

## On-Line Prediction Method for Water Demands in City Water Supply Network System

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### ABSTRACT

A method to predict on-line the water demands, hydraulic pressures and pipe discharges in city water supply network is proposed. This prediction algorithm is developed (Ueda et al (1986)) based on the Kalman filtering theory, where the equation of continuity and linearized equation of head loss are transformed into observation equation. Sensor information on some of the pipe discharges, hydraulic pressures and valve openings in the network are the observable variables in the prediction algorithm. This prediction method is applied to the Fukuoka City water supply network to investigate in detail not only its accuracy but also its basic characteristics.

KEYWORDS: On-line Prediction, Water Demands, Water Supply Network, Kalman Filter.

### 1 INTRODUCTION

Because of the severe water shortage experienced during the 1978 drought, the Fukuoka City government (Kyushu, Japan) decided to set up a water supply network control system. In the present control system, sensor information from 99 pressure gauges, 47 flowmeters and 123 automatic valves are continuously sent to the control center where valve openings are adjusted depending on the situation in the network given by the sensor information. The purposes of the control are to minimize leakage and to maintain appropriate hydraulic pressures for the consumers. By controlling the distribution of hydraulic pressures in the network, pipe-breaking could be lessened and water could be conserved. However, to achieve this kind of control, it is necessary to accurately estimate the distribution of hydraulic pressures in the network and to properly control the valves. In this case, on-line predictions of water demands and hydraulic pressures at all pipe nodes and pipe discharges with high accuracy appear indispensable.

In this paper, we propose a method to predict water demands, pipe discharges and hydraulic pressures in all pipes in the network. This method is based on the Kalman filtering theory. The filter uses continuous sensor information on pipe discharges, hydraulic pressures and valve openings, which are sent to the control center, from some measurement points in the network. To demonstrate the accuracy of the method, it is applied to Block No.9 of the 21-block water supply network in Fukuoka City. Moreover, the basic characteristics of the method such as the effect of changing valve openings and arrangement of pressure gauges and flow meters in the network on the accuracy of the prediction are investigated in detail.

### 2 DEVELOPMENT OF THE ALGORITHM FOR THE PREDICTION METHOD

Water supply network has two basic hydraulic equations: one is the equation of continuity, Eq.1, and another is the equation of head loss, Eq.2. As for the equation of head loss, we use the Hazen-Williams empirical equation, which is often applied in pipe network analysis.

$$\sum_j Q_{ij}(k) = -q_i(k) \quad (1)$$

$$H_i(k) - H_j(k) = r_{ij}^{-1/\alpha} |Q_{ij}(k)|^{1/\alpha-1} Q_{ij}(k) + \frac{8f_{vij}}{g\pi^2 d_{ij}^4} |Q_{ij}(k)| Q_{ij}(k) \quad (2)$$

The second term in Eq.2 is required only if there is a valve between nodes. Here  $k$  is time step;  $Q_{ij}$  ( $m^3/\text{hour}$ ) is pipe discharge from node  $i$  to node  $j$ , and  $Q_{ij} = -Q_{ji}$ ;  $| \cdot |$  means absolute value;  $q_i$  ( $m^3/\text{hour}$ ) is water demand at node  $i$ ,  $H_i$  (m) and  $H_j$  (m) are hydraulic pressures at nodes  $i$  and  $j$ ;  $\alpha=0.54$ ;  $f_{vij}$  is valve loss coefficient;  $g$  ( $m/\text{hour}^2$ ) is acceleration of gravity;  $r_{ij}$  is a constant, and  $r_{ij} = r_{ji}$ ,

$$r_{ij} \triangleq 0.27853 c_{ij} d_{ij}^{2.63} l_{ij}^{-0.54} \quad (m^{2.46}/\text{hour}) \quad (3)$$

where  $c_{ij}$  ( $m^{0.37}/\text{hour}$ ) is velocity coefficient,  $d_{ij}$  (m) is diameter of the pipe, and  $l_{ij}$  (m) is pipe length.

