Lotze's Theory of Concepts and Its prospects

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1 Introduction

The subject of this paper is Hermann Lotze's theory of concepts in his *Logik* written in 1874.[1] His *Logik* is one of the most influential philosophical texts of the second half of the 19th century, so its importance in the history of philosophy is unquestioned. As is well known, his influence is very widespread, ranging from Neo-Kantianism and phenomenology to early analytic philosophy and pragmatism [6, 11,12, 15,16].

In recent years, such as [2] and [18], many works which extensively deal with his life and philosophy have been published. Furthermore, there is even a magazine that features a special issue entitled "Lotze's Back" in 2018.[2] In this sense, it can be said that the movement of "Lotze Renaissance" is increasing. But even in such a reevaluation, it seemed to me that his theory of concepts, especially his remarks on the structure and formation of the concept has received little attention (compared to his famous doctrine of "validity").[3]

Hearing the name of Lotze, many people who belong to analytic philosophy may recall the controversy between Sluga and Dummett. The controversy that began in the 1970s was about the historical background of Frege's philosophy, especially, the influence of Lotze on Frege. I will not go into the details of it, but, following G. Gabriel[4], I would like to say a few words about it to clarify the standpoint of this article.

In [17], Sluga argues that Frege's philosophy, especially his idea concerning "objectivity" and "reality", is under the influence of Lotze (while Dummett denies this in [4]). Sluga's argument is mainly based on Bruno Bauch's paper "Lotzes Logik und ihre Bedeutung im deutschen Idealismus", written in 1918. As Sluga quotes, Bauch wrote that even Frege couldn't escape Lotze's influence.

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[1] Lotze's *Logik* written in 1874 is an elaboration of his earlier logical text in 1843. In this paper, *Logik* refers to the 1874 version. Every section number without the document name in the following citations refer to citations from *Logik*, vol. I. The English translation is based on Bosanquet's, or it has been changed by my preference.

[2] *Philosophical Reading*, 10-2, 2018. It contains nine article on Lotze's philosophy. But unfortunately, none of them deals with his theory of concepts in *Logik*.

[3] As far as I know, the only exception is the work by Jeremy Heis. In his series of studies such as [9,10], he argues that Lotze was an important step in the "history of the theory of concepts" from Kant to Frege and Cassirer.

[4] [6], p. 39.

> Of everything that has followed in the area of logic from Hegel to the present day, there is nothing that has surpassed Lotze's logical achievements in value. [...] His influence reveals itself in every important figure in the area of logic no matter what philosophical direction he might belong to. If he has any claim to significance in logic, he cannot have remained uninfluenced by Lotze. ([Bauch1918], p. 45, quoted in [17], p. 53.)

Furthermore, according to Sluga, Bauch stressed four points of Lotze's thought that are presumably significant in understanding Frege: (1) Lotze's anti-psychologism, (2) his distinction between an object of knowledge and its recognition, (3) his reformulation of the Platonic theory of ideas as an ontology free theory of objectivity (i.e. his theory of validity), and (4) his account of concepts as functions.[5] Our concern lies in (4) while the controversy between Sluga and Dummett mainly concerns (3).[6]

It is commonly known that, for Frege, a concept is a function from one or more objects (or concepts if it is higher-order one) to truth-values. Lotze also considers concepts to be "functions" and argues that they should be expressed using functional symbols. Though, as we will see, there is a great difference between what they mean by "concept as function". Therefore, as Heis correctly points out, "one can see a 'substantial influence' of Lotze on Frege [...] only if one seriously distorts Frege's or Lotze's philosophy".[7] For this reason, I agree with Dummett (and Heis) and take a cautious attitude about the substantial influence between them.

However, it should be noted here that, even if we deny the substantial influence on Frege, this does not mean that Lotze's theory of concepts, and hence his logic, does not reach the current standards (based on Frege's quantified logic), and therefore the study of Lotze's philosophy is at best of historical interest. If we examine the remarks in his theory more closely, we can see that his theory has no less insight than Frege's. Furthermore, we can recognize the central features of his theory are adopted in the conceptual models used in current information science and cognitive science.

