A certain estimate appearing in the Atkinson-type formula of cusp L-functions

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Jutila[1] gave a new proof of Atkinson’s formula, which is an estimation for the mean square of Riemann’s zeta-function on the critical line, using the method of Laplace transforms. He also demonstrated that cusp L-functions attached to $\text{SL}_2(\mathbb{Z})$ and weight $k$ have an analogue of Atkinson’s formula:

$$E(T) = \int_0^T Z \left( \frac{1}{2} + it \right) dt$$

Remark 1: The analytic function $Z(s), (s \in \mathbb{C})$ is a product of the cusp L-function and an entire function.

Remark 2: In this case, we have no "main" term.

We are interested in the error term $E(T)$ and already showed an estimation analogue of Heath-Brown[2]. Today we will show $E(T)$ changes its sign as an analogue of a result of Heath-Brown and Tsang[3].