Eq.2 can be linearized by using Taylor series expansions. Considering the first-order term of the expansions and rearranging, the following equation is obtained.

$$H_i(k+1) - H_j(k+1) - f_{ij}(k) Q_{ij}(k+1) = H_i(k) - H_j(k) - f_{ij}(k) Q_{ij}(k) \quad (4)$$

where

$$f_{ij}(k) \triangleq (1/\alpha) r_{ij}^{-1/\alpha} |Q_{ij}(k)|^{1/\alpha-1} + \frac{16f_{vij}}{g\pi^2 d_{ij}^4} |Q_{ij}(k)| Q_{ij}(k) \quad (\text{hour}/m^2) \quad (5)$$

Eq.1 applies to all nodes, while Eq.4 applies to all pipes in the network.

On the water demand  $q_i$  at every node in Eq.1, we suppose it can be expressed with some functions. In this paper, it is assumed that the water demands change every 12 hours and every 24 hours (Jinno et al (1986)); the water demands can then be expressed by periodic-stochastic model. For example, the water demand at node  $i$ ,

$$q_i(k) = M_i + a_{i1} \sin 2\pi F_{i1} k + b_{i1} \cos 2\pi F_{i1} k + a_{i2} \sin 2\pi F_{i2} k + b_{i2} \cos 2\pi F_{i2} k + v_i(k) \quad (6)$$

where  $M$  ( $m^3/\text{hour}$ ) is the mean of the water demand fluctuation,  $F_1$  and  $F_2$  are the frequency components,  $a_1, b_1$  and  $a_2, b_2$  ( $m^3/\text{hour}$ ) are the amplitudes of  $F_1$  and  $F_2$ ,  $v_i$  is assumed to be independent zero-mean Gaussian white noise  $N(0, \sigma_i^2)$ .

The prediction method uses the Kalman filtering algorithm with the following system and observation equations

$$x(k+1) = \Phi(k)x(k) + u(k) \quad (7)$$

$$y(k) = H(k)x(k) + T(k) + w(k) \quad (8)$$

where  $k$  is time step,  $x$  is system state vector,  $\Phi$  is known transition matrix,  $u$  is assumed to be independent, zero mean system noise vector,  $y$  is observation vector,  $H$  is known observation matrix,  $w$  is assumed to be independent, zero mean observation noise vector.

In applying Kalman filter to the water supply network, pipe discharges and hydraulic pressures observed hourly from flow meters and pressure gauges in the network composed the observation vector  $y$  in Eq.8. Considering that the frequency