In this paper, I will review the outline of Lotze's theory of concepts in *Logik* and examine its philosophical prospects. This paper will proceed as follows. In section 2, I will present the outline of Lotze's theory of concepts mainly discussed in his *Logik*, I-C, "The Formation of the Concept". Its basic framework essentially consists in his remarks on abstraction and structures of concepts, and subordinate relations derived from them. In section 3, I will give two candidates for formalizing his theory: one is the framework of "relational model" in information science, and the other is that of "conceptual space", which Peter Gärdenfors proposes in cognitive science. Finally, in section 4, I will give a brief comment on some remaining issues toward the reevaluation of Lotze's theory of concepts.

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[5] [17], p. 53.

[6] As to (4), Bauch wrote as follows: "The notion of a function which was taken by Lotze from mathematics and made fruitful in logic has received a brilliant development in mathematics again on the basis of Logic. The altogether classical proof of this is the mathematical work of Frege. The interrelation of logic and mathematics prepared by Lotze also explains the tight connection between Laze and Kantian philosophy, not least with reference to the notion of function. For it is through Lotze [...] that Kant' s idea that transcendental laws or forms, just as much as judgments, are really functions and that concepts rest on functions receives its further elaboration and reformulation." ([Bauch1918], pp. 47-8, quoted in [17], p. 57.)

[7] [9], p. 301.

2 The Outline of Lotze's Theory of Concepts[8]

The basic framework of Lotze's theory of concepts is based on his considerations on abstraction and structures of concepts. As Heis points out, we can understand them as objections to the traditional understanding of concepts.[9]

2.1 "Abstraction" does NOT mean a mere Omission

The first consideration concerns the operation of "abstraction". According to the traditional understanding, which probably originates in Locke,[10] we obtain concepts by *abstraction*, that is, by leaving out what is different in a series of observed particulars. For instance, we form the concept of a triangle by observing particular triangles each of which is either acute, obtuse, or right, and then removing the particular features which differentiate them, resulting in a concept which neither acute, obtuse nor right.[11] In this sense, according to the traditional account, abstraction is nothing but a mere *omission* or *negation* of differences between particulars and hence consists in the reduction of their content, that is, the number of the marks that they have. In his *Substanzbegriff und Funktionsbegriff*, Cassirer described the operation of abstraction understood in this way as the procedure to obtain a common feature a from a series $a\alpha_1\beta_1, a\alpha_2\beta_2, a\alpha_3\beta_3, \cdots$.[12]

Lotze argues that this traditional account does not capture our practice of thinking. He points out that the actual procedure of abstraction involves not only negation in the above sense, but also a kind of "affirmation".

> Gold, silver, copper, and lead differ in colour, brilliancy, weight, and density; but their universal, which we call metal, is not found upon comparison by simply leaving out these differences without compensation. Clearly it is no sufficient definition of metal to say negatively, it is neither red nor yellow nor white nor grey; the affirmation, that it has at any rate some colour, is equally indispensable; it has not indeed this or that specific weight, this or that degree of brilliancy, but the idea of it would either cease to have any meaning at all, or would certainly not be the idea of metal, if it contained no thought whatever of weight, brilliancy, and hardness. (§23)

What we do in forming the concept of metal from particular metals (gold, silver, copper, etc.) is *not* simply to omit or negate the differences between them such as their colors (yellow, white, red, etc.) so that metal in itself has no color. We also need to affirm that it has *some* color. Similarly, when we form the concept of an animal by abstraction from dogs, birds, fish, etc., we not only deny their particular modes of self-motion such as walking, flying and swimming, but also affirm that any animal has *some* mode of self-motion.

Thus Lotze proposes that the procedure of abstraction should be understood as that of *substitution* rather than omission.

