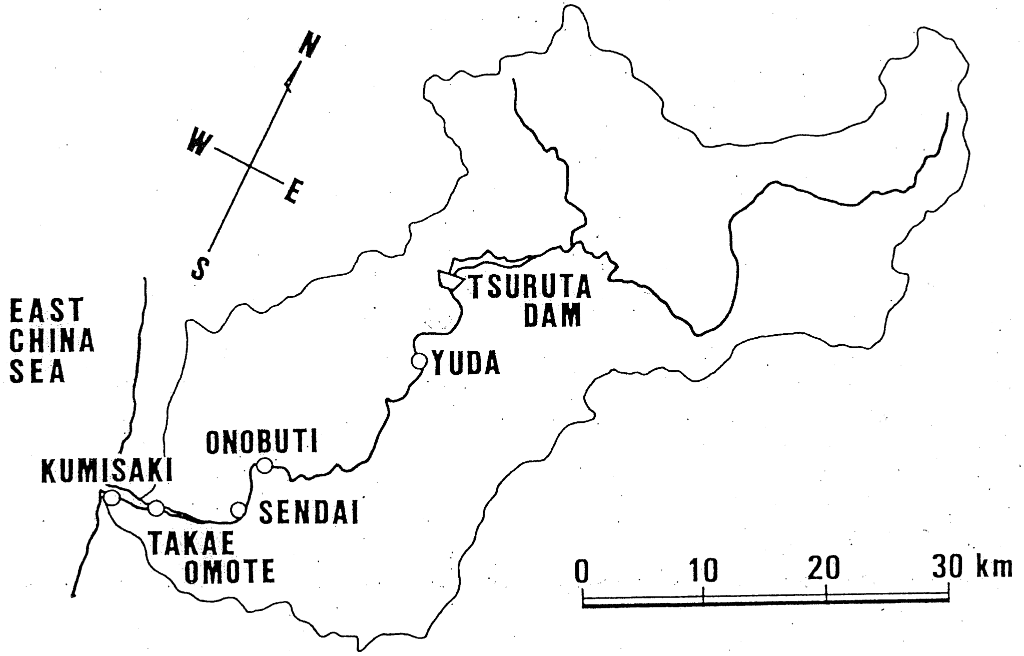

FIGURE 1 : Sendai River Basin.

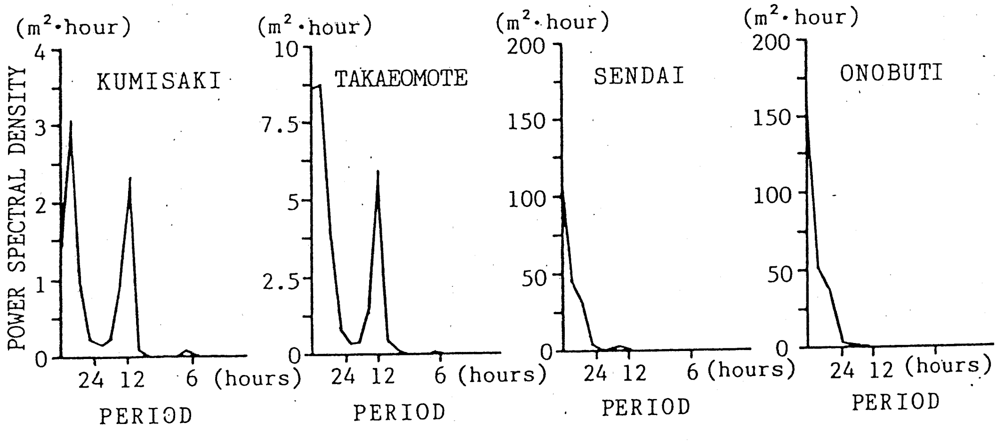

FIGURE 2 : MEM power spectrum in the tidal reach.

