1. Errortum

⋆ (a) I wrote.
(b) But I should have written.
(c) Dates
(d) Other comments
(1) Index 27. (page xvii) Add
The Kronecker delta function defined on a set $X$ is given by $\delta_{jk} \equiv \begin{cases} 1 & (j = k), \\ 0 & (j \neq k). \end{cases}$
for $j, k \in X$.
(2) 29 Oct. 2019
(3) I should have expanded the range of the set on which $\delta$ is defined!
(4) Index 27. (page xvii) Add
When two normed spaces $X, Y$ are isomorphic, we write $X \approx Y$.
(5) xiii, Notation in This Book, line 2 from below
(a) $E$ is integrable over $f$
(b) $f$ is integrable over $E$
(c) 29 Oct. 2019
(6) Page 25
(a) and
$\tilde{H}^d_0(E_1 \cup E_2)$
(b) 29 Oct. 2019
(c) and
$\tilde{H}^d_0(E_1 \cap E_2)$
(d) 29 Oct. 2019
(7) page 120, Lemma 1.22
(a) Let $1 \leq p, q \leq \infty$.
(b) Let $1 \leq p, q<\infty$.
(c) 29 Oct. 2019
(d) Clearly there is a counterexample if $p + q = \infty$.
(8) (a) page 357
(b) $(f, g)_H = \frac{1}{I(\mathcal{U})} \langle f, \varphi^{(j)} \cdot g \rangle$
(c) $(f, g)_{H_j} = \frac{1}{I(\mathcal{U})} \langle f, \varphi^{(j)} \cdot g \rangle$
(d) 29 Oct. 2019
(9) page 358, It should be remarked that (3.67) is used to deduce (3.68).
(c) 29 Oct. 2019
(10) page 359, “=” in (3.73) should have been $\equiv$.
(c) 29 Oct. 2019
(11) page 359, “=” in (3.74) should have been $\equiv$.
(c) 29 Oct. 2019
(12) page 359, “≤” in line 3 from below in page 359 should have been ≲.
(c) 29 Oct. 2019
(13) page 362, I should have mentioned that k in the quantity W^i in the last line of page 362 moves over all k ∈ K_j as well as j ∈ N.
(c) 29 Oct. 2019
(14) page 363, I did not use (3.72) in line 5 from above in page 363.
(c) 29 Oct. 2019
(15) page 365, “2” in two lines below (3.88) is of course “2”.
(c) 29 Oct. 2019
(16) page 368, In Proposition 3.8, the subindices l should have been j.
(c) 29 Oct. 2019
(17) page 368, line 9 from above
(a) to partition
(b) partition
(18) page 368, line 6 from above
(a) Q_l
(b) Q_j
(c) 29 Oct. 2019
(19) page 371, (3.93).
(a) Q_l
(b) Q_j
(c) 29 Oct. 2019
(20) page 379, Exercise 3.34
(a) Let f ∈ H^p(\mathbb{R}^n).
(b) Let f ∈ H^p(\mathbb{R}^n) in the sense that \mathcal{M} f ∈ L^p(\mathbb{R}^n).
(c) 9 Nov. 2019
(21) page 570, Proof of Theorem 5.3.
(a) Hestones
(b) Hesteness
(c) 29 Oct. 2019
(22) page 624, Lemma 5.10
(a) a sectorial operator on a Banach space X.
(b) a sectorial operator on a Banach space X as in Definition 5.13, and let 0 < η < μ < π.
(c) 29 Oct. 2019
(d) I should have clarified the role of μ.
(23) page 629, The second line of the proof of Lemma 5.12. Add “for all v ∈ Dom(D).”
(24) page 630, The second line of the proof of Proposition 5.11
(a) are dense in H_0
(b) are H_0.
(c) 29 Oct. 2019
(d) H_0 itself!
(25) page 631, Theorem 5.22 Add: We assume that |\arg(⟨Lu, u⟩_H)| < \frac{\pi}{2} − \theta^* for all u ∈ Dom(L) for some 0 < \theta^* < \frac{\pi}{2}.
(c) 29 Oct. 2019
(26) page 631, Theorem 5.23 Add: if |\arg(⟨Lu, u⟩_H)| < \frac{\pi}{2} − \theta^* for all u ∈ Dom(L) for some 0 < \theta^* < \frac{\pi}{2}.
(c) 29 Oct. 2019
(27) page 632, line 4 from above.
(a) L_a u \to u
THEORY OF BESOV SPACES, ERROTUM 3

(b) \( L_a u \rightarrow Lu \).