[8] The critical appendix to Boolean logic that was added in the 2nd edition of *Logik* (1880) is based on his theory of concepts for the most part. It is very interesting that Frege was in the midst of writing a critical essay [5] to Boolean logic when the second edition of *Logik* was published. Heis discussed the historical relationship between their criticism in [9], Appendix 8.

[9] cf. [9], chapter 3, section 3.

[10] [13], III-iii-6.

[11] cf. [16], p. 152.

[12] [3], p. 29.

> [...] the universal is produced, not by simply leaving out the different marks p^1 and p^2, q^1 and q^2, which occur in the individuals compared, but by substituting for those left out the universal marks [die allgemeinen Merkmale] P and Q, of which p^1p^2 and q^1q^2 are particular kinds. (ibid.)

For Lotze, abstraction is the procedure of substituting for different marks that appear in a series of partuicular things being compared —for example, the marks corresponding to the colors peculiar to gold, silver, etc.—the higher mark which has these marks as its species—in the present case, the mark "some color" or "colored".[13]

The relationship between "particular marks (die besondern Merkmale)"[14], that is, the marks peculiar to the particulars being abstracted (p^1p^2, q^1q^2)and universal marks introduced by abstraction (P,Q) can be thought as one between variables and values they can take. Again, Cassirer described the operation characterized in this way as the procedure of obtaining axy from a series $a\alpha_1\beta_1, a\alpha_2\beta_2, a\alpha_3\beta_3, \cdots$ by using the variables x and y over the sets $\alpha = \{\alpha_1, \alpha_2, ...\}$ and $\beta = \{\beta_1, \beta_2, ...\}$ respectively.[15]

As is clear from Cassirer's formulation, for Lotze, abstraction no longer means the reduction in the *number* of marks. Thus he argues that "[o]f the true universal, [...] its content is always precisely as rich, the sum of its marks precisely as great, as that of its species themselves"(§31), hence the inverse ratio of intension and extension, which is a central conclusion of the traditional understanding, doesn't hold in general. Therefore, for him, the system of our concepts does not form a unimodal pyramid beginning in the broad base of all singular concepts and ending in the all-embracing concept of "the thinkable" or "something".[16]

2.2 "Concept as Function" in Lotze

The second consideration concerns the structures of concepts. Again, according to the traditional understanding, each concept is composed of its constituents (marks) in terms of *conjunction*. For example, the concept ⟨gold⟩ can be represented as ⟨yellow⟩ *and* ⟨gold⟩. In a concept thus formed, as we can see, its marks are simply *listed* or *summed*, hence they are assumed to be *independent* of each other.

Against this understanding, Lotze points out that the structures of our concepts are more complex than the traditional understanding assumes. He often explains this by using the example of the concept ⟨triangle⟩.

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[13] Lotze thinks abstraction in this way because he recognizes the element of *necessity* that the traditional model could not capture. For example, various species in animals *necessarily* have their own way of self-moving although their ways may differ from one another. However, the traditional model of abstraction cannot respect the necessity in this sense. For the marks representing their mode of self-motion are omitted by abstraction due to differences between them, together with the marks representing features whose presence is *contingent* among the species (such as the features of their body-hair). This idea is indicated in his remark on when the traditional model of abstraction (i.e. abstraction as a mere omission) should be applied. "The simple process of leaving out only takes place when one of two individuals compared actually possesses no species of a mark P, of which some species is a *necessary* mark of the other. Thus we suppose, whether rightly or wrongly does not matter, that we cannot find in plants any trace of sensation and self-movement, both of which are essential to all animals; we do therefore form the universal idea of organic being from a comparison of plant and animal by leaving out these marks without compensation. " (§23, my emphasis)

[14] cf. §35.

[15] [3], p. 29.

[16] §33. According his theory of concepts, Lotze speculates that the system will take the form of a multi-modal "mountain-chain" with all singular concepts as its common base.

