Date 08 . 8 . 14 No.

高木俊輔

論文紹介

de Fernex – Mustata
arxiv:0710.4978

Kollár
arxiv:0805.0756

§ Intro

k: alg. closed field of char. 0

X: smooth variety $/k$

$Y \subset X$ closed

$\rightsquigarrow \mathrm{lct}(X,Y) \in \mathbb{Q} \subset \mathbb{R}$

$n \in \mathbb{N}$

$$T_n(k) := \left\{ \mathrm{lct}(X,Y) \;\middle|\; \begin{matrix} X: \mathrm{sm}/k \text{ of dim } n \\ Y \subset X \text{ closed} \end{matrix} \right\} \subseteq \mathbb{R}$$

Conj 1. (smooth variety of ACC conjecture)

$\forall n \in \mathbb{N}$

T_n has no accumulation pts from below.

Conj 2 (smooth version of Accumulation Conj)

$\forall n \in \mathbb{N}$

$\{\text{accumulation pts in } T_n\} = T_{n-1}$

Today's Goal

Thm A

$\forall n \in \mathbb{N}$, T_n is closed in $\mathbb{R}$

Thm B (Weak version of Conj2)

$\forall n \in \mathbb{N}$, $\{\text{accumulation pts of } T_n \text{ from above}\}$

$= T_{n-1}$

Strategy

$c_m := \mathrm{lct}(\mathfrak{a}_m) \longrightarrow \exists c \in \mathbb{R}$

$(\mathfrak{a}_m \subset k[x_1, \dots, x_n])$

$\rightsquigarrow$ non standard method $\exists \hat{\mathfrak{a}} \subset {}^*k[[x_1, \cdots x_n]]$

s/t $c = \mathrm{lct}(\hat{\mathfrak{a}})$

(k^* : non standard ext of k)

$T_n^{Ser}(k) := \{ \mathrm{lct}(\hat{\mathfrak{a}}) \mid \hat{\mathfrak{a}} \subset k[[x_1, \cdots, x_n]] \}$

実は $T_n(k) = T_n^{Ser}(k) = T_n^{Pol}(k)$

Rmk

characterization of $\mathrm{lct}(X, Y)$

in terms of $\mathrm{Codim}(Y_m, X_m)$ (Mustaţă)

$\rightsquigarrow$ $T_n(k)$ is independent of k

(k: alg. closed)

Rmk (Kollár)

B-C-H-M $\rightsquigarrow$ Conj 2

§ Log canonical thresholds

Temkin

$\exists$ log resol for excellent integral schemes of char. 0

(smooth var/k or $\mathrm{Spec}\, k[[x_1, \cdots, x_n]]$)

X: regular excellent scheme of char. 0

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$0 \neq \mathfrak{a} \subsetneq \mathcal{O}_X$

$\pi : \hat{X} \longrightarrow X$ log resol of $\mathfrak{a}$

$K_{\hat{X}/X} = \sum x_i E_i$

$F = \sum a_i E_i \qquad \mathfrak{a}\mathcal{O}_{\hat{X}} = \mathcal{O}_{\hat{X}}(-F)$

$$\mathrm{lct}(\mathfrak{a}) = \min\left\{\frac{x_i+1}{a_i}\right\}$$

$\xi \in X$ not nec. closed pt

$$\mathrm{lct}_\xi(\mathfrak{a}) = \min\left\{\frac{x_i+1}{a_i} \;\middle|\; \pi(E_i) \ni \xi\right\}$$

$$\mathrm{LCT}(\mathfrak{a}) = \bigcup_{\frac{x_i+1}{a_i} = \mathrm{lct}(\mathfrak{a})} \pi(E_i)$$

$$\left(= \mathrm{Zeros}\left(\mathcal{J}\left(\mathfrak{a}^{\mathrm{lct}(\mathfrak{a})}\right)\right)\right)$$

$$\mathrm{LCT}_\xi(\mathfrak{a}) = \bigcup_{\frac{x_i+1}{a_i} \leq \mathrm{lct}_\xi(\mathfrak{a})} \pi(E_i)$$

$$\left(= \mathrm{Zeros}\left(\mathcal{J}\left(\mathfrak{a}^{\mathrm{lct}_\xi(\mathfrak{a})}\right)\right)\right)$$