(c) 29 Oct. 2019

(28) page 632, line 6 from above.

(a) \( f(L_a)u \)

(b) \( f(L_a)(\text{id}_H + L_a)^{-3}v \)

(c) 29 Oct. 2019

(d) The operator must be \( L_a \) instead of \( L \).

(29) page 632, line 11 from above.

(a) as required.

(b) Since \( (\text{id}_H + L_a)^{-3}w \rightarrow (\text{id}_H + L)^{-3}w \) for any \( w \in \text{Dom}(L) \) and \( \|f(L_a)\|_{B(H)} \leq \|f\|_{L^\infty(\{\Re(z) > 0\})} \), we obtain the desired result.

(c) 29 Oct. 2019

(30) page 634, line 7 from below. Add: such that \( |\arg(\langle Lu, u \rangle_H)| < \frac{\pi}{2} - \theta^* \) for all \( u \in \text{Dom}(L) \) for some \( 0 < \theta^* < \frac{\pi}{2} \).

(c) 29 Oct. 2019

(31) page 635, line 3 from above.

(a) \( |\sqrt{z + b} - \sqrt{z + a}| = \)

(b) \( |\sqrt{z + b} - \sqrt{z + a}| \leq \)

(c) I used the triangle inequality.

(32) page 635, On line before Corollary 5.7, I should have clarified that Corollary 5.7 defines \( \sqrt{\frac{T}{\text{id}_H + L}} \).

(c) 29 Oct. 2019

(33) page 635, Corollary 5.7: Add: such that \( |\arg(\langle Lu, u \rangle_H)| < \frac{\pi}{2} - \theta^* \) for all \( u \in \text{Dom}(L) \) for some \( 0 < \theta^* < \frac{\pi}{2} \).

(c) 29 Oct. 2019

(34) page 635, Definition 5.17, Add: such that \( |\arg(\langle Lu, u \rangle_H)| < \frac{\pi}{2} - \theta^* \) for all \( u \in \text{Dom}(L) \) for some \( 0 < \theta^* < \frac{\pi}{2} \).

(c) 29 Oct. 2019

(35) page 638, Lemma 5.20, Add: Assume that \( L \) is a sectorial operator such that \( |\arg(\langle Lu, u \rangle_H)| < \frac{\pi}{2} - \theta^* \) for all \( u \in \text{Dom}(L) \) for some \( 0 < \theta^* < \frac{\pi}{2} \).

(c) 29 Oct. 2019

(36) page 638, Lemma 5.21, Add: Assume that \( L \) is a sectorial operator such that \( |\arg(\langle Lu, u \rangle_H)| < \frac{\pi}{2} - \theta^* \) for all \( u \in \text{Dom}(L) \) for some \( 0 < \theta^* < \frac{\pi}{2} \).

(c) 29 Oct. 2019

(37) page 638, Theorem 5.25, Add: Assume that \( L \) is a sectorial operator such that \( |\arg(\langle Lu, u \rangle_H)| < \frac{\pi}{2} - \theta^* \) for all \( u \in \text{Dom}(L) \) for some \( 0 < \theta^* < \frac{\pi}{2} \).

(c) 29 Oct. 2019

(38) page 639, line 4 from above

(a) \( \int_0^\infty \frac{\alpha dx}{1 + \alpha x^2} = \sqrt{\alpha} \pi. \)

(b) \( \int_0^\infty \frac{2\alpha dx}{1 + \alpha x^2} = \sqrt{\alpha} \pi. \)

(c) 29 Oct. 2019

(d) \( \int_0^\infty \frac{dt}{t^2 + 1} = \pi \) should have been \( \int_0^\infty \frac{dt}{t^2 + 1} = \frac{\pi}{2} \).