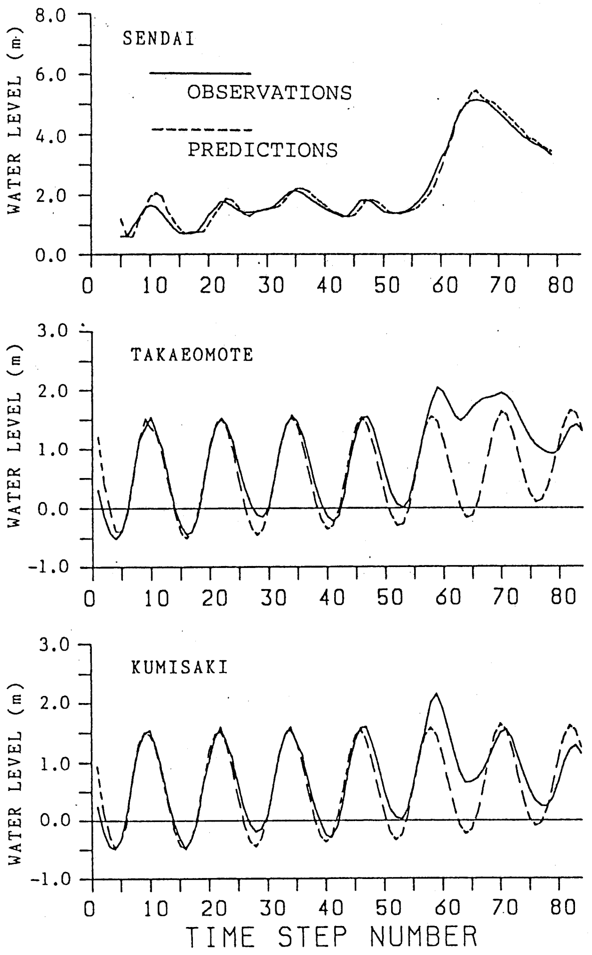

FIGURE 3 : Forecasting performance of the Kalman Filter when system noise $Q = 0$.

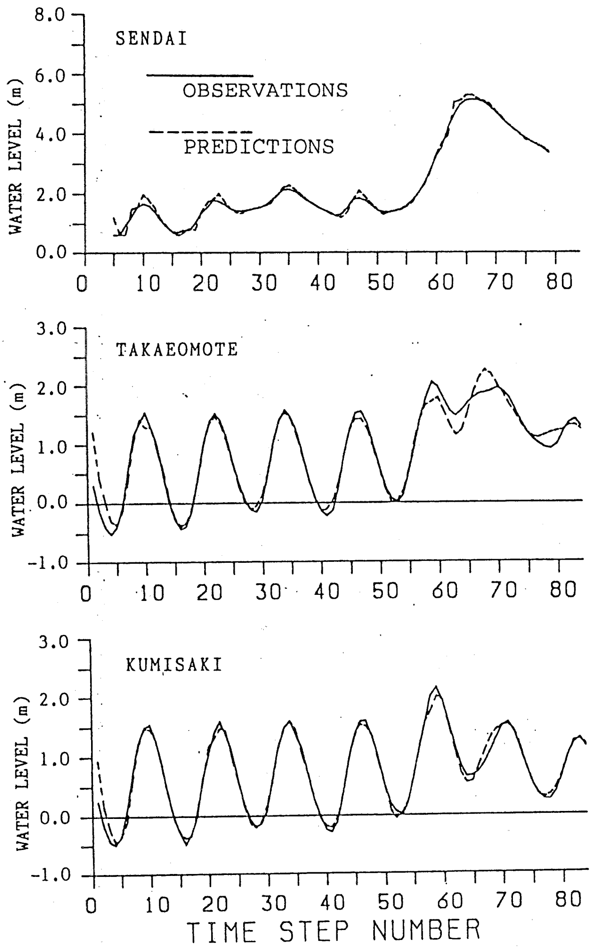

FIGURE 4 : Observed and forecasted water levels for the September, 1976 flood.