Prop 1

$\mathfrak{p}$: def ideal of $\overline{\{\zeta\}}$

Assume $\mathfrak{p} \supset \mathfrak{a}$

$$\mathrm{lct}_\zeta(\mathfrak{a}) \overset{\leq \text{ o.k.}}{=} \lim_{d\to\infty} \mathrm{lct}_\zeta(\mathfrak{a}+\mathfrak{p}^d)$$

If $\overline{\{\zeta\}}$ is an irr. comp of

$L \subset T_\zeta(\mathfrak{a})$

$$\Rightarrow \mathrm{lct}_\zeta(\mathfrak{a}) = \mathrm{lct}_\zeta(\mathfrak{a}+\mathfrak{p}^d) \quad \forall d \gg 0$$

Lem

$$\mathrm{lct}_\zeta(\mathfrak{a}) = \inf_{C_X(E)=\overline{\{\zeta\}}} \frac{\mathrm{ord}_E(K_{Y/X})+1}{\mathrm{ord}_E(\mathfrak{a})}$$

If $\overline{\{\zeta\}}$ is an irr comp of $L \subset T_\zeta(\mathfrak{a})$

$\Rightarrow \exists E$ divisor $/X$, $C_X(E) = \overline{\{\zeta\}}$

$$\mathrm{lct}_\zeta(\mathfrak{a}) = \frac{\mathrm{ord}_E(K_{Y/X})+1}{\mathrm{ord}_E(\mathfrak{a})}$$

proof

$\geq$ を示す.

Fix a log resol $\pi : \hat{X} \longrightarrow X$ of $\mathfrak{a} \cdot P$

a divisor F on $\hat{X}$

s/t $\mathrm{lct}_{\xi}(\mathfrak{a}) = \dfrac{\mathrm{ord}_F(K_{\hat{X}/X}) + 1}{\mathrm{ord}_F(\mathfrak{a})}$

Assume $\pi(F) \not\ni \overline{\{\xi\}}$

$\pi^{-1}(\overline{\{\xi\}}) = \sum a_i E_i$

$\sum E_i + F$ SNC divisor

$\exists F_0$ divisor on $\hat{X}$

$\exists \hat{\xi} \in \pi^{-1}(\xi)$ s/t $F_0 \cap F \ni \hat{\xi}$

$\Rightarrow \pi(F_0 \cap F) = \overline{\{\xi\}}$

$$\begin{array}{ccccc} \hat{X}_1 & \xrightarrow{\pi_1} & \hat{X} & \xrightarrow{\pi} & X \\ & \uparrow & \cup & & \\ & \text{blow-up along } F_0 \cap F & F_0 \cap F \ni \hat{\xi} & & \end{array}$$

$F_1 \subset \hat{X}_1$: irr comp of $\mathrm{Exc}(\pi_1)$
the image of F_1 in $\hat{X} \ni \hat{\xi}$

$$
\begin{array}{lllll}
\mathrm{ord}_{F_1}(K_{\widetilde{X}_1/X}) = & \mathrm{ord}_{F_1}(K_{\widetilde{X}_1/\widetilde{X}}) & + & \mathrm{ord}_{F_1}(\pi_1^* K_{\widetilde{X}/X}) \\
& \quad \| & & \quad \| \\
& \quad 1 & & \mathrm{ord}_{F}(K_{\widetilde{X}/X}) \\
& & & \quad + \\
& & & \mathrm{ord}_{F_0}(K_{\widetilde{X}/X})
\end{array}
$$

