7/5 9:30- 吉岡先生

○ Fourier-Mukai 変換

* Bridgeland, Orlov-Bondal.
* 例.

○ Fourier-Mukai 変換 と vector bundle の安定性

$$\begin{array}{ccc} \mathcal{F} : D(X) & \xrightarrow{\sim} & D(Y) \\ \cup & & \\ \mathrm{Coh}(X) & & \\ \cup & & \\ \text{stable sheaf} & \longmapsto & ? \end{array}$$

いつ安定になるか

<u>Mukai</u>

X : abel. var

$\hat{X}$: dual abel var

$\|$

$\mathrm{Pic}^0(X)$, $X = \mathrm{Pic}^0(\hat{X})$

$P|_{\{\hat{x}\}\times X} \longleftrightarrow \hat{x} \in \mathrm{Pic}^0(X)$

$P|_{\hat{X}\times\{x\}} \longleftrightarrow x \in \mathrm{Pic}^0(\hat{X})$

$$\begin{array}{rccc} \Phi^{P}_{\hat{X}\to X} : & D(\hat{X}) & \longrightarrow & D(X) \\ & \Psi & & \\ & \alpha & \longmapsto & \mathbb{R}\pi_{X*}^{\cdot}\,(P \otimes \pi_{\hat{X}}^{*}(\alpha)) \end{array}$$

$$\begin{tikzcd} & \hat{X}\times X \arrow[dl, "\pi_{\hat{X}}"'] \arrow[dr, "\pi_X"] & \\ \hat{X} & & X \end{tikzcd}$$

<u>他の例</u> X : K3 surface

Y : X 上のある sheaf の moduli

Y : K3 surface

$\mathcal{E}$: univ. family

$\Rightarrow \Phi^{\mathcal{E}}_{Y\to X} : D(Y) \to D(X)$ equiv.

X, Y smooth proj var / k

$$\begin{array}{ccc} & Y\times X & \\ \swarrow^{\pi_Y} & & \searrow^{\pi_X} \\ Y & & X \end{array}$$

$P \in D(Y\times X)$

$$F = \Phi^{P}_{Y\to X} : D(Y) \to D(X)$$
$$\alpha \longmapsto R\pi_{X*}(P \otimes \pi_Y^*(\alpha))$$

(P を kernel とする integral functor)

Adjoint of F

$$G := \Phi^{P^\vee \otimes \pi_X^*(\omega_X)[\dim X]}_{X\to Y} \quad \text{right adjoint}$$
$$H := \Phi^{P^\vee \otimes \pi_Y^*(\omega_Y)[\dim Y]}_{X\to Y} \quad \text{left adjoint}$$

たとえば

$$\mathrm{Hom}_{D(Y)}(Ga, b) = \mathrm{Hom}_{D(X)}(a, Fb) \quad \text{——}(*)$$

$$\Leftrightarrow \exists\ 1 \xrightarrow{\zeta} F\circ G, \quad G\circ F \xrightarrow{\delta} 1$$

s.t.

$$\underbrace{F \xrightarrow{\zeta} F\circ G\circ F \xrightarrow{\delta} F}_{\mathrm{id}}$$

$$\underbrace{G \xrightarrow{\zeta} G\circ F\circ G \xrightarrow{\delta} G}_{\mathrm{id}}$$

$(*)$

$$\mathrm{Hom}_{D(Y)}(Ga, b) = \mathrm{Hom}_{D(Y)}(R\pi_{Y*}(P^\vee \otimes \pi_X^*(\omega_X)[\dim X] \otimes \pi_X^*(a)), b)$$
$$\underset{\text{duality}}{=} \mathrm{Hom}_{D(Y\times X)}(P^\vee \otimes \pi_X^*(a), \pi_Y^*(b))$$
$$= \mathrm{Hom}_{D(Y\times X)}(\pi_X^*(a), P \otimes \pi_Y^*(b))$$
$$= \mathrm{Hom}_{D(X)}(a, \underbrace{R\pi_{X*}(P \otimes \pi_Y^*(b))}_{= Fb})$$

S : scheme

$$E \in D(S\times Y) \Rightarrow F(E) \in D(S\times X)$$
$$F(E)|_{\{s\}\times X} = F(E|_{\{s\}\times Y})$$

<u>Criterion</u> for an equiv.

<u>Lem</u> (A, m) reg. loc. ring. $\dim A = n$

$$V_\cdot : 0 \to V_0 \to V_1 \to \cdots \to V_n \to 0$$

complex of free A-module

$H^j(V_\cdot)$ artinian A-mod $(\forall j)$

$$\Rightarrow H^j(V_\cdot) = 0 \quad j < n$$

証) $m = (x_1, \cdots, x_n)$

$I_k = (x_1, \cdots, x_k)$

$$0 \to A/I_k \xrightarrow{x_{k+1}} A/I_k \to A/I_{k+1} \to 0 \quad \text{(exact)}$$

$\otimes V_\cdot$

$$0 \to V_\cdot \otimes A/I_k \xrightarrow{x_{k+1}} V_\cdot \otimes A/I_k \to V_\cdot \otimes A/I_{k+1} \to 0 \quad \text{(exact)}$$

$$\rightsquigarrow H^{j-1}(V_\cdot \otimes A/I_{k+1}) \to H^j(V_\cdot \otimes A/I_{k}) \xrightarrow{x_{k+1}} H^j(V_\cdot \otimes A/I_k)$$

$$H^{j-1}(V_\cdot \otimes A/I_{k+1}) = 0 \underset{\text{artinian}}{\Rightarrow} H^j(V_\cdot \otimes A/I_k) = 0$$

$$H^{-1}(V_\cdot \otimes A/I_{j+1}) = 0 \quad (j < n)$$

$$\Rightarrow H^j(V_\cdot) = 0$$

//

Thm (Bridgeland, Bondal-Orlov) Assume char $k=0$

(I) F fully faithful $\Leftrightarrow$

ⓐ $\mathrm{Hom}_{D(X)}(Fk_y, Fk_y) \cong k$

ⓑ $\mathrm{Hom}^i_{D(X)}(Fk_y, Fk_z) = 0$ unless ($y=z$ & $0 \le i \le \dim X$)

ここで $\mathrm{Hom}^i(a,b) = \mathrm{Hom}(a, b[i])$

(II) $Fk_y \otimes \omega_X \simeq Fk_y \; (\forall y \in Y) \Rightarrow F$: equiv

