F閾値について Daisuke Hirose

§1 11日の [H] から

B ·· Blickle
H ·· Hara
Hu ·· Huneke
M ·· Mustaţă
S ·· Smith
T ·· Takagi
W ·· Watanabe
Y ·· Yoshida

R local, reduced, regular ring with max ideal $\mathfrak{m}$
finitely generated over k,
char $k = p > 0$, k perfect
$\mathfrak{a} \subseteq R$ an ideal, $t \in \mathbb{R}_{\geq 0}$

$(R, \mathfrak{a}^t)$: F-pure
$\iff \forall q = p^e >> 0 \ \exists d \in \mathfrak{a}^{\lceil t(q-1) \rceil}$ s.t. $R \xhookrightarrow{\times d^{1/q}} R^{1/q}$ splits
$\iff \forall q >> 0 \quad \mathfrak{a}^{\lfloor t(q-1) \rfloor} \not\subseteq \mathfrak{m}^{[q]}$
Fedder's criterion

$$\mathrm{fpt}(\mathfrak{a}) := \sup\{ t \in \mathbb{R}_{\geq 0} \mid (R, \mathfrak{a}^t) : \text{F-pure} \}$$
$$= \lim_{q \to \infty} \frac{\max\{ r \in \mathbb{N} \mid \mathfrak{a}^r \not\subseteq \mathfrak{m}^{[q]} \}}{q} =: c^{\mathfrak{m}}(\mathfrak{a})$$

Q1. $\exists \lim$?
In fact,
$$= \sup \frac{\max\{ r \in \mathbb{N} \mid \mathfrak{a}^r \not\subseteq \mathfrak{m}^{[q]} \}}{q}$$

Q2. $c^{\mathfrak{m}}(\mathfrak{a}) \in \mathbb{Q}$?
By [BMS].

Q3. メリットはあるのか?
(形式上は) 計算しやすくなる.

A(1) ~ A(3) —— 全て Yes!!

§2 F-threshold.

$(R, \mathfrak{m})$, $\mathfrak{a} \subseteq R$: as in §1. except the assumption R regular.

$J \subseteq R$: an ideal such that $\mathfrak{a} \subseteq \sqrt{J}$

$$C^J(\mathfrak{a}) := \lim_{q\to\infty} \frac{\max\{r \in \mathbb{N} \mid \mathfrak{a}^r \not\subseteq J^{[q]}\}}{q}$$

Remark.

R : regular

$$\mathrm{fpt}(\mathfrak{a}) = \sup\{t \in \mathbb{R}_{\geq 0} \mid \tau(\mathfrak{a}^t) = R\} = C^{\mathfrak{m}}(\mathfrak{a})$$

$\mathfrak{m}$ 以外の J を入れる.

$C^{\tau(\mathfrak{a}^{a})}(\mathfrak{a})$ は

$$\tau(\mathfrak{a}^{a}) \subsetneq \tau(\mathfrak{a}^{a-\varepsilon}),\quad \forall \varepsilon > 0$$

をみたす

$$\{C^J(\mathfrak{a}) \mid J \subset R \text{ an ideal s.t. } \mathfrak{a} \subseteq \sqrt{J}\}$$
$$\Big\updownarrow 1:1$$
$$\{t \in \mathbb{R}_{\geq 0} \mid \tau(\mathfrak{a}^t) \subsetneq \tau(\mathfrak{a}^{t-\varepsilon})\}$$

A(1).

- $F : R \longrightarrow R$, Frobenius, splits
- $\mathfrak{a} = (f)$; principal

[HuMTW] $\Longrightarrow$ O.K.

A(2).

- [BMS]
- [広] R_σ : toric
  σ : simplicial
  $\mathfrak{a}$: $\mathfrak{m}$-primary monomial ideal

A(3). No! (今のところは).

- 計算は(むしろ)難しくなる.
- 幾何学的意味が失われる. (一般には, $\mathrm{fpt}(\mathfrak{a}) \leq C^{\mathfrak{m}}(\mathfrak{a})$)

§3 Example

「いつ Fedder's criterion, $\mathfrak{fpt}(\mathfrak{a}) = c^{\mathfrak{m}}(\mathfrak{a})$, が使えるのか?」

(1) R_σ : Gorenstein toric ring

$\exists\, \mathfrak{a} \subseteq R_\sigma$ s.t. $\mathfrak{fpt}(\mathfrak{a}) = c^{\mathfrak{m}}(\mathfrak{a}) = 1$

$\quad \|$

$(X^{\underline{u}})_k \leftarrow$ ~~ω~~ $\underline{\omega_{R_\sigma}}$ の生成元

canonical module

(2) $\mathfrak{a}$ を $\mathfrak{m}$-primary としてとりたいなら,

$(x, y, z, w) \subseteq k[x, y, z, w]/(xz - yw)$

以外ほとんど見つかっていない.

<u>Proposition.</u> R_σ : toric, σ : simplicial

$\exists\, \mathfrak{a}$: $\mathfrak{m}$-primary ideal s.t. $\mathfrak{fpt}(\mathfrak{a}) = c^{\mathfrak{m}}(\mathfrak{a})$

$\Longrightarrow R_\sigma$: regular.