$$
\begin{array}{ccccc}
\widetilde{X}_2 & \xrightarrow{\pi_2} & \widetilde{X}_1 & \longrightarrow & \widetilde{X} \\
& & \cup & & \\
& & F_1 \cap \widetilde{F} & &
\end{array}
$$

blow-up along $F_1 \cap \widetilde{F}$ ↖ strict transform of F

$$\mathrm{ord}_{F_m}(K_{\widetilde{X}_m/X}) = m(1+\mathrm{ord}_F(K_{\widetilde{X}/X})) + \mathrm{ord}_{F_0}(K_{\widetilde{X}/X})$$

$$\mathrm{ord}_{F_m}(\mathfrak{a}) = m\,\mathrm{ord}_F(\mathfrak{a}) + \mathrm{ord}_{F_0}(\mathfrak{a})$$

$$@ \quad C_X(F_m) = \overline{\{\xi\}}$$

$$\lim_{m\to\infty} \frac{\mathrm{ord}_{F_m}(K_{\widetilde{X}_m/X})+1}{\mathrm{ord}_{F_m}(\mathfrak{a})} = \frac{\mathrm{ord}_F(K)+1}{\mathrm{ord}_F(\mathfrak{a})}$$

$$\qquad\qquad\qquad\qquad\qquad \| $$

$$\qquad\qquad\qquad\qquad\qquad \mathrm{lct}_\xi(\mathfrak{a}) \quad //$$

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<u>proof of prop</u>

Lem $\rightsquigarrow$ $\forall \varepsilon > 0$, $\exists E$ divisor/X

$$C_X(E) = \overline{\{\xi\}}$$

$$\mathrm{lct}_\xi(\mathfrak{a}) + \varepsilon \geq \frac{\mathrm{ord}_E(K)+1}{\mathrm{ord}_E(\mathfrak{a})}$$

$\mathrm{ord}_E(P) \geq 1$

If $d > \mathrm{ord}_E(\mathfrak{a})$

$$\Rightarrow \mathrm{ord}_E(\mathfrak{a} + P^d) = \mathrm{ord}_E(\mathfrak{a})$$

$$\therefore \mathrm{lct}_\xi(\mathfrak{a}) \leq \mathrm{lct}_\xi(\mathfrak{a} + P^d)$$

$$\leq \frac{\mathrm{ord}_E(K)+1}{\mathrm{ord}_E(\mathfrak{a}+P^d)}$$

$$= \frac{\mathrm{ord}_E(K)+1}{\mathrm{ord}_E(\mathfrak{a})}$$

$$\leq \mathrm{lct}_\xi(\mathfrak{a}) + \varepsilon$$

def ideal of $\overline{\{\xi\}}$

Prop 2

Suppose $P \supset \mathfrak{a}$

$\Longrightarrow \mathrm{lct}_{\xi}(\mathfrak{a}) = \mathrm{lct}(\mathfrak{a} \cdot \widehat{\mathcal{O}}_{X,\xi})$

$$
(\because)\quad
\begin{tikzcd}
{}^{\exists}W \arrow[r] \arrow[d] & \widehat{X} \arrow[d, "\text{log resol}"] \\
\mathrm{Spec}\,\widehat{\mathcal{O}}_{X,\xi} \arrow[r] & X
\end{tikzcd}
$$

$W \to \mathrm{Spec}\,\widehat{\mathcal{O}}_{X,\xi}$ も log resol となる.