Rem Y : X 上の sheaf の moduli のとき

Kodaira-Spencer map $(T_Y)_y \xrightarrow{\sim} \mathrm{Hom}^1(Fk_y, Fk_y)$

このときは char >0 でも OK

証) 必要性 $\mathrm{Hom}(k_y, k_y) \cong k$

$\mathrm{Hom}^i(k_y, k_z) = 0$ unless ($y=z$, $0 \le i \le \dim Y$)

F : fully faithful

$\Leftrightarrow \mathrm{Hom}_{D(X)}(Fa, Fb) = \mathrm{Hom}_{D(Y)}(a,b)$

十分性 ここでは Y が simple sheaf の moduli のときに説明する

$$Fk_y = R\pi_{X*}(\underbrace{P \otimes k_y}_{= P|_{\{y\}\times X}}) = P|_{\{y\}\times X}$$

(I) $G\circ F(\mathcal{O}_{\Delta_Y}) \xrightarrow{\delta} \mathcal{O}_{\Delta_Y}$ $\quad (G\circ F \xrightarrow[\delta]{\sim} 1)$

(Δ_Y : diagonal (family of k_y par. by Y)

これが isom を示す ($G\circ F(\mathcal{O}_{\Delta_Y})$ は $G\circ F$ の kernel と一致)

$V_\bullet : 0 \to V_{-n} \to V_{-n+1} \to \cdots \to V_0 \to 0$

complex of loc free sheaves on $\underset{\substack{\uparrow \\ \text{parameter}}}{Y} \times Y$

representing $G\circ F(\mathcal{O}_{\Delta_Y})$

- $G\circ F(\mathcal{O}_{\Delta_Y})$: sheaf, flat $/Y$.

∵ $G\circ F(k_y)$ sheaf, ~~[illegible]~~ $\forall y \in Y$ を示す

$$H^i(i_z^*(G\circ F(k_y))) = \mathrm{Hom}^{i+\dim X}(P_z, P_y \otimes \omega_X)$$

$i_z : \{z\} \hookrightarrow Y$.

$$= \mathrm{Hom}^{-i}(P_y, P_z)$$

$$= 0 \quad \text{unless } (y=z,\ 0 \le -i \le \dim X)$$

$\Rightarrow H^i(G\circ F k_y)$ artinian

$\underset{\text{Lem}}{\Rightarrow}$ ~~[illegible]~~ $H^i(G\circ F k_y) = 0 \quad i<0$

さらに $\mathrm{Supp}\, G\circ F k_y = \{y\}$

これより $G\circ F(\mathcal{O}_{\Delta_Y})|_{\Delta_Y} \xrightarrow[\sim]{\delta} \mathcal{O}_{\Delta_Y}$ をいえる

$G\circ F(\mathcal{O}_{\Delta_Y}) \overset{\delta}{\twoheadrightarrow} \mathcal{O}_{\Delta_Y}$ が injective を言う

$$\begin{tikzcd}
G\circ F(\mathcal{O}_{\Delta_Y}) \arrow[r, "\delta"] \arrow[d] & \mathcal{O}_{\Delta_Y} \arrow[d] \\
G\circ F(\mathcal{O}_{\Delta_Y})|_{y\times Y} \arrow[r, "\delta"] & k_y
\end{tikzcd}$$

.. inj を言う

$G\circ F(\mathcal{O}_{\Delta_Y})|_{y\times Y} = G\circ F(k_y) = \mathcal{O}_Z$

δ : not inj とする

$\mathcal{O}_Z \longrightarrow k[\varepsilon] \longrightarrow k_y$ (ring of dual numbers)

$$\Rightarrow \begin{tikzcd}
F\circ G\circ F(k_y) \arrow[r] \arrow[rr, bend left, "\delta"] & Fk[\varepsilon] \arrow[r] & Fk_y \\
F(k_y) \arrow[u, "\zeta"] \arrow[urr, "1"'] & &
\end{tikzcd}$$

$\Rightarrow Fk[\varepsilon] \underset{\delta}{\twoheadrightarrow} Fk_y$ — (#)

ところが $Fk(\varepsilon) = P \otimes_{\mathcal{O}_Y} k[\varepsilon]$ は $k[\varepsilon]$ 上の simple sheaf の family

よって Kodaira-Spencer map の injectivity によって

$Fk(\varepsilon)$ は 非自明な Fk_y の変形 ⇒ ⓐに矛盾 ⇒ δ : injective.

Rem ① P が sheaf でない時

$GF(k_y) \xrightarrow{\delta} k_y$ を示すには $G \circ F$ をほどこす
$\subset$ flat sheaf

$$\begin{array}{ccc} (GF)(GF)k_y & \longrightarrow & GFk_y \\ \uparrow & \nearrow_{\sim} & \\ GFk_y & & \end{array}$$

$Y \supset U$ open $\quad G\circ F(\mathcal{O}_{\Delta_Y})|_{U\times Y} \simeq \mathcal{O}_Z \quad Z \subset U \times Y$, flat $\downarrow$ U

⇒ $U \to \mathrm{Hilb}_Y$ ⓑ が injective

② Y の非特異性は (a), (b) から従う

まず $\delta : H^0(GFk_y) \longrightarrow k_y$ isom を示す

$$\begin{array}{ccc} H^0(GFk_y) & \longrightarrow & k_y \\ \uparrow & \nearrow_{\delta} & \\ GFk_y & & \end{array}$$

証明は同様

δ isom ⇒ Y smooth.
"intersection theorem"

(II) $1 \xrightarrow[\xi]{\sim} FG$ を示す (十分性)

$a \in D(X)$

$c \to a \xrightarrow{\xi} FGa \to c[1]$ exact

⇒ $Gc \to Ga \xrightarrow{\xi} GFGa \to Gc[1]$ exact

$$\begin{array}{ccc} Ga & \xrightarrow{\xi} & GFGa \\ & \searrow^{\sim} & \downarrow \delta \wr \\ & & Ga \end{array}$$

⇒ $\xi : Ga \to GFGa$ isom ⇒ $Gc = 0$

P flat family, simple

$P \otimes \pi_X^*(\omega_X) \simeq P \otimes \pi_X^* L \quad (\exists L \in \mathrm{Pic}(Y))$