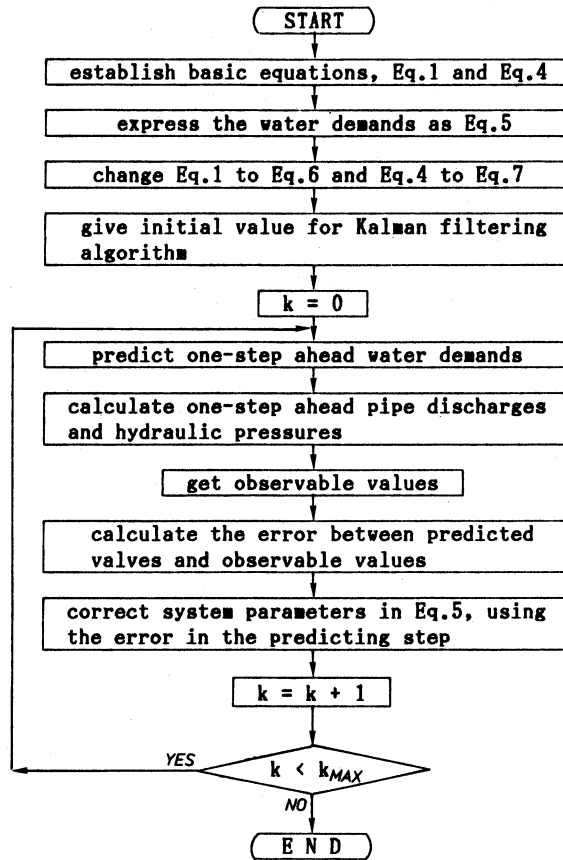


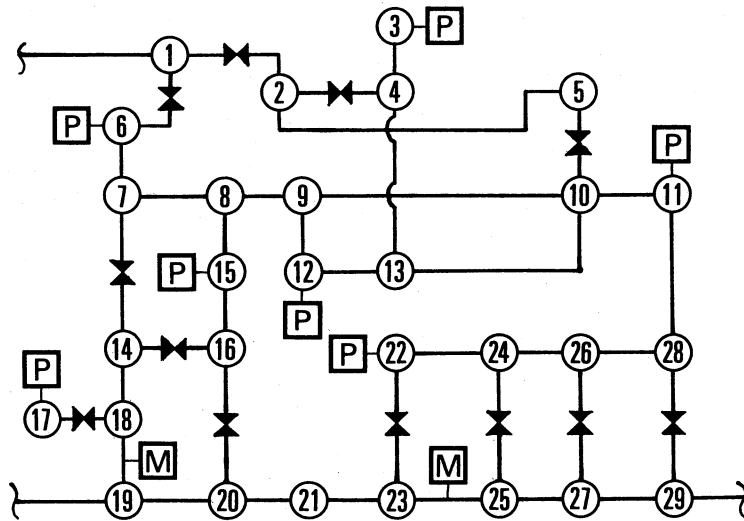
Fig. 1 Algorithm

components ( $F_{i1}$  and  $F_{i2}$ ) are known, the parameters  $M_i$  and  $a_{i1}, b_{i1}$  and  $a_{i2}, b_{i2}$  composed the system state vector  $x$  in Eq.7. These system states  $x$  are estimated optimally on-line using Kalman filter; the optimal  $x$  is estimated by feedback of the error between one-step ahead predicted value and observed value of  $y$ . From the result of this estimation, the one-step ahead prediction of water demand at every node is made by substituting the estimated parameters into Eq.6. Moreover, the discharge at every pipe and hydraulic pressure at every node are calculated from Eq.1 and Eq.4 using the predicted water demands. The algorithm of the prediction method is shown in Fig.1.

### 3 EXAMPLE

Simulation of Block No.9 of the Fukuoka City water supply network (see Fig.2) is done to demonstrate the application of the method and the accuracy of the predictions. The hydraulic pressures at nodes 3, 6, 11, 12, 15, 17 and 22 can be observed hourly, and the pipe discharges between nodes 18 and 19 and between nodes 23 and 25 can also be observed hourly. Table 1 shows the diameters and lengths of pipes shown in Fig.2.

In this block, there are 29 water demands, 29 hydraulic pressures, 37 pipe



(i) : NODE *i*    [P] : PRESSURE GAUGE  
 [M] : FLOW METER    ◀▶ : VALVE

Fig. 2 Block No.9 of the Fukuoka City water supply network

Table 1 Diameters and lengths of pipes in Block No.9 of the Fukuoka City water supply network

Pipe <i>i</i> - <i>j</i>	Diameter(mm) -Length(m)	Pipe <i>i</i> - <i>j</i>	Diameter(mm) -Length(m)
1- 2	800 - 850	14-18	700 - 850
1- 6	300 - 400	15-16	300 - 100
2- 4	250 - 400	16-20	300 - 1100
2- 5	800 - 600	17-18	200 - 600
3- 4	250 - 400	18-19	700 - 150
4-13	250 - 800	19-20	900 - 200
5-10	250 - 100	20-21	900 - 100
6- 7	200 - 800	21-23	800 - 550
7- 8	600 - 750	22-23	300 - 1200
7-14	600 - 1100	22-24	400 - 300
8- 9	400 - 300	23-25	800 - 400
8-15	800 - 200	24-25	300 - 1000
9-10	400 - 350	24-26	350 - 200
9-12	200 - 200	25-27	800 - 550
10-11	250 - 1000	26-27	400 - 600
10-13	250 - 100	26-28	250 - 500
11-28	300 - 600	27-29	600 - 350
12-13	250 - 500	28-29	250 - 800
14-16	250 - 100		