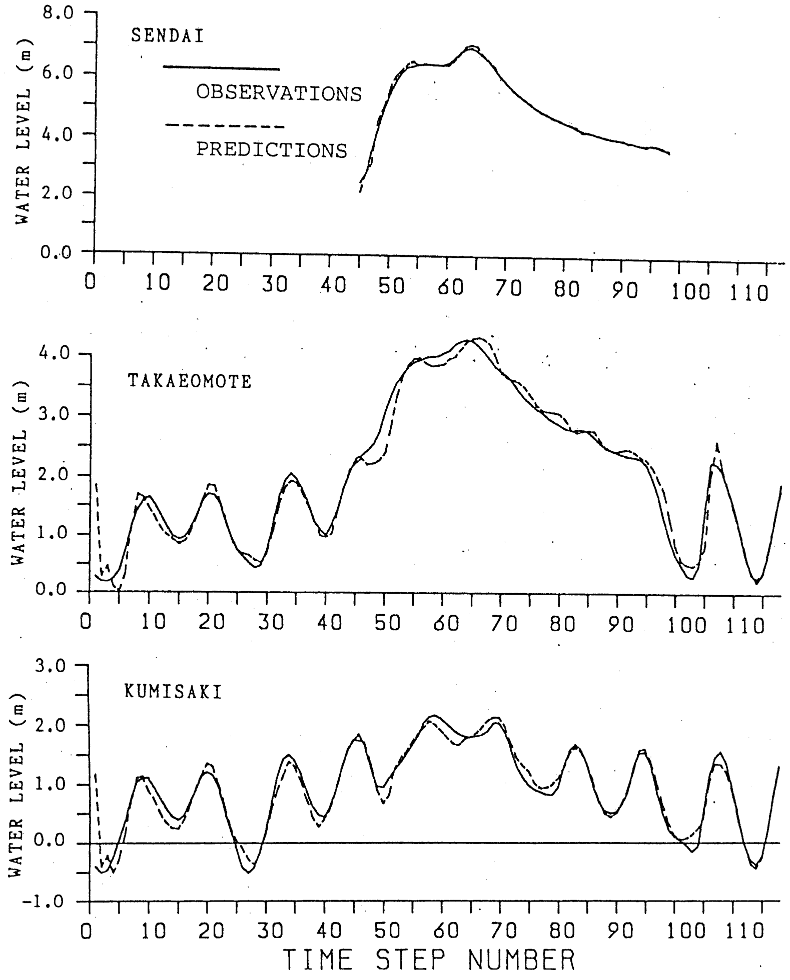

FIGURE 5 : Observed and forecasted water levels for the August, 1971 flood.

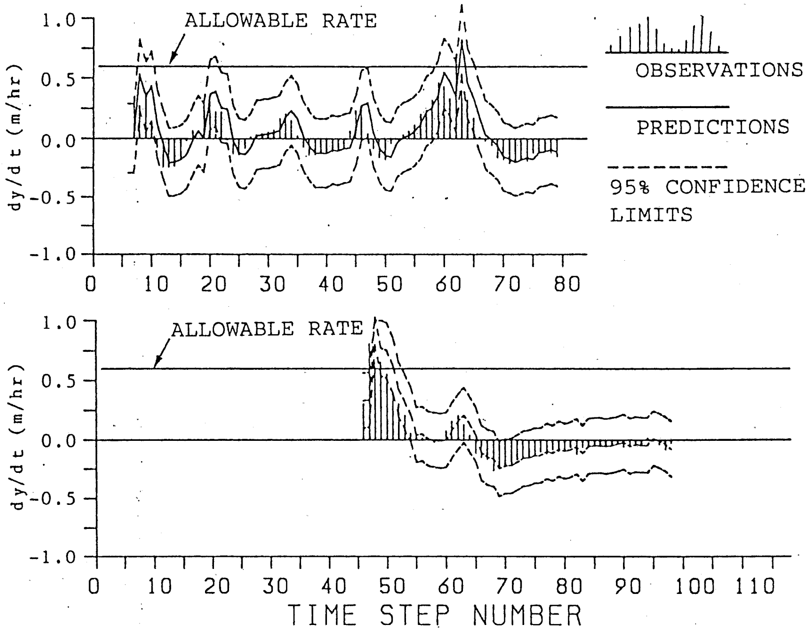

FIGURE 6 : Observed and forecasted rates of water-level changes at Sendai Gauging Station.