$(\Leftarrow P \otimes \pi_X^*(\omega_X)|_{\{y\}\times Y} \cong P|_{\{y\}\times Y})$

$\mathrm{Hom}_{D(X)}(FGa, c[1]) = \mathrm{Hom}_{D(X)}(c[1], FGa \otimes \omega_X[\dim X])^\vee$

$= \mathrm{Hom}_{D(X)}(c[1], F(L \otimes Ga)[\dim X])^\vee$

$= \mathrm{Hom}_{D(Y)}(\underset{\substack{\| \\ 0}}{Gc[1]}, (L \otimes Ga)[\dim X])^\vee$

$= 0$

$$\begin{array}{ccccc} & & FGa & & \\ & \swarrow & \downarrow \mathrm{id} & \searrow^{0} & \\ a & \to & FGa & \to & c[1] \end{array}$$

∴ $a = FGa \oplus c$

⇒ $\mathcal{O}_{\Delta_X} = FG\mathcal{O}_{\Delta_X} \oplus c$ ⇒ $c = 0$

証

Examples

1 Semi-homog. v.bdls on abel. var

X : n-dim abel

$\hat{X}$ dual P : Poincaré l.b.

E coh sheaf on X $\mathrm{rk}\, E = r > 0$, torsion free

$\mathcal{E} = \{ T_x^* E \otimes P_y \ : \ (x,y) \in X \times \hat{X} \}$ flat family of torsion free sheaves

$\det \mathcal{E}_{(x,y)} = T_x^* (\det E) \otimes P_{ry}$

$\Rightarrow \dim \mathrm{Def}(E) \geq n$

Def. E : semi-homog. if $\dim \mathrm{Def}(E) = n$
($\Rightarrow E$: v.b)

- E semi-homog. rk r $\det E = L$

  $\Rightarrow T_{rx}^* E \cong E \otimes P_{\phi_L(x)}$ $\phi_L : X \longrightarrow \hat{X}$, $x \longmapsto T_x^* L \otimes L^{-1}$

  (∵) $T_x^* E \simeq E \otimes P_y$ ($\exists y \in \hat{X}$) がとれる

  $\Rightarrow T_{rx}^* E \cong E \otimes P_{ry}$

  $T_x^* L \cong L \otimes P_{ry}$

- E semi-homog. rk r $\det E = L$

  $\Rightarrow E$ semi-stable (polarization に依らず)

  更に E simple $\Rightarrow$ E : μ-stable

- E, F simple semi-homog. $\dfrac{c_1(E)}{\mathrm{rk}\, E} = \dfrac{c_1(F)}{\mathrm{rk}\, F}$

  $\Rightarrow \mathrm{Ext}^i(E,F) = 0$ ($\forall i$)
  or $E \simeq F$

Thm Y : moduli of simple semi-homog. v.b. on X
(は abel var になる)
$\exists\ \mathcal{P}$: univ. fam. on $Y \times X$
$\Rightarrow \Phi_{Y \to X}^{\mathcal{P}} : D(Y) \longrightarrow D(X)$ equiv.

X abel surface とする

E : coh sheaf on X $\mathrm{ch}(E) = (r, \xi, a)$

$\chi(E,F) = \xi^2 - 2ra$ R-R thm

E : simple $\Rightarrow$ $\mathrm{Def}(E)$ smooth

E : semi-homog $\Rightarrow$ $\dim \mathrm{Ext}^1(E,E) = 2$

$$\dim \mathrm{Ext}^1(E,E) = \dim \underset{\substack{\parallel \\ 1}}{\mathrm{Hom}(E,E)} + \dim \underset{\substack{\parallel \\ 1}}{\mathrm{Ext}^2(E,E)} - \chi(E,E)$$

simple Serre duality

$\therefore \chi(E,E) = 0$

E, F が $\mathrm{ch}\, E = \mathrm{ch}\, F$ をみたす μ-stable な vector bundle
$E \neq F$
$\Rightarrow \mathrm{Hom}(E,F) = 0 \Rightarrow \mathrm{Ext}^2(E,F) = 0$

$\Rightarrow \chi(E,F) = 0 \Leftrightarrow \mathrm{Ext}^1(E,F) = 0$.

Stable sheaf on K3 surfaces

Mukai Lattice X : K3 or abelian

$(H^{ev}(X,\mathbb{Z}), \langle\ ,\ \rangle)$

$\langle x, y \rangle = \int_X x_1 \wedge y_1 - x_2 \wedge y_0 - x_0 \wedge y_2$.

$x = (x_0, x_2, x_4) \in H^0 \oplus H^2 \oplus H^4$
$y = (y_0, y_2, y_4)$

$X : K3 \Rightarrow H^{ev}(X,\mathbb{Z}) \cong (-E_8)^{\oplus 2} \oplus \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}^{\oplus 4}$

$X : \text{abel} \Rightarrow H^{ev}(X,\mathbb{Z}) \cong \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}^{\oplus 4}$

Mukai vector E sheaf on X

$$v(E) = \mathrm{ch}\, E \sqrt{\mathrm{td}(X)}$$

$$= \begin{cases} (\mathrm{rk}\, E,\ c_1(E),\ \chi(E) - \mathrm{rk}\, E) & K3 \\ (\mathrm{rk}\, E,\ c_1(E),\ \chi(E)) = \mathrm{ch}(E) & \text{Abel.} \end{cases}$$

$$\chi(E,F) = -\langle v(E), v(F) \rangle$$

Thm (Mukai) E simple sheaf

$\Rightarrow \mathrm{Def}(E)$ smooth

of $\dim = \langle v(E), v(E) \rangle + 2$

Def $v \in H^{ev}(X,\mathbb{Z})$

$v = (r, \xi, a)$ $\xi \in NS(X)$

$M_H(v)$ = moduli of H stable sheaves E on X s.t. $v(E) = v$

Thm X K3, v : primitive

$M_H(v) \neq \emptyset$ for general H

$\Leftrightarrow \langle v^2 \rangle \geq -2$

このとき $M_H(v)$ sm proj var

特に $\langle v^2 \rangle = 0 \Rightarrow M_H(v)$ K3 surface $\rightsquigarrow$ FM変換

$\langle v^2 \rangle = -2 \Rightarrow$ 1点 $w = (\mathrm{rk}\, v) \cdot v - (0,0,1)$