> The three sides of a triangle are not merely there *as well as* the three angles; they must form the angles by their intersection .... (§28)

> The concept of a triangle does not consist in the fact that we think three angles *and* three sides, but in the fact that three sides intersect one another so as completely to bound a plane space and by this very fact produce the angles. It is this connexion of the sides and angles which makes equiangular unequilateral and rectangular equilateral triangles impossible .... (§126)

For three sides to form a triangle, the length of each side cannot be set independently. They must stand in a certain relation and *determine each other*. This relation can be expressed by triangle inequality. Thus Lotze argues that the constituents of our concepts must also be *mutually dependent* and determine each other. And he says that such interdependency and inter determinacy cannot be expressed in terms of addition (i.e. conjunction), but *functional* symbol.[17]

> [...] as a rule, the marks of a concept are not coordinated as all of equal value, but [...] they stand to each other in the most various relative positions, offer to each other different points of attachment, and so mutually determine each other; and [...] an appropriate symbol for the structure of a concept is not the equation $S = a + b + c + d$, etc., but such an expression as $S = F(a, b, c,$ etc.) indicating merely that, in order to give the value of S, a, b, c, etc, must be combined in a manner precisely definable in each particular case, but extremely variable when taken generally. (§28)

It is in this sense that Lotze thinks concepts are "functions". It will be clear that there is a great difference between what Lotze and Frege mean when they mean "concepts as functions". (*Note to myself: More detailed comparison with Frege should be inserted.*)

2.3 Ground and Structure

Behind the idea of concepts as functions described above is Lotze's view on the role that concepts play in our intellectual activities. He explains what it is for something to be a concept or understood conceptually (begrifflich gefaßt) as follows.

> I speak of any composite matter s as conceptually conceived [begrifflich gefaßt] or as a concept, when it is accompanied by the thought of a universal S, which contains the conditional *ground* of the coexistence [den bedingenden Grund für das Zusammensein] of all its marks and of the form of their connexion. [...] For when we observe a new object s for the first time, and not content with the perfectly clear sensible perceotion of it, go on to ask what it really is, we clearly whant to know the *rule* [Regel] which connects the perceived marks in the observed fact and converts them into a coherent whole [ein zusammengehöriges Ganze] of a definite and predictable [voraussagbaren] character. (§26, my emphasis)

[17] In the following quotation, Lotze doesn't use the word "function (Funktion)" explicitly, but in some later sections (e.g. §110) he refers such structures as "functions" of their constituents. See also [9], chp. 3, §3.

Lotze repeatedly refers to the concept as containing a *rule* or *ground* that explains the coexistence of the perceived marks in those observed things that fall under it.[18] It seems very difficult to draw a single consistent motif for the idea of "concept as rule/ground", which we can go back to Kant.[19] But at leaset the above passage suggests that it can be related to the notion of *prediction*. Indeed, in the same section, Lotze says that a concept is a definite general image taken as "the scheme [das Schema] according to which the connexion of the marks observed here with one another and with the future behaviour to be expected from them [dem künftig von ihnen zu erwartenden]". The question then is how these aspects of concepts connect to their functional structure described in the previous section. Although Lotze hardly expresses this connection explicitly, if we use his favorite example, i.e. the concept ⟨triangle⟩, this connection can be illustrated as follows. (It seems to me that his remarks on the reformulation of the syllogism (§106, 110) indicate the possibility of the following explanation.)

Suppose that we want to know the lengths of three sides a, b, c of *some* figure X and observe the length of a is 2 and that of b is 4. From these observations, we can only infer the trivial fact that their lengths are so. For the fact that X is *some* figure does not tell us the relation between its sides, so we can only assume the lengths of them are *independent* of each other. But once we find that X is a triangle and hence conceive it through the concept ⟨triangle⟩, we can find that the length of c must be within the range $2 < c < 6$. We can do this because the concept tells us that the sides of those objects which fall under it *stand in a certain relation* (i.e. that of triangle inequality), and hence the possible length of c *depends* on the lengths of the two sides already observed.

