The Fourth Taiwan-Japan Joint Conference on Differential Geometry

Level set flow and the set of singular points

Siao-Hao Guo (National Taiwan University)

Abstract:
In this talk we will first introduce the level set flow, which is an extension of the mean curvature flow past its first singular time. Then we will show that how the rate of blow-up of curvature near a singularity is related to the distribution of surrounding singular points.

Limit theorems for the total scalar curvature

Shota Hamanaka (Mitsubishi Electric Advanced Technology R&D Center & OCAMI)

Abstract:
In this talk, we give some limit theorems for total scalar curvatures. More precisely, we show that the lower bound of the total scalar curvatures on a closed manifold is preserved under the convergence of the Riemannian metrics with respect to certain regularity provided that each scalar curvature is nonnegative.

The rigidity of asymptotically flat static spaces

Brian Harvie (National Center for Theoretical Sciences)

Abstract:
Asymptotically flat static spaces are Riemannian manifolds that correspond to static vacuum spacetimes in general relativity. The most important example is the Schwarzschild space, a rotationally symmetric Riemannian manifold corresponding to the Schwarzschild spacetime. A number of important questions about the uniqueness of the Schwarzschild spacetime may be posed as rigidity questions for AF static spaces. These include the famous static black hole uniqueness theorems of Israel and Bunting/Masood-ul-Alam as well as the more recent uniqueness theorems for static spacetimes containing photon surfaces.

In this talk, I will present a new approach to these questions that is based on a Minkowski-type inequality for AF static spaces. Like the Minkowski inequality for convex hypersurfaces in Euclidean space, this Minkowski inequality gives a bound from below on the total mean curvature of the manifold boundary in the AF static setting. First, I will characterize rigidity within this inequality, showing under suitable boundary assumptions that the equality is achieved only by rotationally symmetric regions of Schwarzschild space. As an application, we will solve the problem of static metric extension uniqueness for the Bartnik data of Schwarzschild coordinate spheres. This talk is based on joint work with Ye-Kai Wang of NYCU.

Smooth maps minimizing the energy and the calibrated geometry

Kota Hattori (Keio University)

Abstract:
In this talk, I will explain the generalization of calibrated geometry to smooth maps between Riemannian manifolds. Then I will show several examples of maps that fit our situation and maps that minimize some of the energies.

Results related to prescribing Gaussian curvature and geodesic curvature

Pak-Tung Ho (Tamkang University)

Abstract:
I will first talk about results related to prescribing Gaussian curvature on surfaces. Then I will talk about results related to prescribing Gaussian curvature or geodesic curvature on surfaces with boundary.

Discrete isothermic surfaces in architectural surface design

Yoshiki Jikumaru (Toyo University)

Abstract:
In architectural surface design, a geometric method that achieves both constructibility and mechanical properties must be established. On the other hand, a field called discrete differential geometry, which has been studied actively in recent years, is proving to be effective in achieving these purposes. In this talk, we introduce our recent activities on the concept of discrete isothermic surfaces, introduced by Bobenko and Pinkall, can be applied to architectural design and the feedback it gives to mathematics.

On the fundamental groups of compact aspherical manifolds with parabolic structures

Yoshinobu Kamishima (Josai University)

Abstract:
By a parabolic G-structure we mean a conformal structure, CR-structure, or quaternionic contact structure (qc for short) on a manifold M. In this talk we discuss how the parabolic G-structure determines the topology of closed aspherical G-manifolds X/Γ. More precisely, we shall prove (1) If Γ is virtually solvable, then a standard aspherical CR-manifold (equivalently aspherical Sasaki manifold) is diffeomorphic to the $2n+1$-dimensional Heisenberg infra-nilmanifold $N/\rho(\Gamma)$, where $\rho(\Gamma)$ is a discrete uniform subgroup of the CR-Hermitian group $N \times U(n)$. (2) If $\Gamma/Z(\Gamma)$ admits a torsion free normal subgroup (where $Z(\Gamma)$ is the center of Γ), then a standard aspherical qc-manifold is qc-isomorphic to the $4n+3$-dimensional quaternionic Heisenberg infra-nilmanifold M/Γ where Γ is virtually nilpotent in $M \times Sp(n)$. (1),(2) are strong rigidity results using the existence of Ricci flat Kaehler metrics. This is a joint work with Oliver Baues (University of Fribourg).