Cor $\langle v^2 \rangle = 0$. $\exists$ univ fam $\mathcal{P}$

$\Rightarrow \Phi^{\mathcal{P}}_{Y \to X}$ equiv $(Y = M_H(v))$

$\langle v^2 \rangle = -2$ のとき $M_H(v) = \{E_0\}$

$\mathrm{Hom}(E_0, E_0) = k$

$\mathrm{Ext}^1(E_0, E_0) = 0$

$\mathrm{Ext}^2(E_0, E_0) = k$

$\mathcal{E} := \mathrm{Ker}\,(E_0^\vee \boxtimes E_0 \xrightarrow{ev} \mathcal{O}_\Delta)$ on $X \times X$

$\mathcal{E}|_{x \times X}$ stable sheaf $v(\mathcal{E}|_{x \times X}) = (\mathrm{rk}\, v) v - (0,0,1)$

$$\begin{array}{ccc} M_H(w) & \xleftarrow{\sim} & X \\ \cup & & \cup \\ \mathcal{E}|_{x \times X} & \longleftarrow\!\shortmid & x \end{array}$$

$\Rightarrow \Phi^{\mathcal{E}}_{X \to X} : D(X) \to D(X)$ これを Mukai reflection とよぶ

$$\begin{array}{ccc} D(X) & \xrightarrow{\Phi^{\mathcal{E}}_{X \to X}} & D(X) \\ \downarrow v & & \downarrow v \\ H^{ev}(X,\mathbb{Z}) & \xrightarrow{-R_{v(E_0)}} & H^{ev}(X,\mathbb{Z}) \end{array}$$

$$E \longmapsto \mathbb{R}\pi_{X*}(\mathcal{E} \otimes \pi^*_{M_H(w)}(E))$$

$$0 \to \mathcal{E} \to E_0^\vee \boxtimes E_0 \to \mathcal{O}_\Delta \to 0$$

$\otimes E$

$$0 \to \mathcal{E} \otimes \pi^*(E) \to (E_0^\vee \otimes E) \boxtimes E_0 \to E \to 0$$

$$0 \to \pi_*(\mathcal{E} \otimes \pi^*(E)) \to \mathrm{Hom}(E_0, E) \otimes E_0 \xrightarrow{ev} E \to \cdots$$

$$v(\Phi^{\mathcal{E}}_{X \to X}(E)) = \chi(E_0, E) v(E_0) - v(E)$$

$$= -\underbrace{(\langle v(E_0), v(E) \rangle v(E_0) + v(E))}_{\substack{(-2)\text{-reflection} \\ R_{v(E_0)}}}$$

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$\mathcal{A}$ abelian category
$K(\mathcal{A})$ the category of complexes of $\mathcal{A}$

$K(\mathcal{A})$ の object は complex.

$$E^\cdot = [\cdots \longrightarrow E^{i-1} \xrightarrow{d^{i-1}} E^i \xrightarrow{d^i} E^{i+1} \to \cdots]$$

$$E^i \in \mathcal{A} \qquad d^i \cdot d^{i-1} = 0.$$

$K(\mathcal{A})$ の morphism
Ψ
$E^\cdot, F^\cdot$

$$\mathrm{Hom}(E^\cdot, F^\cdot) = \{ f^\cdot : E^\cdot \longrightarrow F^\cdot \text{ chain map} \}/\sim$$
$$d^i_F \cdot f^i = f^{i+1} \cdot d^i_E$$

$f^\cdot \sim g^\cdot \overset{\text{def}}{\Longleftrightarrow} \exists k = (k^n)$ with $k^n \in \mathrm{Hom}_{\mathcal{A}}(E^n, F^{n-1})$
s.t.
$$f^n - g^n = d^{n-1} \cdot k^n + k^{n+1} \cdot d^n$$

$K(A) \ni E^\cdot \longmapsto E^\cdot[n] \in K(A)$ translation

$$(E^\cdot(n))^p = E^{n+1}$$

$$d_{E^\cdot[n]} = (-1)^n d_E.$$

$$\mathrm{Hom}_{K(\mathcal{A})}(E^\cdot, F^\cdot) \ni f \longmapsto \mathrm{Cone}(f) \in K(\mathcal{A})$$

$$\mathrm{Cone}(f) = (E^\cdot[1])^i \oplus F^\cdot$$

$$d_{\mathrm{Cone}(f)}(a^{i+1}, b^i) = (-d_E \cdot a^{i+1},\ f^{i+1}(a^{i+1}) + d_F(b^i))$$

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<u>Fact</u> $T : K(\mathcal{A}) \ni E^\cdot \longmapsto E^\cdot[1] \in K(\mathcal{A})$ translation
と triangle $E^\cdot \longrightarrow F^\cdot \longrightarrow \mathrm{Cone}(f)$

によって $K(\mathcal{A})$ は triangulated category となる.

$E_0[-1] \longrightarrow E_1^\cdot \longrightarrow E_2^\cdot \longrightarrow E_3^\cdot$ triangle.

$$\Longrightarrow \ \to H^i(E_1^\cdot) \longrightarrow H^i(E_2^\cdot) \to H^i(E_3^\cdot) \longrightarrow \cdots$$
$$H^i(E_3^\cdot) = H^\cdot(E_1[1])$$

$F^\cdot \in K(\mathcal{A})$

$$\cdots \longrightarrow \mathrm{Hom}_{K(\mathcal{A})}(F^\cdot, E_1^\cdot) \longrightarrow \mathrm{Hom}_{K(\mathcal{A})}(F^\cdot, E_2^\cdot) \longrightarrow \mathrm{Hom}_{K(\mathcal{A})}(F^\cdot, E_3^\cdot)$$
$$\longrightarrow \mathrm{Hom}_{K(\mathcal{A})}(F^\cdot, E_1^\cdot[1])$$

$$\cdots \longrightarrow \mathrm{Hom}_{K(\mathcal{A})}(E_3^\cdot, F) \longrightarrow \mathrm{Hom}_{K(\mathcal{A})}(E_2^\cdot, F) \longrightarrow \mathrm{Hom}_{K(\mathcal{A})}(E_1^\cdot, F)$$
$$\longrightarrow \mathrm{Hom}_{K(\mathcal{A})}(E_1, F[1])$$

は完全列.