discharges, and three inflows from outside the network. The outline of the simulation is described as follows. First, we give values (randomly chosen) to the parameters  $M_i, a_{i1}, b_{i1}, a_{i2}, b_{i2}$  and  $\sigma_i$  in Eq.6, where these values will be called 'true values' of the parameters. (The initial values of state parameters  $x$  in the prediction algorithm are taken as 50% of their true values.) Then given these quantities and known  $F_{i1}$  and  $F_{i2}$ , the 29 water demands are calculated. Here, calculation is done for 288 steps with one step equal to 10 minutes.

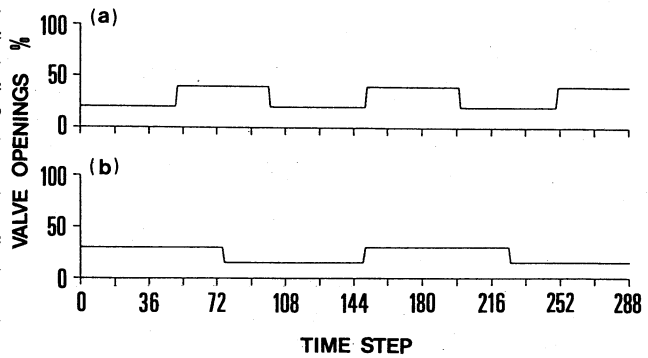


Fig. 3 Valve openings

The total number of equations is 66 equal to 29 equations of head loss (Eq.4) plus 37 equations of continuity (Eq.1). Nevertheless, the number of unknown quantities to be estimated excluding 29 water demands is 69 which is broken down into 29 hydraulic pressures, 37 pipe discharges and three inflows. Unless three quantities are given, the network flow can not be solved uniquely. We choose two inflows at nodes 1 and 19 and one hydraulic pressure at node 1 as the three quantities (see Fig.2). In the analysis of the network, the coefficient of valve loss,  $f_{vij}$  is calculated using the following equation;

$$f_{vij}(\theta) = \begin{cases} 165226 \times 10^{-0.18\theta} & (0 \leq \theta < 12.5) \\ 3696 \times 10^{-0.06\theta} & (12.5 \leq \theta < 45) \\ 221 \times 10^{-0.03\theta} & (45 \leq \theta \leq 100) \end{cases} \quad (9)$$

where  $\theta$  (%) is valve openings.

The valve openings between nodes: 1 and 2, 2 and 4, 7 and 14, 16 and 20, 22 and 23, 26 and 27 change as shown in Fig.3 (a), and those between nodes 1 and 6, 5 and 10, 14 and 16, 17 and 18, 24 and 25, 28 and 29 change as shown in Fig.3 (b).

In the one-step ahead prediction of pipe discharges, hydraulic pressures and water demands, the one-step ahead valve openings are supposed to be known as they are decided based on the present situation in the network.

With these assumptions, the simulation results in 37 pipe discharges and 29 hydraulic pressures and three inflows to be used as the 'observed' data. The prediction method proposed in this paper will be applied to these 'observed' data, and in the next section, its effectiveness and usefulness will be discussed through the results of the prediction exercise.

## 4 RESULTS AND DISCUSSION ON THE BASIC CHARACTERISTICS OF THE METHOD

### 4.1 Discussion on the Results of the Prediction

The result of the calculation of the linearization errors shows that they are below 0.1% of the true values. Hence the results of the predictions can be discussed without any difficulty caused by ignoring the linearization errors.