(39) page 639, Theorem 5.26, Add: Assume that \( L \) is a sectorial operator such that \( |\arg(\langle Lu, u \rangle_H)| < \frac{\pi}{2} - \theta^* \) for all \( u \in \text{Dom}(L) \) for some \( 0 < \theta^* < \frac{\pi}{2} \).
(c) 29 Oct. 2019

(40) page 639, (5.93)
(a) $id_H + 2\varepsilon id_H$
(b) $(1 + 2\varepsilon)id_H$
(c) This is just a matter of a marginal typo.

(41) page 639, (5.94)
(a) $id_H + 2\varepsilon id_H$
(b) $(1 + 2\varepsilon)id_H$
(c) 29 Oct. 2019
(d) This is just a matter of a marginal typo.

(42) page 640, Corollary 5.9
(a) $\sqrt{L}u = \frac{1}{\pi} \int_0^\infty L(id_H + t^2L)^{-1}udt.$
(b) $\sqrt{L}u = \frac{2}{\pi} \int_0^\infty L(id_H + t^2L)^{-1}udt.$
(c) 29 Oct. 2019
(d) $\int_0^\infty \frac{dt}{t^2 + 1} = \pi$ should have been $\int_0^\infty \frac{dt}{t^2 + 1} = \frac{\pi}{2}.$

(43) page 656, Theorem 5.36. Remove ”Assume that the boundary is smooth.”
(c) 29 Oct. 2019

(44) page 660, Lemma 5.29
(a) $\lambda |\xi|^2$
(b) $\lambda^2 |\xi|^2$
(c) 29 Oct. 2019

(45) page 662, lines 3 and 4 from below
(a) $3e^{2k} - 8e^k + 4 \geq e^k - 1$ for $k \geq 2,$ we conclude
$$e^{\alpha} \int_F |u^t(x)|^2 dx \leq \int_{\mathbb{R}^n} |u^t(x)|^2 \exp(\alpha \eta(x)) dx \leq \int_{\mathbb{R}^n} |u^t(x)|^2 dx \leq \int_E |f(x)|^2 dx.$$ (b) and
$$\|u^t \exp(\alpha \eta)\|_{L^2} \leq \|u^t(\exp(\alpha \eta) - 1)\|_{L^2} + \|u^t\|_{L^2} \leq \frac{1}{2} \|u^t \exp(\alpha \eta)\|_{L^2} + \|u^t\|_{L^2},$$
we conclude
$$e^{\alpha} \int_F |u^t(x)|^2 dx \leq \int_{\mathbb{R}^n} |u^t(x)|^2 \exp(\alpha \eta(x)) dx \leq 4 \int_{\mathbb{R}^n} |u^t(x)|^2 dx \leq 4 \int_E |f(x)|^2 dx.$$ (c) 23 Nov. 2019

(46) page 739, line 6 from below, (twice) Proof of Lemma 6.2
(a) $m^{(1+\varepsilon)}(w)$
(b) $m_Q^{(1+\varepsilon)}(w)$
(c) 29 Oct. 2019
(d) There are two places.

(47) page 720 line 14 from above, Proof of Theorem 6.4.
(a) $q = \frac{p}{1 + \varepsilon}$
(b) $q = \frac{p}{1 + \varepsilon}$
(c) 29 Oct. 2019
(d) I should have used $w^{-\frac{1}{p'}} A' \subset A'' \subset A_\infty$ together with Lemma 6.2.

(48) page 720 line 16 from above Proof of Theorem 6.4 (Major change).
(a) there exists $\delta > 0$ such that $\frac{w(A)}{w(Q)} \leq \left( \frac{|A|}{|Q|} \right)^\delta,$ whenever $A$ is a subset of a cube $Q.
(b) there exists a constant $D > 0$ such that \( \frac{w(E)}{w(Q)} \leq D \left( \frac{|E|}{|Q|} \right)^{\frac{1}{1+\varepsilon}} \), whenever $E$ is a subset of a cube $Q$.