2.4 Subordinate Relations

Given the considerations on abstraction and the structure of concepts described above, what is the relation between a concept and its subordinates like? Lotze illustrates what it is like as follows.

> [...] the universal concept, the genus, contains a number of marks in a merely indefinite and even universal form; these are represented in the species by definite values or particular characterisations, and finally in the singular concept all indefiniteness vanishes, and each universal mark of the genus is replaced by one fully determined in quantity, individuality, and relation to others. (§31)

We can see that (1) concepts consist of "indefinite" universal marks and (2), in its subordinates (i.e. those which belong to it), each universal mark is replaced with a certain "definite" particular mark which it takes as "value". Therefore, each subordinate of a concept can be expressed as a combination of certain values that each universal mark of it can take.

Let's take an example. Suppose that our concept of animal has the mode of self-motion, reproduction, breathing as its universal marks, each of which has its values (particular marks) as follows.[20]

[18] cf. §27, 120

[19] cf. Kant, XI, pp. 95-6, A106. See also [8], §1.1.

[20] cf. §23.

universal mark	*particular mark*
self-motion	`run`, `fly`, `swim`, ...
reprduction	`viviparous`, `egg`, `division`,...
breathing	`lung`, `gill`, `skin`,...

Then, we can express the subordinates of this concept, that is, the animal species such as dog, bird, fish, etc., as the triples of values in the following way.

$$\begin{aligned} \text{dog} &= \langle \texttt{run}, \texttt{viviparous}, \texttt{lung} \rangle \\ \text{bird} &= \langle \texttt{fly}, \texttt{egg}, \texttt{lung} \rangle \\ \text{fish} &= \langle \texttt{swim}, \texttt{egg}, \texttt{gill} \rangle \end{aligned}$$

The values which the subordinates of a concept take can still be concepts or include some indeterminate (universal) marks in its component marks. For instance, the value `fly` may have very various modes of flight peculiar to the concepts of individual birds. However, according to Lotze, the values which appear in the singular concept, the lowest element in the subordinate relations, are completely determined.

If we understand subordinate relations in this way, we can understand more specifically what is meant by "functional" structures of concepts. As we have seen in §126, the concept of a triangle makes equiangular unequilateral and rectangular equilateral triangles *impossible*. This means that the structure of it imposes certain constraints on the possible values its subordinates (the concept of individual triangles) can take, thereby eliminates certain combinations of values as impossible ones. Thus we can think the "functional" structure of a concept, that is, the interdependency of its marks, consists in excluding certain value-combinations of their subordinates as impossible ones. Thus the concept ⟨triangle⟩ does not admit the combinations of the side lengths such as $\langle 1, 3, 5 \rangle$, which fail to hold triangle inequality. Conversely, if we count the combination $\langle \texttt{fly}, \texttt{egg}, \texttt{gill} \rangle$ in the concept ⟨aminal⟩ described above as impossible ones, we are assuming that there is some "functional" structure in it whether explicitly or not. (In this case, perhaps we see some constraint between the modes of self-motion and breathing.)

3 Formalizing Lotze's Theory: Two candidates

So far, we have seen the basic framework of Lotze's theory of concepts in *Logik*. In this section, I will give two promising candidates for formalizing his theory: one is the framework of "relational model" in information science, and the other is that of "conceptual space" in cognitive science.