Topological recursion, exact WKB analysis, and the (uncoupled) BPS Riemann-Hilbert problem

Omar Kidwai (OIST & University of Birmingham)

Abstract:
The notion of BPS structure formalizes some of the output of the study of four-dimensional N=2 QFTs, as well as the Donaldson-Thomas theory of CY3 triangulated categories. Bridgeland formulated a certain Riemann-Hilbert-like problem associated to such a structure, seeking jumping functions in the $\hbar$ plane with given asymptotics — these appear in the description of complex hyperkahler metrics associated to the CY3 category, and physically correspond to the "conformal limit". Starting from a quadratic differential on a Riemann surface X, I'll recall how to associate a BPS structure to it, and explain, in the simplest examples, how to produce a solution to the Riemann-Hilbert problem using topological recursion, quantum curves, and Borel resummation. Based on joint work with K. Iwaki.

Sharp result for the spherical metric on flat tori with conical angle 6π at two symmetric points

Ting-Jung Kuo (National Taiwan Normal University)

Abstract:
We investigate the following curvature equation:

$$\Delta u + e^u = 8\pi(\delta_0 + \delta_{((\omega_k)/2)}) \tag{1}$$

in E_τ, where $\tau \in H(1)$. Here E_τ represents a flat torus and $(\omega_k)/2$ is one of the half periods of E_τ. Our primary objective is to establish a necessary and sufficient criterion for the existence of a non-even family of solutions of equation (1). Remarkably, this is equivalent to determining the presence of solutions for the equation with a single conical singularity. As an application, we offer explicit descriptions for the entire structure of solutions to equation (1) in the context of both rectangle tori and rhombus tori.

On the Convergence Properties of Sasaki-Ricci Flows

Chieh Lin (Chongqing University of Technology)

Abstract:
First, we introduce some history about the canonical Kähler metrics and the fundamentals of Sasakian geometry. And then we will discuss our main results about the convergence properties of the Sasaki-Ricci flow in the Cheeger-Gromov sense on a closed quasi-regular Sasakian manifold with the well-formed space of leaves of the Reeb foliation by following the ideas developed by Tian-Zhang in 2013. It is based on joint work with Shu-Cheng Chang and Chin-Tung Wu.

Symplectic fillings of unit cotangent bundles of spheres

Takahiro Oba (Osaka University)

Abstract:
This talk is concerned with symplectic fillings of contact manifolds. I will first give basic definitions and an overview of results on the topology of symplectic fillings, mainly focusing on uniqueness results of diffeomorphism types of fillings. After that, I will present a joint work with Myeonggi Kwon, in which we give some topological restrictions on symplectic fillings of unit cotangent bundles of spheres S^n and show the uniqueness of diffeomorphism types of them, under some condition, for the case $n = 3$.

The real part of a holomorphic function is harmonic

Thomas Raujouan (Kobe University)

Abstract:
Because the real part of a holomorphic function is harmonic, minimal surfaces can be described in terms of holomorphic data. This is the core of the Enneper-Weierstrass representation (1866), which has been used extensively to construct minimal surfaces as conformal immersions into the Euclidean 3-space. Since then, several Weierstrass-type algorithms have been developed in order to translate the method into other ambient spaces or for other constant mean curvatures. We will give an overview of these methods and show how they have been used to construct new examples of constant mean curvature surfaces and investigate their properties.

