Seshadri constants and Okounkov bodies.

§0. Intro

X : proj var / $\mathbb{C}$

L : ample line bdl

$$\swarrow \qquad\qquad \searrow$$

Seshadri const.　　　　Okounkov bodies $\subset \mathbb{R}^n$

の 2つが 定まる。

(toric geom における polytope の一般化 にもなっている)

thm The Okounkov body of L has some information of the Seshadri constant of L.

§1. Seshadri constants.

Def $\left(\begin{array}{l} X : \text{proj var} \\ L : \text{ample} \quad , \ p \in X \end{array} \right.$

$$\varepsilon(X, L; p) \underset{\text{def}}{=} \inf_{C} \frac{L \cdot C}{m_p C} \quad (>0)$$

($m_p C$: multiplicity at p)

$$= \max \{ t > 0 \mid \pi^* L - tE : \text{nef} \}$$

$$\pi : \hat{X} \to X, \quad E \subset \hat{X}, \quad p \in X$$

(b-up at p)

$$= \lim_{k \to \infty} \frac{j(kL, p)}{k}$$

$$j(kL, p) = \max \{ j \geq 0 \mid H^0(kL) \twoheadrightarrow L \otimes \mathcal{O}/m_p^{j+1} \}$$

$\varepsilon(X, L, 1) = \varepsilon(X, L, p)$
for general pt $p \in X$.

Lem L: big $\rightsquigarrow$ $\varepsilon(X, L; 1)$ can be defined.

§2. Okounkov bodies

X: proj var
$p \in X$: smooth
L: big
$z = (z_1, \dots, z_n)$: local coords at p

$$\nu : \mathcal{O}_{X,p} \setminus 0 \longrightarrow \mathbb{N}^n$$
$$f = \sum_{u \in \mathbb{N}^n} a_u z^u \longmapsto \min\{u \mid a_u \neq 0\}$$

$L_p \simeq \mathcal{O}_p$, fix.

$$H^0(kL) \setminus 0 \to L_p^{\otimes k} \setminus 0 \xrightarrow{\sim} \mathcal{O}_p \setminus 0 \xrightarrow{\nu} \mathbb{N}^n$$

image $\{ \nu(kL) \} \subset \mathbb{Z}^n$.

$$\Delta_z(L) := \overline{\bigcup_{k>0} \tfrac{1}{k} \nu(kL)} \subset \mathbb{R}^n$$

§3. Result

Def $\Delta \subset \mathbb{R}^n$ closed convex body

we define $s(\Delta) > 0$ as follows: $(n \geq 2)$

<u>When</u> Δ = rational polytope

$$S(\Delta) = \varepsilon(X_\Delta, L_\Delta; 1)$$

L_Δ: $\mathbb{Q}$-div

X_Δ: (toric var associated to Δ.)

general case,

$$\Delta_1 \subset \Delta_2 \subset \cdots \qquad \Delta = \overline{\bigcup \Delta_i}$$

$\Delta_1 \subset \Delta_2 \subset \cdots$: rational

$$S(\Delta) := \lim_{i\to\infty} S(\Delta_i) \geq 0.$$

<u>Thm</u> X: proj var
L: big
ξ, p: as above

$$\rightsquigarrow \varepsilon(X, L; 1) \geq S(\Delta_\xi(L))$$

<u>Idea of proof</u>

$$\Gamma := \bigcup_{k\geq 0} \{k\} \times \nu(kL) \subset \mathbb{N} \times \mathbb{N}^n$$

if Γ is f.g.

$$(X, L) \rightsquigarrow (X_{\Delta_\xi(L)}, L_{\Delta_\xi(L)}) \text{ 退化.}$$

$\varepsilon(X, L; 1)$ ⟷ (X, L), $S(\Delta_\xi(L))$ ⟷ $L_{\Delta_\xi(L)}$

ここで non case には、 $\displaystyle\lim_{k\to\infty} \frac{s(kL, p)}{k}$ を使う。

三内

No. 1

Date 2011・7・7

Graded category and its applications.

Thm (Quillen)

A : Noether ring

$\mathrm{mod}\,A$ = the cat of f.g modules

$$K(\mathrm{mod}\,A) \xrightarrow{\sim} K(\mathrm{mod}\,A[t]).$$

$$\mathrm{mod}\,A \longrightarrow \mathrm{mod}\,A[t]$$

$$M \longmapsto M \otimes_A A[t]$$

$\mathrm{mod}\,A$ を abel cat $\mathcal{A}$ にして同じことができる。

このとき、

$\mathrm{mod}\,A[t] \overset{対応}{\rightsquigarrow} \mathcal{A}[t]$ の polynomial category.