Fig.4 shows some of the results of the predictions of the 29 water demands, 37 pipe discharges, 29 hydraulic pressures and three inflows. In this figure, the solid line is the observed values, and the circled line represents the predicted values obtained by the proposed method. It is clear that  $q_{11}$ , the water demand,  $Q_{18,19}$ , the observable pipe discharges,  $H_{12}$ , the observable hydraulic pressures and  $H_{28}$ , the unobservable hydraulic pressures, are predicted with high accuracy. The predicted values of the pipe discharges and the hydraulic pressures follow the fluctuations of the observed values, where these fluctuations depend on the changes of valve openings as shown in Fig.3. This illustrates the influence

of giving one-step ahead valve openings as one information for the prediction. On the other hand,  $q_9$  and  $q_{19}$ , the water demands,  $Q_{9,10}$ , the unobservable pipe discharge, and  $Q_{0,19}$  (the suffix 0 means outside the network;  $Q_{0,19}$  means inflow at node 19) are predicted with some bias. Actually, the results of the predictions of  $q_{19}$  and  $Q_{0,19}$  are not so good; the bias remains to the last. In predicting both water demand and inflow at the same node, sensor information from flow meters and pressure gauges can not distinguish water demand from inflow. Except for these unobservable quantities, other quantities (both observable and unobservable) not shown in Fig.4 converged to their true values.

Fig.5 shows the identification of parameters for the case of  $q_{11}$  in Eq.6. In this figure, broken line is the true parameter values. It is obvious that each parameter converges to the true value after around 80 steps. It is supposed that the small bias in the identification of parameters causes the prediction error in Fig.4. Moreover, it is confirmed that the prediction could be done more accurately, if the initial values of the parameters are closer to their true values.

#### 4.2 Effect of the Arrangements of Pressure Gauges and Flow Meters

In order to discuss the effect of the arrangements of observation flow meters and pressure gauges, the prediction method is applied to the case where the three inflows are assumed observable. Fig.6 shows some of the results of the prediction in cases where  $Q_{0,1}$ ,  $Q_{0,19}$ , and  $Q_{0,29}$  are observable. As compared with the results shown in Fig.4, it is clear that the accuracy of the prediction is increased for the observable quantities. Thus the effect of observable inflows appears very significant. Considering these results, it can be proposed that, for a more accurate prediction, the water supply network should be divided into blocks where the observable pipe discharges will become inflows to the next block.

By referring to Fig.7 and Fig.8, we discuss the difference between the effect of flow meters and that of pressure gauges on the prediction accuracy. Fig.7 shows the water demand,  $q_9$  and the pipe discharge,  $Q_{10,9}$ , when a flow meter is set between