以下(必要なら)簡単のため
$\mathcal{A}$ has enough injectives と仮定する

(i.e. $\forall E \in \mathcal{A}\ \exists I \in \mathcal{A}$; inject object
$\exists E \longrightarrow I$ monomorphism)

Def. $f^{\cdot} : E^{\cdot} \to F^{\cdot}$ morphism on $K(\mathcal{A})$

f is a quassi-isomorphism

def ⇔ $\forall p$, $H^p(E^{\cdot}) \xrightarrow{H^p(f)} H^p(F^{\cdot})$; isom

Def. ($D(\mathcal{A})$ の定義)
$D(\mathcal{A})$ の objects = $K(\mathcal{A})$ の objects

$$\mathrm{Hom}_{D(\mathcal{A})}(E^{\cdot}, F^{\cdot}) = \varinjlim_{\substack{\hat{E}^{\cdot} \to E^{\cdot} \\ q\text{-isom}}} \mathrm{Hom}_{K(\mathcal{A})}(\hat{E}^{\cdot}, F^{\cdot})$$

$$= \varinjlim_{\substack{F^{\cdot} \to \hat{F}^{\cdot} \\ q\text{-isom}}} \mathrm{Hom}_{K(\mathcal{A})}(E^{\cdot}, \hat{F}^{\cdot})$$

$F^{\cdot} \in K(\mathcal{A})$ かつ $\exists i_0$ s.t. $F^i = 0$ $i \le i_0$

($I^{\cdot}$ complex of injective objects
$F^{\cdot} \longrightarrow I^{\cdot}$ quasi-isom
をとる

$$\mathrm{Hom}_{D(\mathcal{A})}(E^{\cdot}, F^{\cdot}) = \mathrm{Hom}_{K(\mathcal{A})}(E^{\cdot}, I^{\cdot})$$

Notation $K^+(\mathcal{A}) = \{ E^{\cdot} \in K(\mathcal{A}) \mid \exists i_0$ s.t. $H^i(E^{\cdot}) = 0\ \forall i \le i_0 \}$

$K^-(\mathcal{A}) = \{$ 〃 $\mid$ 〃 $\forall i \ge i_0 \}$

$K^b(\mathcal{A}) = K^+(\mathcal{A}) \cap K^-(\mathcal{A})$

$D^+(\mathcal{A}) = \{ E \in D(\mathcal{A}) \mid \exists i_0$ s.t. $H^i(E) = 0\ \forall i \le i_0 \}$

$D^-(\mathcal{A})$ $D^b(\mathcal{A})$ も同じように定義する.

$Q : K^*(\mathcal{A}) \longrightarrow D^*(\mathcal{A})$ $(* = \phi, +, -, b)$
$E^{\cdot} \longmapsto E^{\cdot}$
natural functor.

Prop-Def $F : K^*(\mathcal{A}) \longrightarrow K(\mathcal{B})$ exact functor.
$(* = \phi, +, -, b)$

(i.e. $\forall E_1^{\cdot} \to E_2^{\cdot} \to E_3^{\cdot}$ triangle
$\Rightarrow F(E_1^{\cdot}) \to F(E_2^{\cdot}) \longrightarrow F(E_3^{\cdot})$
$F \circ T = T \circ F$)

Assume
$\exists L \subseteq K^*(\mathcal{A})$ triangulated subcategory

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s.t.

(1) $\forall E^\cdot \in K^*(\mathcal{A})$ $\exists I^\cdot \in L$

$\varphi : E^\cdot \longrightarrow I^\cdot$ quasi-isom

(2) $\forall I^\cdot \in L$ $H^i(I^\cdot) = 0\ (\forall i) \Rightarrow H^i(F(I^\cdot)) = 0\ (\forall i)$

then.

$\exists\, \mathbb{R}F : D^*(\mathcal{A}) \longrightarrow D(\mathcal{B})$; exact functor

(right derived functor of F)

$\exists\, \xi : Q \circ F \longrightarrow \mathbb{R}F \cdot Q$ morphism of functors

s.t. $\forall G : D^*(\mathcal{A}) \longrightarrow D(\mathcal{B})$ exact functor

$\forall \zeta : Q \circ F \longrightarrow G \cdot Q$ morphism

$\exists !\, \eta : \mathbb{R}F \longrightarrow G$

s.t.

$\zeta = (\eta \cdot Q) \circ \xi$

$$\begin{tikzcd} Q \circ F \arrow[r, "\xi"] \arrow[d, "\zeta"'] \arrow[dr, phantom, "\circlearrowleft"] & \mathbb{R}F \circ Q \arrow[dl, "\eta \cdot Q"] \\ G \cdot Q & \end{tikzcd}$$

<u>Remark</u> $E^\cdot \in D^*(\mathcal{A})$, take $I^\cdot \in L$

$\varphi : E^\cdot \longrightarrow I^\cdot$ quasi-isom

$\Longrightarrow \mathbb{R}F(E^\cdot) \cong F(I^\cdot)$

(Usually L = the category of injective complex of $\mathcal{A}$)

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<u>Example</u> (1). $F : \mathcal{A} \longrightarrow \mathcal{B}$ left exact functor

$F : K^\cdot(\mathcal{A}) \longrightarrow K^\cdot(\mathcal{B})$ exact functor

$(E^\cdot) \longrightarrow (F(E^\cdot))$

has a right derived functor

$\mathbb{R}F : D^\cdot(\mathcal{A}) \longrightarrow D(\mathcal{B})$

$\underset{\text{triangle}}{E_1^\cdot \longrightarrow E_2^\cdot \longrightarrow E_3^\cdot} \Rightarrow H(\mathbb{R}F(E_1^\cdot)) \longrightarrow H(\mathbb{R}F(E_2^\cdot)) \longrightarrow H(\mathbb{R}F(E_3^\cdot))$

exact seq.

(2) $E^\cdot, F^\cdot \in K(\mathcal{A})$ に対し

$Hom^\cdot(E^\cdot, F^\cdot) \in K(ab)$ を

$Hom^n(E^\cdot, F^\cdot) = \prod_{p \in \mathbb{Z}} Hom_{\mathcal{A}}(E^p, E^{p+n})$

$d^n = \prod_p (d_E^{p-1} + (-1)^{n+1} d_F^{p+n})$

$d^n(\{f^p\}) = \{f^p d_E^{p-1} + (-1)^{n+1} d_F^{p-1} \circ f^p\}$

$Hom^\cdot(E^\cdot, -) : K^\cdot(\mathcal{A}) \longrightarrow K(ab)$ exact functor

これ derived functor を

$\mathbb{R}Hom^\cdot(E^\cdot, -) : D^+(\mathcal{A}) \longrightarrow D(ab)$ と書く.

