LINEAR G_a-ACTIONS AND HILBERT'S FOURTEENTH PROBLEM

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Abstract. The classical action of SL_2 on the vector space of binary forms, together with its restricted $\mathbb{G}_a$-action, were of central interest in 19th Century invariant theory. Gordan (1868) showed that the invariants of the SL_2-action are finitely generated; the Mauer-Weitzenböck Theorem (1899/1932) shows the same for the $\mathbb{G}_a$-action. Hilbert (1900) famously asked if the invariant ring of a general algebraic group action is finitely generated, and Nagata (1957) provided the first counterexamples. We begin by looking at the linear $\mathbb{G}_a$-actions – though finitely generated, their invariant rings remain largely unknown. Then we will look at examples of $\mathbb{G}_a$-actions on $\mathbb{A}^n$ where the invariant rings are not finitely generated, and show how these lead to linear actions of unipotent groups whose invariant rings are not finitely generated.

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