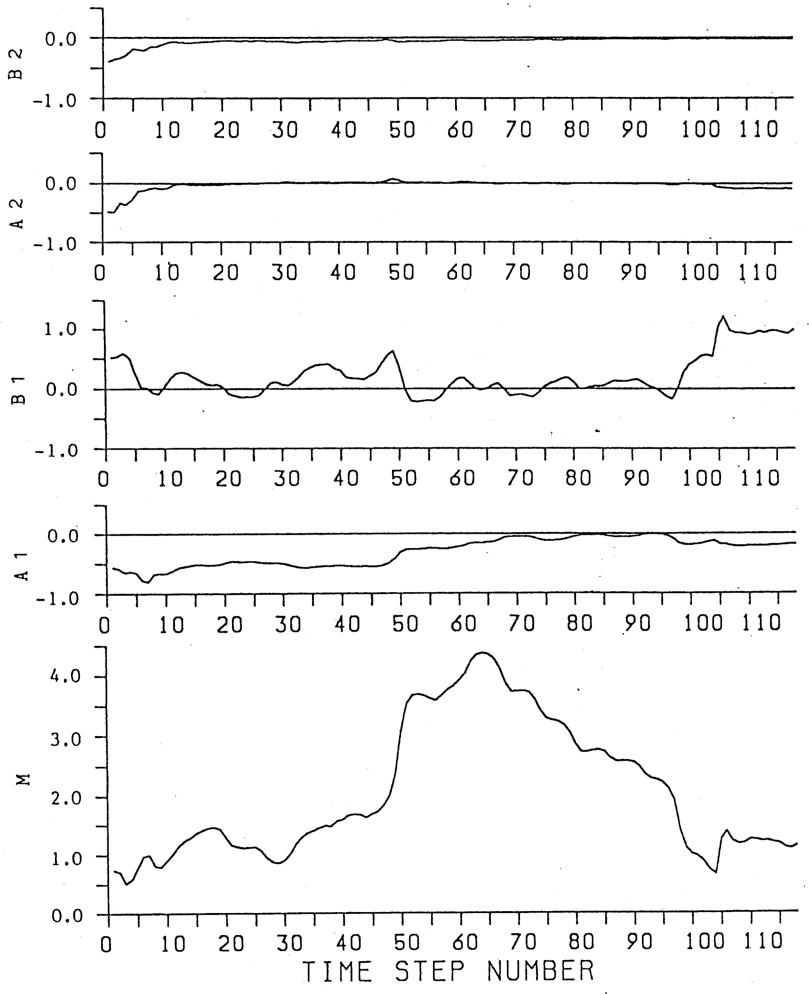

FIGURE 7 : Identification of periodic function model coefficients for the August, 1971 flood, Takaeomote Gauging Station.

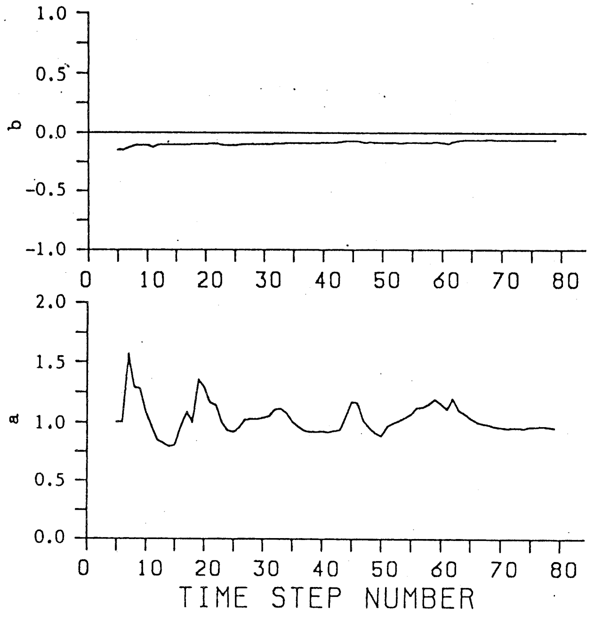

FIGURE 8 : Identification of ARMAX coefficients for the September, 1976 flood, Sendai Gauging Station.