(c) 29 Oct. 2019

(49) page 720 line 18 from above, Proof of Theorem 6.4 (Major change).
(a) As a consequence there exists $0 < \alpha_0, \beta_0 < 1$ such that $w(E) \leq \beta_0 w(Q)$ for all measurable sets $E$ and $Q$ such that $Q$ is a cube containing $E$ and that $|E| \leq \alpha_0 |Q|$.
(b) If we set $\beta_0 = D\alpha_0^{\frac{1}{1+\varepsilon}}$ with $0 < \alpha_0 \ll 1$, then we have such that $w(E) \leq \beta_0 w(Q)$ for all measurable sets $E$ and $Q$ such that $Q$ is a cube containing $E$ and that $|E| \leq \alpha_0 |Q|$.

(c) 29 Oct. 2019

(50) page 720 line 19 from above, Proof of Theorem 6.4.
(a) If we contrapose this fact, then there exists $0 < \alpha, \beta < 1$ such that
\( w(E) \leq \beta w(Q) \)
for all measurable sets $E$ and $Q$ such that $Q$ is a cube containing $E$ and that $|E| \leq \alpha |Q|$.

(b) If we contrapose this fact and set $\alpha \equiv 1 - \beta\alpha_0$ and $\beta \equiv 1 - \alpha\alpha_0$.

(c) 29 Oct. 2019

(51) page 720 line 20 from above, Proof of Theorem 6.4.
(a) Denote by $M_{\text{dyadic}, Q}$ the dyadic maximal operator with respect to $Q$.
(b) Denote by $M_{\text{dyadic}, Q, w}$ the dyadic maximal operator with respect to $Q$ with weight $w$.

(c) 29 Oct. 2019

(52) page 720 line 25 from above, Proof of Theorem 6.4.
(a) $\{ x \in Q : M_{\text{dyadic}, Q, w}(x) > \lambda \}$
(b) $\{ x \in Q : M_{\text{dyadic}, Q, w}[w^{-1}](x) > \lambda \}$

(c) 29 Oct. 2019

(53) page 720 line 26 from above, Proof of Theorem 6.4.
(a) $\lambda \Omega_k \prod_{j=1}^{\infty} |\lambda_j \Omega_k|$
(b) $\lambda \prod_{j=1}^{\infty} |\lambda_j \Omega_k|$
(a) \((t^{-1}(x - z))^m\)
(b) \((t^{-1}(y - z))^{2m}\)
(c) 4 Nov. 2019
(d) This is by no means a mistake but it is somewhat strange.

(60) page 840, line 12 from above
(a) \(\leq t^{-n}\)
(b) \(\lesssim t^{-n}\)
(c) 4 Nov. 2019

(61) page 840, line 12 from above
(a) \((t^{-1}(y - z))^{2m}\)
(b) \((t^{-1}(y - z))^m\)
(c) 9 Nov. 2019

(62) page 841 (6.21), line 11 from below
(a) \(\gamma_t \cdot S^n_t\)
(b) \((\theta_t(e_1), \theta_t(e_2), \ldots, \theta_t(e_n)) \cdot S^n_t\)
(c) 9 Nov. 2019

(63) page 841 (6.231), lines 4 and 7 from below (three places)
(a) \(\|G - \gamma_t \cdot S^n_t G\|_2 \lesssim t\|\nabla G\|_{L^2} n\)
(b) \(\|G - (\theta_t(e_1), \theta_t(e_2), \ldots, \theta_t(e_n)) \cdot S^n_t G\|_2 \lesssim t\|\nabla G\|_{L^2} n\)
(c) 9 Nov. 2019

(64) page 842, title of § 6.7.2.3
(a) Applications of Gaffney-type estimates to commutators
(b) Commutator estimates
(c) 9 Nov. 2019

(65) page 842, line 2 from above
(a) \(-t \left\{ (M_A \nabla h) \cdot t \text{div}(\text{id}_{L^2} + t^2 L)^{-1} \right\}\)
(b) \(+t \left\{ (M_A \nabla h) \cdot t \text{div}(\text{id}_{L^2} + t^2 L)^{-1} \right\}\)
(c) 9 Nov. 2019

(66) page 846, line 3 from below
(a) We first control \(U_t \circ P_t\), as follows.
(b) We define \(P^n_t \in B(L^2(\mathbb{R}^n))\) naturally from \(P_t \in B(L^2(\mathbb{R}^n))\). We first control \(U_t \circ P^n_t\), as follows.
(c) 9 Nov. 2019