$\Big(\underset{\text{q-iso}}{E \longrightarrow} \underset{\text{injective complex}}{I^\cdot} \quad \mathbb{R}Hom^\cdot(E^\cdot, F^\cdot) = Hom^\cdot(E^\cdot, I^\cdot) \Big)$

<u>Fact</u>

$H^i(\mathbb{R}Hom^\cdot(E^\cdot, F^\cdot)) \cong Hom_{D(\mathcal{A})}(E^\cdot, F^\cdot[i])$

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(3) X scheme
$Mod(\mathcal{O}_X) = \mathcal{O}_X$-module のなす abel cat.
$E^\cdot \in K(Mod(\mathcal{O}_X))$

$$\mathcal{H}om^\cdot(E^\cdot, -) : K^+(Mod(\mathcal{O}_X)) \longrightarrow K(Mod(\mathcal{O}_X))$$
$$F^\cdot \longmapsto \mathcal{H}om^\cdot(E^\cdot, F^\cdot)$$
exact functor

これの derived functor を
$\mathbb{R}\mathcal{H}om^\cdot(E^\cdot, -)$ と書く.

$H^i(\mathbb{R}\mathcal{H}om^\cdot(E^\cdot, F^\cdot)) = \mathcal{E}xt^i(E^\cdot, F^\cdot)$ と書く

性質

$$\Gamma : Mod(\mathcal{O}_X) \longrightarrow ab.$$
$$E \longmapsto \Gamma(X, E)$$

$$\mathbb{R}\Gamma \circ \mathbb{R}\mathcal{H}om^\cdot = \mathbb{R}\,Hom^\cdot$$

$E_1^\cdot \longrightarrow E_2^\cdot \longrightarrow E_3^\cdot$ triangle
$\Rightarrow \mathcal{E}xt^i(F^\cdot, E_1) \longrightarrow \mathcal{E}xt^i(F^\cdot, E_2) \longrightarrow \mathcal{E}xt^i(F^\cdot, E_3)$
exact.

Notation
X noetherian scheme ($* = \phi, +, -, b$)
$D^*_{qc}(X) = \{ E \in D^*(Mod(\mathcal{O}_X)) \mid {}^\forall i \ \ H^i(E^\cdot) \text{ is quasi coh} \}$
$D^*_c(X) = \{ \quad " \quad \mid \quad " \quad \}$

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Fact $Qc(X)$ = quasi-coherent sheaf の category
$D^+(Qc(X)) \xrightarrow{\sim} D^+_{qc}(X)$ は category equivalence.
$E^\cdot \longrightarrow E^\cdot$

この Fact を使うと: ${}^\forall E^\cdot \in D_c^b(X)$
${}^\exists F^\cdot = (F^i)$ bdd complexes of coherent $\mathcal{O}_X$-module
(${}^\forall i$ F^i coherent)
s.t.
$E^\cdot \cong F^\cdot$ in $D_c^b(X) = D^b(Coh(X))$ とも書ける.

$F^\cdot, G^\cdot \in K(Mod(\mathcal{O}_X))$
$F^\cdot \otimes G^\cdot \in K(Mod(\mathcal{O}_X))$ を
$$(F^\cdot \otimes G^\cdot)^n = \bigoplus_{p+q=n} F^p \otimes G^q$$
$$d = d_F \otimes 1_G + (-1)^n 1 \otimes d_G$$

$$F^\cdot \otimes - : K(Mod(\mathcal{O}_X)) \longrightarrow K(Mod(\mathcal{O}_X))$$
$$E^\cdot \longmapsto E^\cdot$$

$F^\cdot \otimes -$ の left derived functor を $F^\cdot \overset{L}{\otimes} -$ と書く

$$F^\cdot \overset{L}{\otimes} : D^-(Mod(\mathcal{O}_X)) \longrightarrow D(Mod(\mathcal{O}_X))$$

$f : Y \longrightarrow X$ morphism of schemes.
$$f^* : K(Mod(\mathcal{O}_X)) \longrightarrow K(Mod(\mathcal{O}_Y))$$
$$E^\cdot \longmapsto f^*(E^\cdot)$$

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f^* の left exact functor を
$\mathbb{L}f^* : D^-(Mod(\mathcal{O}_X)) \longrightarrow D(Mod(\mathcal{O}_Y))$ と書く.

Remark
$E^\cdot \in D^-(Mod(\mathcal{O}_X))$ に対し
$\exists F \in K^-(Mod(\mathcal{O}_X))$ with $\forall F^i$ flat $/\mathcal{O}_X$

$F^\cdot \xrightarrow{\exists} E^\cdot$ quasi-isom.

$G^\cdot \overset{\mathbb{L}}{\otimes} E^\cdot \cong G^\cdot \otimes F^\cdot$
$\mathbb{L} f^*(E^\cdot) \cong f^*(F^\cdot)$

Th (Grothendieck - Serre duality)
X : proper $/k$ ← 仮
$\exists \omega_X \in D^+_{qc}(X)$ dualizing complex
$\exists \theta : \mathbb{R}Hom^\cdot(F^\cdot, \omega_X^\cdot) \cong \mathbb{R}Hom^\cdot(\mathbb{R}\Gamma(F^\cdot), k)$
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad \| $
$\qquad\qquad\qquad\qquad\qquad\qquad \mathbb{R}\Gamma(F^\cdot)^\vee$
(functorial isom)
for $F^\cdot \in D_{qc}(X)$

特に. X : smooth $/k$ of dim n のとき.