3.1 Relational Model

We begin to see that universal marks can be understood as functions in the normal set-theoretical sense. More specifically, we can represent a universal mark as a function from the set of subordinates of the concept which contains it to the set of its particular marks. For example, in the case of ⟨animal⟩, we can define the universal

mark of self-motion as a function $\mathbf{Sm} : \mathscr{A} \to \mathscr{S}$ such that

$$\begin{aligned} \mathbf{Sm}(\text{dog}) &= \texttt{run} \\ \mathbf{Sm}(\text{bird}) &= \texttt{fly} \\ \mathbf{Sm}(\text{fish}) &= \texttt{swim} \\ &\vdots \end{aligned}$$

where $\mathscr{A} = \{\text{dog}, \text{bird}, \text{fish}, \cdots\}$ and $\mathscr{S} = \{\texttt{run}, \texttt{fly}, \texttt{swim}, \cdots\}$.

Now, we can represent a concept as the *product map* of these functions. Thus the concept of animal is given as a function $\mathbf{Animal} = \langle \mathbf{Sm}, \mathbf{Rep}, \mathbf{Br} \rangle : \mathscr{A} \to \mathscr{S} \times \mathscr{R} \times \mathscr{B}$ (where $\mathbf{Rep}, \mathbf{Br}$ and $\mathscr{R}, \mathscr{B}$ are defined in the same way as $\mathbf{Sm}$ and $\mathscr{S}$). Then conceptual understanding [Begreifen] (e.g. thinking dogs through the concept of animal) can be represented by the functional application, for example, as follows.

$$\begin{aligned} \mathbf{Animal}(\texttt{dog}) = \langle \mathbf{Sm}, \mathbf{Rep}, \mathbf{Br} \rangle (\texttt{dog}) &= \langle \mathbf{Sm}(\texttt{dog}), \mathbf{Rep}(\texttt{dog}), \mathbf{Br}(\texttt{dog}) \rangle \\ &= \langle \texttt{run}, \texttt{viviparous}, \texttt{lung} \rangle \\ \mathbf{Animal}(\texttt{bird}) &= \langle \texttt{fly}, \texttt{egg}, \texttt{lung} \rangle \\ \mathbf{Animal}(\texttt{fish}) &= \langle \texttt{swim}, \texttt{egg}, \texttt{gill} \rangle \\ &\vdots \end{aligned}$$

The "functional" structure of a concept is represented by the logical formulae which express the relations between the functions corresponding to the universal marks of the concept. For example, the concept of a triangle, whose universal marks correspond to the functions $\mathbf{L}_a, \mathbf{L}_b, \mathbf{L}_c$ that output the lengths of its three sides a, b, c respectively, has the following "functional" structure. ($\mathscr{T}$ is the set of individual triangles.)

$$\forall x_{\in \mathscr{T}} (|\mathbf{L}_b(x) - \mathbf{L}_c(x)| < \mathbf{L}_a(x) < \mathbf{L}_b(x) + \mathbf{L}_c(x))$$

In fact, it is very common in "Knowledge Representation (KR)", the field of information science to represent our concepts as logical structures of functions. In Knowledge Representation, such structures are used to represent our concepts or knowledge, and they are called "relational models" or "relational data". The above **Animal** is one the simplest relational models.

In Relational Database Theory, which studies various operations on relational models, each relational model is normally represented in terms of tabular structures. In this theory, what amounts to"universal marks" is called "attributes". For instance, **Animal** is represented by the following table.

	Attributes		
Animal	`Selfmotion`	`Reproduction`	`Breathing`
dog	run	vivioparous	lung
bird	fly	egg	lung
fish	swim	egg	gill

The constraint on the combinations of values that each attribute can take is one of the most basic constraints studied in this theory, called a "tuple constraint". [21] The "functional" structures of concepts we have seen above is an example of this constraint.

[21] [1], p. 95.

3.2 Conceptual Space

There is another candidate for formalizing Lotze's theory. It is the framework called "conceptual space", which Peter Gärdenfors proposes in cognitive science.

In this framework, as its name indicates, concepts are treated as specific regions in multidimensional spaces. A conceptual space consist of a number of "quality dimensions", which denote basic features by which concepts and objects can be compared. In conceptual spaces, *points* denote objects, and *regions* denote concepts.[22] In [7], Gärdenfors give interesting analysis to properties of "natural" categories such as our color-concepts and "prototype effects" of concepts such as ⟨bird⟩ by using the geometric structures which these concepts have in conceptual spaces.