(67) page 846, Lemma 6.33
(a) \(U_t \circ P_t\)
(b) \(U_t \circ P^n_t\)
(c) 9 Nov. 2019

(68) page 847, line 5 from below
(a) \(U_t \circ P_t \circ Q_s\)
(b) \(U_t \circ P^n_t \circ Q^n_s\)
(c) 9 Nov. 2019

(69) page 847, line 4 from below
(a) where \(Q_s\) is given by (6.209)
(b) where \(Q^n_s \in B(L^2(\mathbb{R}^n), L^2(\mathbb{R}^n))\) is defined naturally from \(Q_s\) given by (6.209)
(c) 9 Nov. 2019

(70) page 848, line 3 from above
(a) Let us write \(E\) for the identity matrix of size \(n\). We define \(P^n_t \in B(L^2(\mathbb{R}^n))\) naturally from \(P_t \in B(L^2(\mathbb{R}^n))\), using the identity
(b) Let \(g \in \text{Dom}(L)\). We calculate
(c) 9 Nov. 2019

(71) page 848, lines 4–8 from above
Recall that we considered $U_t \circ P_t$ in Lemma (b)

$$
\theta_t \nabla g = \theta_t \nabla g - \gamma_t \cdot (P_t^2 \nabla g) + \gamma_t \cdot \theta_t \nabla (P_t^2 g)
$$

Recall that we considered $U_t \circ P_t^n$ in Lemma (c) 9 Nov. 2019

\[ \text{(72) page 849, line 7 from below} \]

(a) $\gamma_t(x) \cdot m_Q(G)$
(b) $(\theta_t(e_1)(x), \theta_t(e_2)(x), \ldots, \theta_t(e_n)(x)) \cdot S^n_t G$
(c) 9 Nov. 2019

\[ \text{(73) page 849, lines 4 and 7 from below} \]

(a) $\gamma_t \cdot S^n_t G$
(b) $(\theta_t(e_1), \theta_t(e_2), \ldots, \theta_t(e_n)) \cdot S^n_t G$
(c) 9 Nov. 2019

\[ \text{(74) page 849 line 11 from below} \]

(a) $\gamma_t \cdot S^n_t$
(b) $(\theta_t(e_1), \theta_t(e_2), \ldots, \theta_t(e_n)) \cdot S^n_t$
(c) 9 Nov. 2019

\[ \text{(75) page 848 line 9 from above, page 848 line 10 from above, page 848 line 1 from below, page 850 line 4 from above (twice), page 850 line 5 from above,} \]

(a) $P_t^2 \nabla g$
(b) $P_t^2 \nabla g$
(c) 9 Nov. 2019

\[ \text{(76) page 848, (6.239)} \]

(a) $\gamma_t \cdot P_t^2 \nabla g$
(b) $(\theta_t(e_1), \theta_t(e_2), \ldots, \theta_t(e_n))P_t^{n-2}\nabla g$
(c) 9 Nov. 2019

\[ \text{(77) page 849, (6.231), lines 4 and 7 from below} \]

(a) $\|\theta_t \cdot G - \gamma_t \cdot S^n_t G\|_2 \lesssim t\|\nabla G\|_{(L^2)^n}$
(b) $\|\theta_t - \theta_t(e_1, \theta_t(e_2), \ldots, \theta_t(e_n)) \cdot S^n_t G\|_2 \lesssim t\|\nabla G\|_{(L^2)^n}$
(c) 9 Nov. 2019

\[ \text{(78) page 849, line 7 from below} \]

(a) $\gamma_t(x) \cdot m_Q(G)$
(b) $(\theta_t(e_1)(x), \theta_t(e_2)(x), \ldots, \theta_t(e_n)(x)) \cdot m_Q(G)$
(c) 9 Nov. 2019

\[ \text{(79) page 850, lines 5,8 and 11 from above} \]

(a) $\gamma_t(x) \cdot S_1 \nabla f_{Q,w}^p(x)$
(b) $(\theta_t(e_1)(x), \theta_t(e_2)(x), \ldots, \theta_t(e_n)(x)) \cdot S_1 \nabla f_{Q,w}^p(x)$
(c) 9 Nov. 2019

\[ \text{(80) page 851, line 13 from above} \]

(a) $\| \cdot \|_2$
2. Missing lemmas/assertions

Here I will collect what I should have added in my book.