$\omega^\cdot_X = \Omega_X^n[n]$

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(自分の話)
X K3 or abelian surface $/\mathbb{C}$
Th (Mukai) The moduli space of simple sheaves on X is smooth and has a symplectic form

これを $D^n_c(X)$ の objects の moduli の場合に示す.
$\qquad\qquad \| $
$\qquad D^n(Coh(X))$

$M_X^{pre} : \binom{noeth}{Sch}/\mathbb{C} \longrightarrow (Sets)$

$S \longmapsto \left\{ E^\cdot \in D^-_c(X \times S) \;\middle|\; \begin{array}{l} \forall s : \text{geometric pt of } S \\ E \overset{\mathbb{L}}{\otimes} k(s) \in D^b_c(X)_s \\ Ext(E^\cdot \overset{\mathbb{L}}{\otimes} k(s), E^\cdot \overset{\mathbb{L}}{\otimes} k(s)) \cong \begin{cases} 0 & i=-1 \\ k(s) & i=0 \end{cases} \end{array} \right\} \Big/ \sim$

$E^\cdot \sim F^\cdot$
$\Longleftrightarrow \exists L$: line bdl on S
s.t.
$E^\cdot \otimes_S L \cong F^\cdot$ in $D(X \times S)$

M_X = étale sheafification of M_X^{pre}

Th (—) M_X is representible by an algebraic space locally of finite type $/\mathbb{C}$
(quasi-separated)

Th X (proj) K3 or abelian surface $/\mathbb{C}$
$\Rightarrow M_X$ は smooth で $\exists$ symplectic form on M_X.

(Sketch of proof)

$x = [E^\bullet] \in M_X$ で M_X が smooth になるための

obstruction class

$\omega(E^\bullet) \in \mathrm{Ext}^2(E^\bullet, E^\bullet)$

$$\begin{array}{ccc} \omega(E^\bullet) \in \mathrm{Ext}^2(E, E) & \xrightarrow[\sim]{tr} & H^2(X, \mathcal{O}_X) \cong \mathbb{C} \\ \wr\wr & & \downarrow S \\ \mathrm{Ext}^0(E^\bullet, E^\bullet) & \xrightarrow{\sim} & H^0(\mathcal{O}_X)^\vee \end{array}$$

natural map.

Claim

$tr(\omega(E^\bullet))$ は $[\det E^\bullet] \in \mathrm{Pic}(X)$ で $\mathrm{Pic}\, X$ が smooth になるための obstruction class に一致してる。

Claim より $tr(\omega(E^\bullet)) = 0$ $\omega(E) = 0$

$\therefore M_X$ は smooth

$$\underline{\mathrm{Ext}^1(E^\bullet, E^\bullet) \times \mathrm{Ext}^1(E^\bullet, E^\bullet)} \longrightarrow \mathrm{Ext}^2(E^\bullet, E^\bullet) \xrightarrow{tr} \mathbb{C}$$

$\|$

$T_{M_X}[E^\bullet]$

tangent space

Ω が symplectic form を定める。

$d\Omega = 0$ (← Huybrechts-Lehn の教科書)

16:00 - 戸田先生

Fourier-Mukai transform and canonical divisors

§1. Intro

Y … smooth proj. var $/\mathbb{C}$

$D^b(X) := D^b(\mathrm{Coh}\, X)$ ← X の幾何学的情報を多く含む。

特に、K_X の情報

$S_X := \otimes\, \omega_X[\dim X] : D^b(X) \xrightarrow{\sim} D^b(X)$

Serre functor ($\mathrm{Hom}(a, b) \cong \mathrm{Hom}(b, S_X(a))^*$)

$\Phi : D^b(X) \xrightarrow{\sim} D^b(Y)$

$\Rightarrow \Phi \circ S_X \cong S_Y \circ \Phi$ ($\dim X = \dim Y$ が示せる)

Def. $FM(X) := \{\text{smooth proj. var. } Y \mid \exists \Phi : D^b(X) \xrightarrow{\sim} D^b(Y)\}$

↑ Fourier-Mukai partner と呼ぶ。

Example

① $\begin{cases} A \text{.. A.V. } \hat{A} \text{. dual} \Rightarrow \hat{A} \in FM(A) \\ X \text{.. K3 surface } M^H \text{.. moduli space of stable sheaves w.r.t. } H \text{ fine } \dim M_H = 2 \end{cases}$

$\Rightarrow M^H \in FM(X)$

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② $X \dashrightarrow X^+$ 3-dim flop　X, X^+ : smooth

$\Rightarrow X^+ \in FM(X)$

③ K_X or $-K_X$ ample $\Rightarrow FM(X) = \{X\}$

問題 $Y \in FM(X) \Rightarrow X$ と Y はどのような関係にあるのか？

知られている結果 $\dim X \leq 2$ 又は $\dim X$ … 一般 $\kappa(X) = \dim X$

Main idea $^\exists E \in |mK_X|$

$\Rightarrow$ E を使って、この問題をより低次元の間に帰着できる.

§2. Correspondence of canonical divisors

$Y \in FM(X)$　$\Phi : D^b(X) \xrightarrow{\sim} D^b(Y)$

S_X … categorial invariant　$(d = \dim X = \dim Y)$

$$\begin{array}{ccc} Nat(id_X, S_X^m[-dm]) & \xrightarrow{\sim} & Nat(id_Y, S_Y^m[-dm]) \\ \otimes \omega_X^{\otimes m} & & \otimes \omega_Y^{\otimes m} \\ \cup & & \cup \\ H^0(X, mK_X) & \dashrightarrow & H^0(Y, mK_Y) \end{array}$$

$H^0(X, mK_X) \xrightarrow{\sim} H^0(Y, mK_Y)$

$$\begin{array}{ccc} |mK_X| & \xrightarrow{\sim} & |mK_Y| \\ \Psi & & \Psi \\ E & \longmapsto & E^\Phi \end{array}$$

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$D_E^b(X) := \{a \in D^b(X) \mid \mathrm{Supp}\, a := \bigcup_i \mathrm{Supp}\, H^i(a) \subset E\}$

$D_{E^\Phi}^b(Y) :=$ 同様

lemma. Φ は $D_E^b(X)$ を $D_{E^\Phi}^b(Y)$ にうつす

∵) $a \in Coh(X) \cap D_E^b(X)$ をとる.

$\sigma_E \in H^0(X, mK_X)$, $\mathrm{div}(\sigma_E) = E$

$\sigma_E(a) : a \xrightarrow{\times \text{loc. eq. of } E} a \otimes O(mK_X)$

$^\exists N \gg 0$　$\sigma_E^N(a) : a \to a \otimes O(NmK_X)$ .. 0-map

$H^0(X, mK_X)\ \sigma_E \longmapsto \sigma_{E^\Phi} \in H^0(Y, mK_Y)$

$\Rightarrow \sigma_{E^\Phi}^N(\Phi(a)) : \Phi(a) \xrightarrow{\times \text{loc. eq. of } NE^\Phi} \Phi(a) \otimes O(NmK_Y)$ .. 0-map

$\mathrm{Supp}\, \Phi(a) \subset E^\Phi$　□

$\Phi|_{D_E^b(X)} : D_E^b(X) \to D_{E^\Phi}^b(Y)$

$D_E^b(X)$ と $D^b(E)$

↑ $D^b(E)$ と formal nbd の情報含む

もし $D^b(E) \simeq D^b(E^\Phi)$ が分かれば, $\dim E < \dim X$ なので, E と E^Φ

の関係から, X と Y の関係も分かるだろう.