Construction of vacuum solutions of the Einstein equations using almost abelian Lie groups

Yuichiro Sato (Kogakuin University)

Abstract:
A spacetime is a Lorentzian manifold, then it is a solution of the vacuum Einstein equations without cosmological constant if and only if it is Ricci-flat. In this talk, we find new solutions to the vacuum Einstein equations without cosmological constant using almost abelian Lie groups. Firstly, we consider globally hyperbolic spacetimes in which almost abelian Lie groups act on the Cauchy hypersurfaces isometrically and simply transitively. Previous studies and our results show that such solutions are completely found for Lie groups with zero-dimensional moduli spaces of left-invariant Riemannian metrics. Secondly, we classify Ricci-flat Lorentzian almost abelian Lie groups. This talk is mainly based on the joint work arXiv:2304.10193 with Takanao Tsuyuki (Kogakuin Univ.).

Special Lagrangian desingularization and infinite-time singularity in Lagrangian mean curvature flow

Wei-Bo Su (National Center for Theoretical Sciences)

Abstract:
In this talk, I will explain the construction of a solution to Lagrangian mean curvature flow with infinite-time singularity. Our method is rooted in a parabolic gluing construction originally introduced by Brendle and Kapouleas. I will highlight the crucial role played by the obstruction of special Lagrangian deformation as the driving force behind the singularity formation. This talk is based on a joint work with Chung-Jun Tsai and Albert Wood.

Solutions of the tt^*-equations constructed from the $(SU_2)_k$-fusion ring, and Smyth potentials

Tadashi Udagawa (Waseda University)

Abstract:
Cecotti and Vafa introduced the tt^* equations(topological-antitopological fusion equations), whose solutions describe massive deformations of supersymmetric conformal field theories. From a mathematical point of view, the tt^* equations give examples of pluriharmonic maps into the symmetric space $GL_n(R)/O_n$. As a special case, the tt^*-equations include the sinh-Gordon equation, and more generally the tt^*-Toda equations which were studied by Guest, Its, and Lin. In this talk, we consider another series of examples. Solutions are constructed directly from a finite number of solutions to the radial sinh-Gordon equation, but the construction itself is quite different from the case of the tt^*-Toda equations. It involves the $(SU_2)_k$-fusion algebra, an object which has a prominent role in conformal field theory. The idea of the construction is due to Cecotti and Vafa, but we give a precise mathematical formulation and a description of the "holomorphic data" corresponding to the solutions by using the DPW method. Furthermore, we show that a natural equivalence relation on the representations of SU_2 corresponds to an equally natural notion of gauge equivalence on harmonic maps.

Positivity for holomorphic vector bundles

Kuan-Ru Wu (National Tsing Hua University)

Abstract:
It was conjectured by Kobayashi in the 70s that an ample vector bundle should carry a positively curved Finsler metric. In this talk, we will show that under some curvature assumptions the conjecture is true. The proof relies on our earlier convexity result for Finsler metrics. If time permits, we will also show that Kobayashi positivity is preserved under tensor product. This latter statement gives more evidence on the validity of Kobayashi's conjecture.

Discrete Weierstrass-type representations, and beyond

Masashi Yasumoto (Tokushima University)

Abstract:
Weierstrass-type representations are powerful tools to construct various surfaces, and to analyze their behaviors. It was recently shown that these representation formulae can be obtained by using integrable transformations. In this talk, building upon recent progress on discrete integrable transformations, we introduce that discrete Weierstrass-type representations can be also described in a way. This result covers all the known representation formulae, and new discrete surfaces arise. This is based on joint work with Mason Pember and Denis Polly. If time permits, we will introduce our attempt toward a discrete version of analytic extensions of discrete zero mean curvature surfaces in Minkowski space. This is an ongoing project with Shintaro Akamine, Joseph Cho, Wayne Rossman, and Seong-Deog Yang.

Special Hamiltonian S^1-actions on symplectic 4-manifolds

Mei-Lin Yau (National Central University)

Abstract:
In this talk we consider symplectic 4-manifolds M with vanishing first Chern class which admit a Hamiltonian S^1-action together with an equivariant Maslov condition on orbits of the group action. We call such spaces special Hamiltonian S^1-spaces. We classify all exact special Hamiltonian S^1-spaces and show that all of them admit the structure of a Stein surface. Moreover, we will discuss the classification of special Hamiltonian S^1-actions on 1-connected Stein surfaces up to equivariant symplectomorphisms.