2.1. Page 628. In addition to the conclusions presented in Theorem 5.21, I should have proved that
\[ |\arg(\langle D^* ADx, x \rangle_H)\| < \frac{\pi}{2} - \theta \quad (x \in \text{Dom}(D^* AD)) \]
for some 0 < \(\theta\) < \(\frac{\pi}{2}\) independent of \(x \in \text{Dom}(D^* AD)\). This is achieved as follows:

Let \(x \in \text{Dom}(D^* AD)\). Then
\[ \langle D^* ADx, x \rangle_H = \langle ADx, Dx \rangle_H, \]
since \(ADx \in \text{Dom}(D^*).\) Since
\[ \Re(\langle ADx, Dx \rangle_H) \gtrsim (\|Dx\|_H)^2, \quad |\langle ADx, Dx \rangle_H| \sim (\|Dx\|_H)^2, \]
we obtain the desired result.


2.2. Page 632. (5.84) should have been proved. I omitted the proof. However, this can be found in the proof of Theorem 5.23.


2.3. Page 834. I would like to explain why (6.190) and (6.191) yield (6.192). It will be a solution to Exercise 6.90 at the same time.

This can be explained as follows: Since \(H^1(\mathbb{R}^n) \subset \text{Dom}(\sqrt{L}) \cap \text{Dom}(\sqrt{L^*})\), we have
\[ (\|\nabla f\|_{(L^2)^n})^2 \lesssim |\langle A \nabla f, \nabla f \rangle_{(L^2)^n}| \]
for all \(f \in \text{Dom}(L)\). If \(f \in \text{Dom}(L)\), then we have
\[ (\|\nabla f\|_{(L^2)^n})^2 \lesssim |\langle Lf, f \rangle_{(L^2)^n}| = |\langle \sqrt{L} \sqrt{L} f, f \rangle_{(L^2)^n}| = |\langle \sqrt{L} f, \sqrt{L^*} f \rangle_{(L^2)^n}|, \]
where we have used \(\sqrt{L^*} = (\sqrt{L})^*\) for the last inequality. By virtue of (6.191), we obtain
\[ (\|\nabla f\|_{(L^2)^n})^2 \lesssim \|\sqrt{L} f\|_{L^2} \|\nabla f\|_{(L^2)^n} \]
for all \(f \in \text{Dom}(L)\). Thus,
\[ \|\sqrt{L} f\|_{L^2} \lesssim \|\nabla f\|_{(L^2)^n} \]
for all \(f \in \text{Dom}(L)\). If we are given \(f \in \text{Dom}(\sqrt{L})\), then we consider \(f_j \equiv (\text{id}_H + j^{-1} L)^{-1} f\) for \(j \in \mathbb{N}\). We know that
\[ \|\nabla f_j - \nabla f_k\|_{(L^2)^n} \sim \|\sqrt{L} (f_j - f_k)\|_{L^2}. \]
Thanks to Lemma 5.21, $\sqrt{Lf_j} \to \sqrt{Lf}$. Thus, $\{\nabla f_j\}_{j=1}^{\infty}$ converges in $(L^2(\mathbb{R}^n))^n$. Since $f_j \to f$ in $L^2(\mathbb{R}^n)$ as is seen from

$$f_j - f = \sqrt{L}(\text{id}_H + j^{-1}L)^{-1}\sqrt{Lf} = O(j^{-\frac{1}{2}}),$$

we obtain $f \in \text{Dom}(\nabla) = H^1(\mathbb{R}^n)$ and $\nabla f_j \to \nabla f$. Thus by taking the limit in

$$\|\nabla f_j\|_{(L^2)^n} \sim \|\sqrt{L}f_j\|_{L^2},$$

we obtain $\|\nabla f\|_{(L^2)^n} \sim \|\sqrt{L}f\|_{L^2}$.


3. References

(1) See [1] for the Hesteness extension.
(2)
(3)

References