Thm 1 $\mathcal{A}$: Noeth. abelian cat with projectives

$$K(\mathcal{A}) \simeq K(\mathcal{A}[t])$$

- $\mathcal{A}$ = abel cat
- $\mathrm{End}\,\mathcal{A}$: the cat of end morphism $(\overset{\cup}{X}, \varphi)$
- $\mathrm{Lex}\,\mathcal{A}$ = the cat of left exact func

(𝒜 では可算直和は存在しないかもしれないが、$\mathrm{Lex}\,\mathcal{A}$ ではとれる。)

- $\mathrm{noeth}\,\mathcal{A}$ = the full sub cat of noeth obj in $\mathcal{A}$

Def • $\mathcal{A}$: noeth $\overset{\text{def}}{\Longleftrightarrow}$ noeth $\mathcal{A} = \mathcal{A}$

• $\mathcal{A}[\lambda] \underset{\text{def}}{=}$ noeth End Lex $\mathcal{A}$

base change に対応する functor は…

$$\mathcal{A} \hookrightarrow \mathrm{Lex}\,\mathcal{A} \longrightarrow \mathrm{End}\,\mathrm{Lex}\,\mathcal{A}$$
$$a \mapsto \mathrm{Hom}(-, a) \mapsto \Big(\bigoplus_{i \in \mathbb{N}} \bar{a} t^i, t\Big)$$

(Noe object は Noe obj にうつることが知られている)

A : Noe ring $\mathcal{A} := \mathrm{mod}\, A$

$\rightsquigarrow \mathcal{A}[\lambda] = \mathrm{mod}\, A[\lambda]$

$$(\tilde{M}, \tilde{t}) \longleftarrow\!\shortmid\; M \quad (\in)$$
$$t : M \longrightarrow M$$

graded cat (次数付で変数をつけ加える)

Def $\langle n \rangle$: $\mathrm{Obj}\langle n \rangle = \mathbb{N}$

$$\begin{array}{ccccc} 1 & \underset{n\text{本}}{\overset{\longrightarrow}{\underset{\longrightarrow}{\vdots}}} & 2 & \underset{n\text{本}}{\overset{\longrightarrow}{\underset{\longrightarrow}{\vdots}}} & 3 \quad \cdots \end{array}$$

$$(x^1_1, \ldots, x^1_n), \quad x^2_1, \ldots, x^2_n$$

(合成が可換になるような relation を入れる) $\;(x^n_k x^{n+1}_l = x^n_l \cdot x^{n+1}_k)$

$\mathcal{A}_{gr}[n] := \mathrm{Hom}(\langle n \rangle, \mathcal{A})$

$\mathcal{A}'_{gr}[n] :=$ noeth $\mathcal{A}_{gr}[n]$

No. 3

Date

∘ A : noeth ring

$\mathcal{A} = \mathrm{mod}\, A$

$\mathcal{A}_{gr}[n] = \{\text{graded } A[t_1, \dots, t_n]\text{-mod}\}$

$(\because)$ $a_1 \rightrightarrows a_2 \rightrightarrows \cdots$ $\quad a_i \in \mathrm{mod}\, A$

$\updownarrow$

$\left(\begin{array}{l} \oplus a_i \ : A\text{-mod} \\ \text{そして } t_i \text{ の action を} \\ \quad t_i = \varphi_i^j : a_j \to a_{j+1} \text{ で定める。} \end{array} \right.$

<u>Thm 2</u> $\mathcal{A}$: abelian

$$\mathbb{Z}[\sigma] \otimes_{\mathbb{Z}} K(\mathcal{A}) \xrightarrow{\sim} K(\mathcal{A}'_{gr}[n])$$

(不定元、次数付に対応)

~~prop~~ $\mathcal{A}'_{gr}[2] \big/ \mathcal{A}'_{gr,nil}[2] \simeq \mathcal{A}[t]$