Prop 3

$0 \neq \mathfrak{a} \subsetneq K[[X_1, \cdots, X_N]]$

$\Rightarrow {}^{\exists}k$: alg. closed field

${}^{\exists}n \leq N$

${}^{\exists}\mathfrak{b} \subset (X_1, \cdots, X_n)$ in $k[X_1 \cdots X_n]$

s/t $\mathrm{lct}(\mathfrak{a}) = \mathrm{lct}_0(\mathfrak{b})$

If $\dim \mathrm{LCT}(\mathfrak{a}) > 0 \Rightarrow n \lneq N$

Proof

ξ : generic pt of an irr. comp of $\mathrm{LCT}(\mathfrak{a})$

Cohen's str Thm

$\widehat{\mathcal{O}}_{X,\xi} \simeq k_0[[X_1, \cdots, X_n]]$

$(X = \mathrm{Spec}\,K[[X_1, \cdots, X_N]]$, $\widehat{\mathfrak{a}}$: image of $\mathfrak{a}$ in $k_0[[X_1, \cdots, X_n]])$

Prop 2 $\rightsquigarrow \mathrm{lct}(\widehat{\mathfrak{a}}) = \mathrm{lct}_{\xi}(\widehat{\mathfrak{a}})$

$= \mathrm{lct}(\mathfrak{a})$

$\mathrm{LCT}(\widehat{\mathfrak{a}}) = \{\widehat{\mathfrak{m}}\}$ $\quad (\widehat{\mathfrak{m}} = (X_1, \cdots, X_n) \subset k_0[[X_1, \cdots, X_n]])$

Prop 1 $\rightsquigarrow$ $\mathrm{lct}(\hat{\mathfrak{a}}) = \mathrm{lct}(\hat{\mathfrak{a}} + \hat{\mathfrak{m}}^d)$ $(\forall d \gg 0)$

$\mathfrak{b}_0 := (\hat{\mathfrak{a}} + \hat{\mathfrak{m}}^d) \cap k_0[x_1, \dots, x_n]$

$\mathfrak{b}_0 k_0[[x_1, \dots, x_n]] = \mathfrak{a} + \hat{\mathfrak{m}}^d$

$k := \overline{k_0}$

$\mathfrak{b} := \mathfrak{b}_0 k[x_1, \dots x_n]$

$$\therefore \mathrm{lct}(\hat{\mathfrak{a}} + \hat{\mathfrak{m}}^d) = \mathrm{lct}_0(\mathfrak{b}_0)$$

$$\mathrm{lct}(\mathfrak{a}) = \mathrm{lct}_0(\mathfrak{b}) \quad //$$

$T_n^{pol}(k) = \{\mathrm{lct}_0(\mathfrak{a}) \mid \mathfrak{a} \subset (x_1, \dots, x_n) \text{ in } k[x_1, \dots, x_n]\}$, $T_n^{ser}(k) = \{\mathrm{lct}(\mathfrak{a}) \mid \mathfrak{a} \subset k[[x_1, \dots, x_n]]\}$

Prop

$$T_n(k) = T_n^{pol}(k) = T_n^{ser}(k)$$

$(\because)$ $T_n^{pol}(k) \subset T_n(k)$

$T_n(k) \subset T_n^{ser}(k)$ $(\because \text{Prop } 2)$

$T_n^{ser}(k) \subset T_n^{pol}(k)$ $(\because \text{Prop } 3)$

T_n^{pol}は kによらない。//

§ nonstandard methods

$\mathbb{N}_0 = \{0, 1, 2, \cdots\}$

Def

$\mathcal{U}$: ultra filter

$\Leftrightarrow$ $\mathcal{U}$ is a collection of subsets of $\mathbb{N}_0$

s/t (1) $\phi \notin \mathcal{U}$

(2) If $A \in \mathcal{U}$ and $B \supset A$
$\Rightarrow B \in \mathcal{U}$

(3) If $A, B \in \mathcal{U}$
$\Rightarrow A \cap B \in \mathcal{U}$

(4) If $\subseteq \mathbb{N}_0$
$\Rightarrow$ A or $\mathbb{N}_0 \setminus A \in \mathcal{U}$

Note

disjoint union

$A = A_1 \amalg \cdots \amalg A_n$

$(A_i \subseteq \mathbb{N}_0)$

If $A \in \mathcal{U} \Rightarrow 1 \leq {}^{\exists!} i \leq n$ s/t $A_i \in \mathcal{U}$