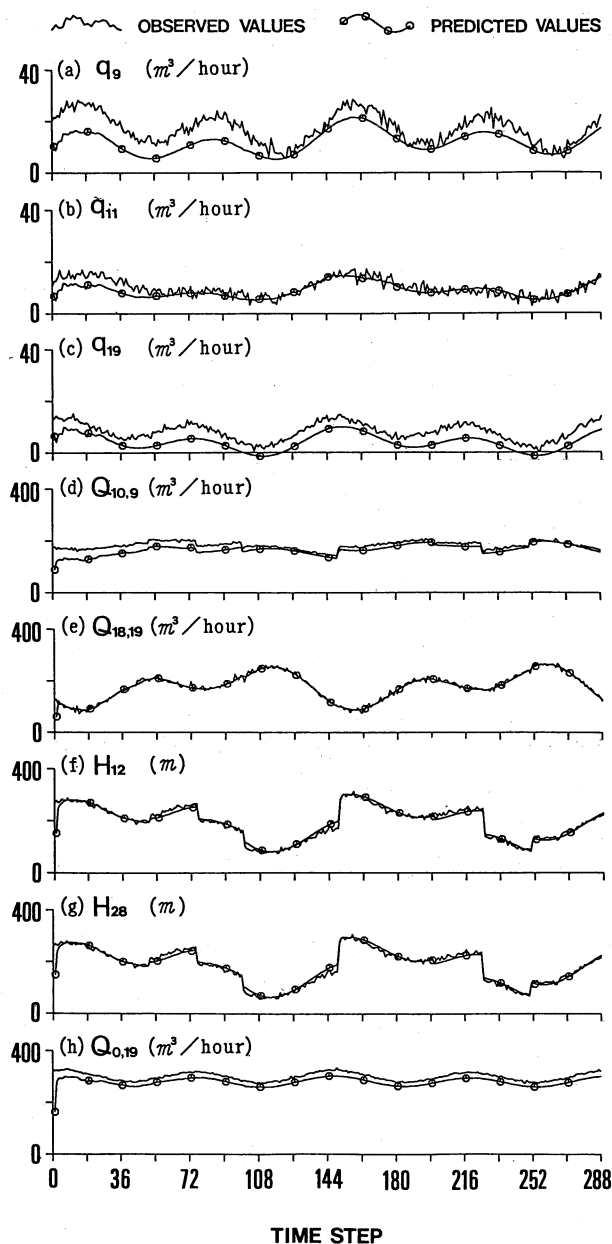


Fig. 4 Results of the prediction

nodes 9 and 10, and Fig.8 shows two pressure gauges installed at nodes 9 and 10. As compared with the results shown in Fig.4, it is concluded that the accuracy of the prediction is increased when a flow meter is set. On the other hand, when setting two pressure gauges, the accuracy of the prediction is the same as in Fig.4. Judging from these results, installing flow meters gives more accurate predictions than installing pressure gauges. In the Fukuoka City water supply network control system, there are more pressure gauges than flow meters; for example, there are seven pressure gauges and two flow meters shown in Fig.2. Only information from pressure gauges is used to control the distribution of hydraulic pressures in the network, and information from flow meters is used just to detect extraordinary pipe discharges. The results of other calculations confirm that more accurate predictions of water demands can be obtained at nodes located downstreams of the flow meters.

## 5 CONCLUSIONS

In this paper, we proposed a method to predict on-line the water demands, hydraulic pressures, and pipe discharges in a water supply network by using sensor information from flow meters and pressure gauges. This method was applied to Block No.9 of the Fukuoka City water supply network to show its effectiveness and usefulness.

The results obtained in this paper show that, by using this method, parameters of the water demand model can be identified accurately and that even if valve openings would change by small fraction, unknown one-step ahead pipe discharges and hydraulic pressures can be predicted accurately. The observable pipe discharges and hydraulic