$\Downarrow$

$$\begin{tikzcd}
0 \arrow[r] & K(\mathcal{A}'_{gr,nil}[2]) \arrow[r] \arrow[d, phantom, "\wr"] & K(\mathcal{A}'_{gr}[2]) \arrow[r] \arrow[dd, phantom, "\wr"] & K(\mathcal{A}[t]) \arrow[r] \arrow[dd, phantom, "\cong"] & 0 \\
 & K(\mathcal{A}'_{gr}[1]) \arrow[d, phantom, "\wr"] & & & \\
0 \arrow[r] & \mathbb{Z}[\sigma] \otimes K(\mathcal{A}) \arrow[r] & \mathbb{Z}[\sigma] \otimes K(\mathcal{A}) \arrow[r] & K(\mathcal{A}) \arrow[r] & 0 \\
 & 1 \otimes a \arrow[r, mapsto] & (1-\sigma) \otimes a & &
\end{tikzcd}$$

2011

7/8 玉原 14:00〜 権業

"Remarks on the non-vanishing Conjecture"

Conj N_d: (X,Δ): d-dim proj lc pair s.t. $K_X+\Delta$: p.eff. $\Delta\in\mathbb{R}$

$\Rightarrow \exists D$: $\mathbb{R}$-div ≥ 0 s.t. $K_X+\Delta\sim_{\mathbb{R}} D$

この話は "Global ACC" & "ACC for lct" を仮定する。(announced by Hacon–McKernan Xu)

Thm1: Conj $N_{\leq d}$ for smooth var with no boundary & abundance in $\leq d-1$

$\Longrightarrow$ Conj N_d

Thm2: X: rationally conn proj. (X,Δ) klt s.t. $K_X+\Delta$: p.eff.

$\Longrightarrow \exists D\geq 0$ s.t. $K_X+\Delta\sim_{\mathbb{R}} D$.

key lemma: (cf. [DHP §8]) (X,Δ): $\mathbb{Q}$-fac dlt pair K_X+D p.eff. $\Delta_i\geq 0$ s.t.

$\lim_{i\to\infty}\Delta_i=\Delta$ & $\Delta_i\leq\Delta_{i+1}$ $K_X+\Delta_i$: not p.eff.

$\Longrightarrow \exists\, \varphi: X \dashrightarrow X' \xrightarrow{f} Y'$ s.t. $K_{X'}+\Delta'\sim_f 0$ (X',Δ'): lc

~~$i\gg 0$ (X,Δ_i) の Mori fibre sp.~~

Rem: when $K_X+\Delta$: p.eff we need ACC conj's when $K_X+\Delta$: not we just use "length of ext"

proof of Thm2:

Induction on dimension We may assume $K_X+(1-\varepsilon)\Delta$: not p.eff. for $\forall\varepsilon>0$ Key lemma $\longrightarrow \exists\,\varphi: X\longrightarrow X' \xrightarrow{f} Y$

s.t. $K_{X'}+\Delta'\sim_f 0$

$$\begin{tikzcd} & W \arrow[dl, "\alpha"'] \arrow[d, "\beta"] \arrow[ddr, bend left, "g"] & \\ X \arrow[r, dashed] & X' \arrow[dr, "f"] & \\ & & Y' \end{tikzcd}$$

F: gen. fibre of g.

$\nu(\beta^*(K_X+\Delta'))=0$

$\alpha^*(K_X+\Delta)+E=\beta^*(K_{X'}+\Delta')+G$

$\rightsquigarrow \nu(\alpha^*(K_X+\Delta)|_F)=0$

$K_W+\Gamma_W=\alpha^*(K_X+\Delta)+E'$ $\rightsquigarrow \nu((K_W+\Gamma_W)|_F)=0$

$$\begin{tikzcd} W \arrow[rr, dashed] \arrow[dr, "g"'] & & W' \arrow[dl, "g'"] \\ & Y & \end{tikzcd}$$

$\overset{\text{Lai}}{\rightsquigarrow}$ good mm over Y'

$K_{W'}+\Gamma_W\sim_{g',\mathbb{Q}} 0$

$\underset{\text{Ambro's can bdl}}{\rightsquigarrow}$ $K_{W'}+\Gamma_W\sim_{\mathbb{Q}} g^*(\underbrace{K_{Y'}+\Delta_{Y'}}_{\text{klt}})$

Y': RC $\rightsquigarrow K_{Y'} + \Delta_{Y'} \geq 0 \rightsquigarrow K_{W'} + \Gamma_{W} \geq 0 \rightsquigarrow K_X + \Delta \geq 0$ $\square$

Global ACC

$d \in \mathbb{N}$ & $I \subseteq [0,1]$ DCC. $\exists I_0 \subseteq I$ finite if $\cdot$ (X, Δ): d-dim proj lc

$\cdot\ K_X + \Delta \equiv 0$ $\quad \cdot\ \Delta \in I$

$\Rightarrow \Delta \in I_0$

Acc for lct

$d \in \mathbb{N}$ $\Gamma \subseteq [0,1)$: DCC $\quad S \subseteq \mathbb{R}_{\geq 0}$ finite $\{ \mathrm{lct}(X, \Delta; D) \mid (X, \Delta)$. lc $\dim X = d$

$\Delta \in \Gamma$, $D \in S \}$

: ACC.

