

Spherical functions on p -adic homogeneous spaces

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Let \mathbb{G} be a linear algebraic group and \mathbb{X} a \mathbb{G} -homogeneous affine algebraic variety both defined over a p -adic field k , where we assume a minimal k -parabolic subgroup \mathbb{B} of \mathbb{G} has an open orbit \mathbb{X}^{op} .

A nonzero K -invariant function Ψ on $X = \mathbb{X}(k)$ is called a *spherical function on X* if it is a common $\mathcal{H}(G, K)$ -eigen function, where $\mathcal{H}(G, K)$ is the Hecke algebra of $G = \mathbb{G}(k)$ with respect to a maximal compact open subgroup K . Spherical functions on homogeneous spaces are an interesting object to investigate and basic for the study of harmonic analysis on G -space X . Also they have a natural close relation to Igusa local zeta functions.

The explicit formula of spherical functions for group cases are given by I. G. Macdonald[M] and also by W. Casselman[C] by representation theoretical method.

In the talk, after reviewing the formulation of Casselman, we give an *expression of spherical functions* based on the data of G and their functional equations with respect to (a subgroup of) the Weyl group, which is a refinement of a result in [H1] inspired by a technique used by O. Offen[O]. Then we formulate *functional equations* attached to a simple root, and explain they are reduced to those of p -adic local zeta functions of small prehomogeneous vector space of limited type.

References

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