Formally, a conceptual space S is represented as $D_1 \times \cdots \times D_n$, where $D_1, ..., D_n$ are quality dimensions. A point in S, i.e., an object in the space, is represented by a vector $v = \langle d_1, ..., d_n \rangle$ such that $d_1 \in D_1, ..., d_n \in D_n$.

As its formal description clearly shows, the framework of concept space is closely related to that of relational model. In particular, quality dimensions correspond to attributes, hence universal marks. Furthermore, since to think a concept as a *specific* region in its conceptual space is to *exclude* the points that are not included in the region, the geometric structure a concept has in its conceptual space is considered to express the tuple constraint in its relational model, its"functional" structure in Lotzean sense.

4 Toward the Reevaluation of Lotze's philosophy

As we have seen in the previous section, Lotze's theory of concepts can be connected with the frameworks currently adopted in information science and cognitive science. It seems to me that this fact shows a remaining issue toward the reevaluation of his philosophy. It concerns his place in post-Kantian philosophy.

In recent years, information science and cognitive science have taught us that our concepts serve as interfaces between ourselves and the environment that surrounds us. For example, in Relational Database Theory, our concept of the book may be represented as the logical structure consisting of very different and heterogeneous attributes such as title, author, genre, price, ISBN, and even names of salespersons, etc..[23] Each of these attributes reflects how we interact with books and the various activities we do with them. Indeed, we usually purchase, classify, and search for books based on information carried by these attributes. More importantly, we even use these attributes as a criterion for something to be a book. In current book classification system, a book without ISBN will perhaps not be entered into the book database and may no longer be counted as "book".

The fact that Lotze's theory of concepts can be connected naturally with the modern frameworks seems to show that he actually recognized, as they have taught, that our concepts could serve as a basis for classification and exploration or a criterion of application. Indeed, this speculation may be supported by the fact that the remarks on the structures of concepts as seen in this paper recurs at the beginning of III-C in *Logik*, which deals with the methodology of classification.

On the other hand, the connection with modern frameworks raises some problems on Lotze's place in post-Kantian philosophy. As we have seen, Lotze often uses

[22] cf. `https://en.wikipedia.org/wiki/Conceptual_space`
[23] cf. [1], p. 8.

geometrical concepts to illustrate that our concepts have a more complex structure than normally considered. These concepts have been favored by philosophers who equate the possession of a concept with the "generation" of objects belonging to it hence the "existence" of such objects. They have often argued that geometrical concepts correspond to their method of construction and that, for example, to possess the concept of circle is nothing but to be able to draw circles on paper and make them exist. And it is also true that Lotze was in the midst of the development of such thought. Hegel and Cassirer, who appear before and after Lotze as the important figures in the development of post-Kantian thought sketched by Heis in [9] and [10], are just such philosophers. In particular, Cassirer eloquently discusses, in [3], the generative aspect of concepts through contrasting the traditional model of concept formation with his new model, which he calls "*Funktionsbegriff*".[24]

On the other hand, as is clear from the examaple given above, such identification is not usually adopted in the modern frameworks. Then, it will be necessary to examine once again whether it is also true of Lotze. (His *Logik* seems to be more practical that the logical writings of the same period or earlier, which might be because he had a medical degree and was well versed in biology. Perhaps he might be more favor of the modern way of thinking than Hegel and Cassirer's.)

There is no space left to discuss this. However, I believe, Lotze's theory of concepts in *Logik* can certainly be seen as an important link between current studies and Kantian philosophy. It shows that we need further research on his philosophy.

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[24] I would like to examine in another paper the relationship between Cassirer's theory of concepts in [3] and Lotze's theory of concepts, especially, their notions of "function".

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