14:30 ~ 大川

problem: X: sm proj var $/\mathbb{C}$ $|K_X|$: base point free.
$\Rightarrow D^b(\mathrm{coh}(X)) = D(X)$ has no non-trivial semi-orthogonal decomposition (SOD)

Motivation: Conj: $f: X \dashrightarrow Y$ extremal contr. induces an SOD of $D(X)$
SOD: T: triangulated cat. $T = \langle \mathcal{A}, \mathcal{B} \rangle$ is an SOD if
∘ $\mathcal{A}, \mathcal{B} \subseteq T$ full sub triangulated cat.
∘ $\mathrm{Hom}_T(\mathcal{B}, \mathcal{A}) = 0$
∘ $\forall x \in T$ $\exists \Delta$ $b \to x \to a \to b[1]$ $a \in \mathcal{A}$. $b \in \mathcal{B}$.

example: $f: X \to Y$ $Z \subseteq Y$, Z, Y: sm $X = Bl_Z Y$ Z of codim c in Y.
$j: E \hookrightarrow X$ is exc $f_E: E \to Z$.
$\Rightarrow D(X) = \langle D_{-c+1}, \cdots, D_{-1}, f^* D(Y) \rangle$ $D_i = j_* f_E^* D(Z) \otimes \mathcal{O}_X(-iE)$

cor: K_X: not nef $\Rightarrow D(X)$ has an SOD.

Rem!!: Converse is NOT true! i.e. K_X: even if nef, $D(X)$ may decompose!

example: ① X: Enriques ($q = 0$ $K_X \not\cong \mathcal{O}_X$, $K_X^{\otimes 2} \cong \mathcal{O}_X$).
$\to h^1(\mathcal{O}_X) = 0$. $h^2(\mathcal{O}_X) \underset{\text{serre duality}}{=} h^0(K_X) = 0 \to \mathcal{O}_X$ is an exceptional obj.

Def: $E \in T$: Δ-cat. is exceptional if $\mathrm{Hom}(E, E[i]) \cong H^i(pt, \mathcal{O}) = \begin{cases} \mathbb{C} & i = 0 \\ 0 & i \neq 0 \end{cases}$

E: exceptional $\to T = \langle \langle E \rangle_{tr}^{\perp}, \langle E \rangle_{tr} \rangle$ is an SOD.
For $\mathcal{A} \subseteq T$. $\mathcal{A}^{\perp} = \{ x \in T \mid \mathrm{Hom}(a, x) = 0.\ \forall a \in \mathcal{A} \}$

② X: Godeaux surf. (K_X: ample, $q = 0$. $h^0(K_X) = 0$)
$\to \mathcal{O}_X$: exc.

Fact: $K_X \cong \mathcal{O}_X \to$ exists no SOD.

Evidence:
prop1: X curve, $g \geq 1$ ($\Leftrightarrow |K_X|$ base point free)
$\Rightarrow D(X)$ has no SOD.

Prop 2: In the same situation of the problem then $D(X)$ has no except obj.

proof of prop2: $0 \neq E \in D(X)$ $\quad \mathrm{Hom}(E, E[\dim X]) \cong \mathrm{Hom}(E, E \otimes k_X)^\vee \neq 0$ if k_X is base point free. $\to$ exists no exceptional obj.

