P-ADIC AND MOTIVIC ZETA FUNCTIONS

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We will introduce Igusa's p-adic zeta function $Z_p(f;s)$, associated to a polynomial f in $\mathbb{Z}[x_0,\ldots,x_m]$ and a prime number p. It is a rational function in p^{-s}, where s is a complex variable, and it is defined by means of a certain p-adic integral. The p-adic zeta function contains information on the number of solutions of the congruences $f(x) \equiv 0 \mod p^n$, and its poles reflect the asymptotic behaviour of these numbers as n tends to ∞. One of the most intriguing open problems around this object is the so-called p-adic monodromy conjecture, stating that the poles of $Z_p(f;s)$ are closely related to the eigenvalues of monodromy on the nearby cohomology of f at complex points of the hypersurface $f=0$, for almost all primes p.

Denef and Loeser observed that the behaviour of $Z_p(f;s)$ for varying p is controlled by a "motivic" object, the so-called motivic zeta function $Z(f;s)$ associated to f. This object is defined over the field of rational numbers, and "specializes" to $Z_p(f;s)$ for almost all primes p. They also formulated the motivic monodromy conjecture, replacing the poles of $Z_p(f;s)$ by those of $Z(f;s)$. This motivic conjecture implies the p-adic one, since no new poles appear under the specialization $Z(f;s) \rightsquigarrow Z_p(f;s)$.

The motivic zeta function is a rational function over a certain Grothendieck ring of varieties, and can be defined by means of a motivic integral. We will give a short introduction to the theory of motivic integration and motivic zeta functions. If time permits, we will also go into the following topics:

- p-adic and motivic zeta functions in several variables, and a generalization of the monodromy conjecture, where the nearby cycles are replaced by the Alexander complex,
- "twisted" versions of the p-adic and motivic zeta functions w.r.t. a multiplicative character χ,
- a hint at a new approach to motivic zeta functions and the monodromy conjecture, using the framework of non-archimedean analytic geometry.