Def $\mathcal{U}$: ultra filter

$\mathcal{U}$ is non-principal $\Leftrightarrow$ 「If $A \subseteq \mathbb{N}_0$ s/t $\#(\mathbb{N}_0 \setminus A) < \infty$
$\Rightarrow A \in \mathcal{U}$ 」

Zorn's Lemma

$\rightsquigarrow$ $\exists$ non-principal ultrafilter $\mathcal{U}$ on $\mathbb{N}_0$

Fix $\mathcal{U}$

Property $P(m)$ holds for almost all m

$\underset{\text{def}}{\Longleftrightarrow}$ $\{m \in \mathbb{N}_0 \mid P(m) \text{ holds}\} \in \mathcal{U}$

Def

$\{A_m\}_{m \in \mathbb{N}_0}$ seq of sets

$(a_m), (b_m) \in \prod_{m \in \mathbb{N}_0} A_m$

$(a_m) \sim (b_m) \Leftrightarrow a_m = b_m$ for almost all m

$[A_m] := \prod A_m / \sim$

ultra product of $\{A_m\}$ w.r.t $\mathcal{U}$

$[a_m] :=$ the image of (a_m) in $[A_m]$

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If $A_m = A$ for $\forall m \in \mathbb{N}_0$

$\Rightarrow {}^*A := [A_m]$ non standard ext of A

$$\begin{array}{ccc} A & \hookrightarrow & {}^*A \quad \text{inj} \\ \Psi & & \Psi \\ a & \longmapsto & [a] \end{array}$$

① A: commutative $\Rightarrow$ ${}^*A$: comm ring.

$$[a_m] + [b_m] = [a_m + b_m]$$

$$[a_m] \cdot [b_m] = [a_m \cdot b_m]$$

② k: (alg closed) field

$\Rightarrow {}^*k$: (alg closed) field

k: field

$g \in k[x_1, \dots, x_n]$

$\rightsquigarrow g$ is a func $\mathbb{N}_0^n \longrightarrow k$.

$$\begin{array}{ccc} \mathbb{N}_0^n & \longrightarrow & k \\ \Psi & & \Psi \\ (m_1, \dots, m_n) & \longmapsto & (\text{coeff of } x_1^{m_1} \cdots x_n^{m_n} \text{ in } g) \end{array}$$

$F := [f_m] \in {}^*(k[x_1, \dots, x_n])$

$\rightsquigarrow F: {}^*(\mathbb{N}_0^n) \longrightarrow {}^*k$

$$\begin{array}{c} {}^*(\mathbb{N}_0^n) \\ \cup \\ \mathbb{N}_0^n \end{array}$$

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$$F|_{\mathbb{N}_0^n} : \mathbb{N}_0^n \longrightarrow {}^*k$$

$$\rightsquigarrow \quad \hat{f} \in {}^*k[[X_1, \cdots, X_n]]$$

$$\left(\hat{f} = \sum F(m_1 \cdots m_n) X_1^{m_1} \cdots X_n^{m_n}\right)$$

$$\begin{array}{ccc} {}^*(k[X_1 \cdots X_n]) & \xrightarrow{\rho} & k[[X_1 \cdots X_n]] \\ \Psi & & \Psi \\ F & \longmapsto & \hat{f} \end{array}$$

$$\begin{array}{ccc} ({}^*k)[X_1 \cdots X_n] & \hookrightarrow & {}^*(k[X_1 \cdots X_n]) \\ \Psi & & \Psi \\ \sum [a_m^{(m_1 \cdots m_n)}] X_1^{m_1} \cdots X_n^{m_n} & \longmapsto & \left[\sum a_m^{(m_1 \cdots m_n)} X_1^{m_1} \cdots X_n^{m_n}\right] \\ & & \downarrow \\ & & {}^*k[[X_1 \cdots X_n]] \\ & & \Psi \\ & & \sum [a_m^{(m_1 \cdots m_n)}] X_1^{m_1} \cdots X_n^{m_n} \end{array}$$

