A large class of relations for multiple zeta values called the derivation relations was proved by K. Ihara, M. Kaneko and D. Zagier. They also suggested a possible extension of the derivation relations and Kaneko recently formulated it explicitly as a conjecture.

In this talk, we give a proof of the conjecture by reducing it to a class of relations studied by G. Kawashima. On the way, we find some algebraic properties of the extended derivation operator “$\partial_n^{(c)}$” ($n \geq 1, c \in \mathbb{Q}$) on the non-commutative polynomial ring $\mathbb{Q}(x, y)$, which has been devised by modeling a Hopf algebra developed by A. Connes and H. Moscovici.