Lemma (川谷): $0 \neq E \in D^b(\mathrm{Coh}\,X)$ L: line bundle base point free
$\Rightarrow \mathrm{Hom}(E, E \otimes L) \neq 0$

pf: E: sheaf $\to$ trivial

in general $m := \inf\{i \mid h^i(E) \neq 0\}$

$\exists\, \Delta$:

$$\begin{tikzcd}
\tau_{\geq m+1} E[-1] \arrow[r] \arrow[d, "\otimes s"'] & \tau_{\leq m} E \arrow[r] \arrow[d, "\otimes s"] & E \arrow[r] \arrow[d, "\otimes s \neq 0"] & \tau_{\geq m+1} E \arrow[d, "\otimes s"] \\
\tau_{\geq m+1} E[-1] \otimes L \arrow[r] & (\tau_{\leq m} E) \otimes L \arrow[r] & E \otimes L \arrow[r] & \tau_{\geq m+1} E \otimes L
\end{tikzcd}$$

sheaf

14:00〜 真瀬

Families of K3 hypersurfaces in toric and non-toric Fano 3-fold /$\mathbb{C}$

§1. Introduction, §2 Main Theorem, §3 Idea of proof

§1.

fix $\mathbb{P}^3 \supset l$ line, $\mathbb{P}^3 \supset C$ sm plane cubic

$\sigma : \underset{\underset{E}{\cup}}{X'} \longrightarrow \mathbb{P}^3$ blow up along l. 〜 X'. sm toric Fano 3-fold

$\pi : \underset{\underset{D}{\cup}}{X} \longrightarrow \mathbb{P}^3$ blow up along C. X : sm non-toric Fano 3-fold

$H = \mathcal{O}_{\mathbb{P}^3}(1)$

$S' \in |-K_{X'}|$ generic $S \in |-K_X|$

$\mathrm{Pic}(S') = \begin{pmatrix} 4 & 1 \\ 1 & -2 \end{pmatrix} \overset{\text{isometry}}{\simeq} \mathrm{Pic}(S) = \begin{pmatrix} 4 & 3 \\ 3 & 0 \end{pmatrix}$

$\{\sigma^* H, E\} \longrightarrow \{\pi^* H, D\}$

$\sigma^* H \longmapsto \pi^* H$

$E \longmapsto \pi^* H - D$

§2

Main Theorem For any $S' \in |-K_{X'}|$, at most ADE sing's,

$\exists S \in |-K_X|$ s.t. $S' \sim S$ and vice versa

Rmk (i) $\dim |-K_{X'}| = 29$ $\dim |-K_X| = 22$

(ii) "$S' \sim S$" means $\{\tau(\tilde{S}')\} = \{\tau(\tilde{S})\} \subseteq \Omega$

§3.

<u>Step 1</u> <u>Lem</u> Let $\varphi : Y \longrightarrow \mathbb{P}^3$ blow up along sm. plane curve Σ of degree 1, 2, 3.

If $M \in |-K_Y|$ has at most ADE sing's, then

(i) $\varphi(M) \in |-K_{\mathbb{P}^3} - \Sigma|$

~~(ii)~~ $M,\ \varphi(M)$: sm

$\Rightarrow \varphi|_M : M \to \varphi(M)$ 1:1 onto ⌋

$S \in |-K_X|$ sm $\rightsquigarrow \underset{\mathrm{sm}}{\pi(S)} \supset C$

elli curve

$\$:= \{ S \in |-K_{\mathbb{P}^3} - C| \mid S : \mathrm{sm}\ \ \exists C' \neq C \ \text{ s.t. } C' \subseteq S \}$

<u>Step 2.</u> $\$$ is open dence in $|-K_{\mathbb{P}^3} - C|$

<u>Step 3</u> $\$ = \{ S \in |-K_{\mathbb{P}^3} - C| : \mathrm{sm} \}$

<u>Step 4</u>

$$\begin{array}{ccc} |-K_{X'}| & & |-K_X| \\ \rotatebox{90}{$\in$} & & \rotatebox{90}{$\in$} \\ S' & \underset{\exists g \in \mathrm{Aut}(\mathbb{P}^3)}{\overset{\exists f \in \mathrm{Aut}(\mathbb{P}^3)}{\rightleftarrows}} & S \\ \mathrm{ADE} & & \mathrm{ADE} \end{array}$$

14 : 30 ～ 高木

$\mathbb{A}$-schemes & its applications)

§0 Intro

Topology (Cpt Haus) : complete (Tychonoff)
($\exists$ limit)

$$\xrightarrow[\text{AFT}]{\text{Freyd's}} \exists\beta : (\text{Top}) \rightleftarrows (\text{Cpt Haus}) : U$$

$$X \longmapsto \beta X$$

Stone - Čech cpt'tion

Compare (Sch) : not complete! e.g. $\prod_{\mathbb{N}} \mathbb{P}^1$

§1. Main results

Def X : top space is coherent $\Longleftrightarrow$ X : sober, q-cpt, q-sep
$\exists$ q-cpt open basis
(e.g. underlying sp of noeth schemes)