Defn

$\{\mathfrak{a}_m\}_{m\in\mathbb{N}}$ $\mathfrak{a}_m \subset k[X_1 \cdots X_n]$

$A := [\mathfrak{a}_m] := \{ F = [f_m] \in {}^*(k[X_1 \cdots X_n]) \mid f_m \in \mathfrak{a}_m$ for almost all $m\}$

A is an ideal of ${}^*(k[X_1 \cdots X_n])$

$\hat{\mathfrak{a}} := \langle P(A) \rangle$

ideal of power series ass. to A

$G = [g_m] \in {}^*(k[X_1, \cdots, X_n])$

If G has bounded degree

(i.e. $\exists\, d \in \mathbb{N}$, $\deg g_m \leq d$ for almost all m)

$\Rightarrow$ G can be identified with $\exists g \in {}^*k[X_1, \cdots, X_n]$

$$P(G) = \sum_{\Sigma m_i \leq d} [g_m(m_1, \cdots, m_n)] X_1^{m_1} \cdots X_n^{m_n}$$
$$\underset{g}{\text{\rotatebox{90}{$\in$}}}$$

$B = [b_m] \subset {}^*(k[X_1, \cdots, X_n])$

If B is gen in $\deg \leq d$,

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$b_m = (g_{0,m}, \cdots, g_{r,m})$

$\deg g_{i,m} \leq d$

$r : \binom{n+d}{n} - 1$

$\shortparallel$

$\dim k[x_1 \cdots x_n] \leq d$

$\Rightarrow B = (G_0, \cdots, G_r)$

$G_i := [g_{i,m}]$

$G_i \longleftrightarrow g_i \in {}^*k[x_1, \cdots, x_n]$

ideal of poly. ass. to B.

$b := (g_0, \cdots, g_r) \subset {}^*k[x_1, \cdots, x_n]$

ideal of poly. ass. to B.

Note

$b \, {}^*k[[x_1, \cdots, x_n]] = \tilde{b}$

Def (standard part or shadow)

$r \in {}^*\mathbb{R}$

r is infinitesimal

$\Longleftrightarrow \forall \varepsilon \in \mathbb{R}_+,\ -\varepsilon < r < \varepsilon.$

e.g.

$r = (1, \frac{1}{2}, \frac{1}{3}, \cdots)$

r is finite $\Longleftrightarrow \exists n \in \mathbb{N}$
$-n \leq r \leq n$

e.g.

$r = (1, 2, 3, 4, \cdots)$
infinite.

Note $r \neq 0$

r is infinite $\Longleftrightarrow \frac{1}{r}$ is infinitesimal

$F = \{ r \in {}^*\mathbb{R} \mid r \text{ is finite} \}$

$S = \{ r \in {}^*\mathbb{R} \mid r \text{ is infinitesimal} \}.$

$\Longrightarrow F$ is a valuation ring of ${}^*\mathbb{R}$ with the max. ideal S.

Fact $F/S \simeq \mathbb{R}$

$st : F \longrightarrow \mathbb{R}$

i.e. $\forall r \in {}^*\mathbb{R}$ finite

$\exists\, st(r) \in \mathbb{R}$

s/t $r - st(r)$ is infinitesimal

e.g. $\{c_m\}$ $(c_m \in \mathbb{R})$

$\downarrow$

$\exists c \in \mathbb{R}$ $\quad st([c_m]) = c$

Def

$k = \overline{k}$

$lct_0 : \{ \mathfrak{a} \subset (x_1, \cdots, x_n) \text{ in } k[x_1, \cdots, x_n] \} \longrightarrow \mathbb{R}$

$\rightsquigarrow {}^*lct_0 : \{ A = [\mathfrak{a}_m] \subset (x_1, \cdots, x_n)^*(k[x_1, \cdots, x_n]) \}$