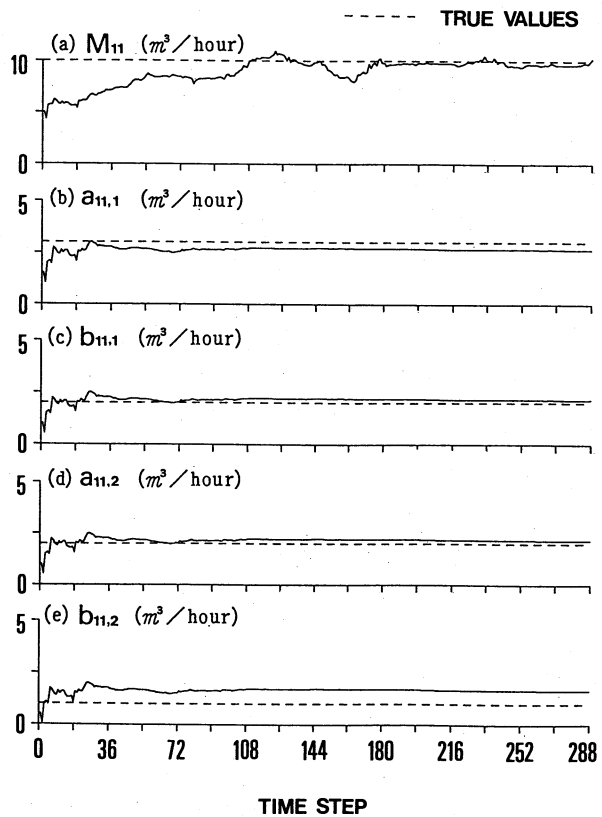


Fig. 5 Identification of parameters for the case of  $q_{11}$

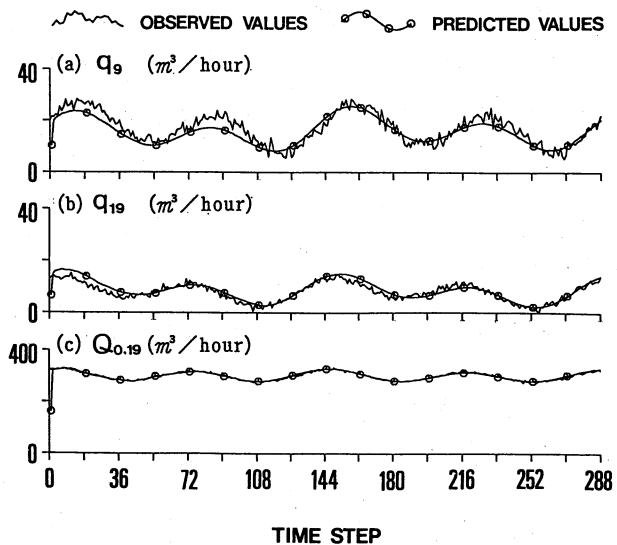


Fig. 6 Results of the prediction in cases where all inflows are observable

pressures were predicted with high accuracy. It became clear that the accuracy of the prediction, especially the predictions of water demands, is improved remarkably if all inflows into the network are observed by flow meters. Moreover, it is confirmed that setting flow meters is more effective than installing pressure gauges in terms of getting high prediction accuracy.

Lastly, to achieve an optimal control of the distribution of hydraulic pressures in the network, a method to determine the optimal valve openings, which is based on the one-step ahead predictions of pipe discharges and hydraulic pressures obtained by the proposed method, needs to be developed.

#### ACKNOWLEDGEMENTS

We would like to express our gratitude to the Water-Works Bureau of the Fukuoka City Government for their kind assistance.

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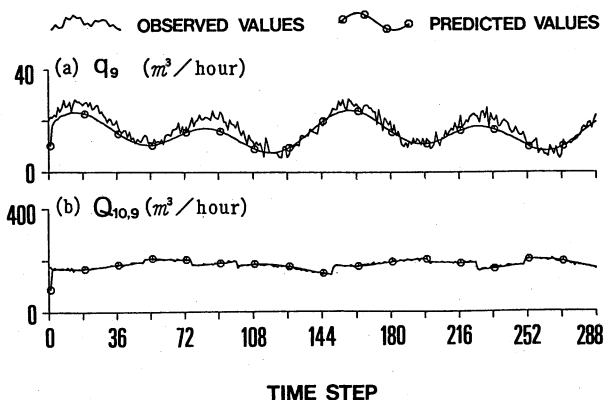


Fig. 7 Results of the prediction in cases when a flow meter is set between nodes 9 and 10

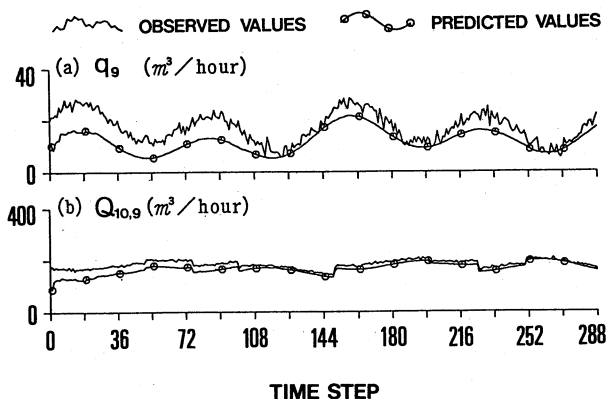


Fig. 8 Results of the prediction in cases when two pressure gauges are set at nodes 9 and 10