(Coh) := cat of coh sp & q-cpt morph

Thm (Stone duality)

(DLat) : cat of distributive lattices

$$\rightsquigarrow (\text{Coh}) \underset{\text{Spec}^\circ}{\simeq} (\text{DLat})^{op}$$

$$X \longmapsto C(X)_{cpt} = \{ Z \subset X : \text{closed}, X \setminus Z : \text{q-cpt} \}$$

Thm [T-] (Coh.sch) $\xrightarrow{\text{f.f.}}$ (Coh.LRS)
f.f. ↘ ↻ ↗ f.f.
($\mathbb{A}$-sch)

preserves { fiber product.
{ finite patching via q-cpt open

$$\mathrm{Spec} : (\mathrm{Ring}) \rightleftarrows (\mathrm{Coh\,Sch})^{op} : \Gamma$$

$$\supset \ \ \big\downarrow \text{f.f.}$$

$$\mathrm{Spec} \searrow\!\!\swarrow \Gamma \quad (\mathcal{A}\text{-sch})^{op}$$

- $(\mathcal{A}\text{-sch})$ complete
  co-complete
  separatedness
  properness

$\rightsquigarrow$

Thm [T-] S : base $\mathcal{A}$-sch

$$ZR_S : (\mathcal{A}\text{-sch}/S) \rightleftarrows (\text{proper } \mathcal{A}\text{-sch}/S) : U$$

$$X \longmapsto ZR_S(X) \ : \text{Zariski-Riemann sp.}$$

$X \xrightarrow{\varepsilon} ZR_S(X)$ If X, S : coh sch, $X \to S$ sp. of f.t.

$\rightsquigarrow$ ε : open imm

Cor $X \to S$: sep of f.t. of coh sch

(Nagata embedding) $\rightarrow$ $X \xrightarrow{\exists} Y$: open imm proper/S

§2 Construction of $\mathcal{A}$-schemes

Def $X \in (\mathrm{Coh}) \rightsquigarrow {}^{\exists}$ canonical (DLat)-valued sheaf τ_X

$$U \longmapsto \varprojlim_{\substack{V \subset U \\ \text{q-cpt}}} C(V)_{cpt}$$

Def $\alpha_1 : (\mathrm{Ring}) \longrightarrow (\mathrm{DLat})$

$$R \longmapsto \{\text{f.g. ideal}/R\} / (\mathfrak{a}^2 = \mathfrak{a})$$

高木

$$\alpha_2^{\text{nat}} : \begin{array}{ccc} R & \longrightarrow & \alpha_1 R \\ \psi & & \psi \\ a & \longmapsto & (a) \end{array}$$

Rmk $\quad \mathrm{Spec}\, R = \mathrm{Spec}^\circ \alpha_1 R$

Def $\quad$ An $\mathbb{A}$-scheme is a triple $X = (|X|, \mathcal{O}_X, \beta_X)$

- $|X|$: coh sp
- $\mathcal{O}_X$: (Ring)-valued sheaf$/X$
- $\beta_X : \alpha_1 \mathcal{O}_X \longrightarrow \tau_X$ "support morph"

s.t. $\quad V \subset U$ q-cpt, open$/X$ $\quad (Z = U \setminus V)$

$$\begin{tikzcd} \mathcal{O}_X(U) \arrow[r, "\mathrm{res}"] \arrow[d] \arrow[dr, phantom, "\circlearrowleft"] & \mathcal{O}_X(V) \\ \mathcal{O}_X(U)_Z \arrow[ur, dotted, "\exists"'] & \end{tikzcd}$$

$\mathcal{O}_X(U)_Z$: localization albng $\{a \in \mathcal{O}_X(U) \mid \beta_X \alpha_2(a) \subset Z\}$

A morphism $X \to Y$ of $\mathbb{A}$-schemes is a pair $(f, f^\#)$

$f : |X| \longrightarrow |Y|$: q-cpt

$f^\# : \mathcal{O}_Y \longrightarrow f_* \mathcal{O}_X$ s.t.

$$\begin{tikzcd} \alpha_1 \mathcal{O}_Y \arrow[r] \arrow[d, "\beta_Y"'] \arrow[dr, phantom, "\circlearrowleft"] & \alpha_1 f_* \mathcal{O}_X \arrow[d, "\beta_X"] \\ \tau_Y \arrow[r, "f^{-1}"'] & f_* \tau_X \end{tikzcd}$$