$\downarrow \qquad\qquad A \mapsto$

${}^*\mathbb{R} \ni [lct_0(\mathfrak{a}_m)]$

Prop 5

$A = [\mathfrak{a}_m] \subset (x_1, \dots, x_n)^{*}\mathbb{R}[x_1 \dots x_n])$

$\hat{\mathfrak{a}} \subset {}^{*}\mathbb{R}[[x_1 \dots x_n]]$ ideal of p.s. ass to A

$\Rightarrow \mathrm{st}({}^{*}\mathrm{lct}_0(A)) = \mathrm{lct}(\hat{\mathfrak{a}})$

Thm B

$\forall n \in \mathbb{N}$

$\{\text{accumulation pts of } T_n\} = T_{n-1}$

proof

$(\supset)$ o.k.

$(\because)$ $T_{n-1} \ni c$

X : sm var of dim $n-1$

$\mathfrak{a} \subset \mathcal{O}_X$ s/t $c = \mathrm{lct}(X, \mathfrak{a})$

$\forall m \in \mathbb{N}$

$$\mathrm{lct}(X \times \mathbb{A}^1, \mathfrak{a} + (t^m)) = c + \frac{1}{m} \xrightarrow{m \to \infty} c$$

座標tでかる.

(C)を示す。

Assume $\exists \{c_m\}$ strictly decreasing

$$\text{s/t} \quad c_m \xrightarrow{m\to\infty} \exists c \in \mathbb{R}$$

$$T_n = T_n^{Pol} \Longrightarrow \exists \mathfrak{a}_m \subset (X_1, \cdots, X_n) \text{ in } k[X_1, \cdots, X_n]$$

$$c_m = \mathrm{lct}_0(\mathfrak{a}_m)$$

$A = [\mathfrak{a}_m] \subset {}^*(k[X_1, \cdots, X_n])$

$\hat{\mathfrak{a}} \subset {}^*k[[X_1, \cdots, X_n]]$ ideal of p.s. ass to A

$$\text{Prop 5} \rightsquigarrow \mathrm{lct}(\hat{\mathfrak{a}}) = \mathrm{st}({}^*\mathrm{lct}_0(A))$$

$$= \mathrm{st}([c_m])$$

$$= c.$$

$\mathfrak{m} := (x_1, \cdots, x_n) \subset k[X_1, \cdots, X_n]$

$\hat{\mathfrak{m}} := (x_1, \cdots, x_n) \subset {}^*k[[X_1, \cdots, X_n]]$

ETS

$\mathrm{lct}(\hat{\mathfrak{a}} + \hat{\mathfrak{m}}^d) \gneq \mathrm{lct}(\hat{\mathfrak{a}})$ for $\forall d \geq 1$

Indeed

Prop 1 $\rightsquigarrow$ $\dim LCT(\mathfrak{a}) > 0$

Prop 3 $\rightsquigarrow$ $c = lct(\hat{\mathfrak{a}}) \in T_{n-1}^{pol} = T_{n-1}$

Fix $\forall d \geq 1$

$A + M^d := [\mathfrak{a}_m + m^d]$

$\subset {}^*(k[x_1 \cdots x_n])$

$\mathfrak{a}_m + m^d$ is gen in deg $\leq d$

claim

$\#\{lct_0(b) \mid b \subset (x_1, \cdots x_n)$ in $k[x_1 \cdots x_n]$ gen in deg $\leq d\} \lneq \infty$

$\exists \lambda \in \mathbb{R}$

s.t. $\lambda = lct_0(\mathfrak{a}_m + m^d)$ for almost all m

$$lct(\hat{\mathfrak{a}} + \hat{m}^d) = St({}^*lct_0(A + M^d))$$

$$= \lambda = lct_0(\mathfrak{a}_m + m^d) \quad \exists m \in \mathbb{N}$$

$$\geq \underset{\substack{\| \\ c_m \\ \vee\!\! + \\ c = lct(\hat{\mathfrak{a}})}}{lct_0(\mathfrak{a}_m)}$$