正確には、$E_i \in |m_i K_X|$ を、$(i=1,2,\dots,N)$ $C \in \pi_0(\bigcap_{i=1}^{N} E_i)$ をとる。

connected component

$\Rightarrow \exists! C^{\Phi} \in \pi_0(\bigcap_{i=1}^{N} E_i^{\Phi})$ s.t. $\Phi|_{D^b_C(X)} : D^b_C(X) \to D^b_{C^{\Phi}}(Y)$

次を仮定する.

① C, C^{Φ} .. complete intersection

② $\mathcal{T}or_i^{\mathcal{O}_{X\times Y}}(H^k(\mathcal{P}), \mathcal{O}_{C\times C^{\Phi}}) = 0$

$\mathcal{T}or_i^{\mathcal{O}_{X\times Y}}(H^k(\mathcal{P}^{\vee}), \mathcal{O}_{C\times C^{\Phi}}) = 0 \quad \forall i>0, \forall k$

($\mathcal{P}$: Φ の kernel)

①, ②は、$|m_i K_X|$ free で、$E_i \in |m_i K_X|$ general number ならOK.

Theorem A. この条件の下、

$\exists \Phi_C : D^b(C) \xrightarrow{\sim} D^b(C^{\Phi})$ 次の図式が可換

$$\begin{tikzcd} D^b(X) \arrow[r, "Li_C^*"] \arrow[d, "\Phi"', "\wr"] & D^b(C) \arrow[r, "i_{C*}"] \arrow[d, "\Phi_C", "\wr"'] & D^b(X) \arrow[d] \\ D^b(Y) \arrow[r, "Li_{C^\Phi}^*"] & D^b(C^{\Phi}) \arrow[r, "i_{C^\Phi *}"] & D^b(Y) \end{tikzcd}$$

$\begin{cases} i_C : C \hookrightarrow X \\ i_{C^\Phi} : C^{\Phi} \hookrightarrow Y \end{cases}$ inclusions

§3. Outline of the proof of Theorem A

$\Phi \circ S_X^{-m}[dm] \xrightarrow{\sim} S_Y^{-m}[dm] \circ \Phi$

kernel $\qquad$ kernel

$\mathcal{P} \otimes \mathcal{O}(-mf^*K_X) \qquad \mathcal{P} \otimes \mathcal{O}(-mg^*K_Y)$

$f: X\times Y \to X$, $g: X \times Y \to Y$ projections

$\exists \varphi_m : \mathcal{P} \otimes \mathcal{O}(-mf^*K_X) \cong \mathcal{P} \otimes \mathcal{O}(-mg^*K_Y)$ (Orlovの定理)

$E \in |mK_X| \quad E^{\Phi} \in |mK_Y|$

Step 1. 次の図式は可換

$$\begin{tikzcd} \mathcal{P} \otimes \mathcal{O}(-mf^*K_X) \arrow[r, "\text{loc. eq. of } E"] \arrow[d, "\varphi_m", "\wr"'] & \mathcal{P} \arrow[r] \arrow[d, equal] & \mathcal{P} \overset{L}{\otimes} \mathcal{O}_{E\times Y} \\ \mathcal{P} \otimes \mathcal{O}(-mg^*K_Y) \arrow[r, "\text{loc. eq. of } E^\Phi"'] & \mathcal{P} \arrow[r] & \mathcal{P} \overset{L}{\otimes} \mathcal{O}_{X\times E^{\Phi}} \end{tikzcd}$$

(idea) Induced diagram of nat. transforms

$$\begin{tikzcd} \Phi \circ S_X^{-m}[dm] \arrow[r] \arrow[d] & \Phi \arrow[d, equal] \\ S_Y^{-m}[dm] \circ \Phi \arrow[r] & \Phi \end{tikzcd}$$

Thm (Orlov)

$\forall \Phi : D^b(X) \cong D^b(Y)$

$\Rightarrow \exists! \mathcal{P} \in D^b(X\times Y)$

$\Phi \cong \Phi^{\mathcal{P}}_{X\to Y}$

+ $\mathcal{P}$ を Φ を使って記述する

→ kernel の間の可換性

Step 2. Step 1 $\Rightarrow$ $\mathcal{P} \otimes \mathcal{O}_{E\times Y} \cong \mathcal{P} \overset{L}{\otimes} \mathcal{O}_{X\times E^{\Phi}}$

$E_i \in |m_i K_X| \quad C \in \pi_0(\bigcap_{i=1}^{N} E_i)$

$P \overset{L}{\otimes} \mathcal{O}_{C\times Y} \simeq P \overset{L}{\otimes} \mathcal{O}_{X\times C^{\Phi}}$

$\quad {}'' \qquad\qquad\qquad\quad {}''$

$i_{C\times Y *} L i^{*}_{C\times Y} P \qquad i_{X\times C^{\Phi} *} L i^{*}_{X\times C^{\Phi}} P$

$\Rightarrow \ \exists P_C \in D^b_C(C\times C^{\Phi})$ s.t. $P \overset{L}{\otimes} \mathcal{O}_{C\times Y} \simeq P \overset{L}{\otimes} \mathcal{O}_{X\times C^{\Phi}}$

technical (①, ②の条件を使う) $\qquad \simeq i_{C\times C^{\Phi} *} P_C$

$\Phi_C := \Phi^{P_C}_{C\to C^{\Phi}}$

Step 3. Φ_C が求める equivalence である.

(i) 次の図式が可換

$$\begin{array}{ccccc} D^b(X) & \to & D^b(C) & \to & D^b(X) \\ \Phi \downarrow\wr & & \Phi_C \downarrow & & \downarrow\wr\, \Phi \\ D^b(Y) & \to & D^b(C^{\Phi}) & \to & D^b(Y) \end{array}$$

(ii) Φ'を使って $\Phi_C : D^b(C^{\Phi}) \to D^b(C)$ を定義する.

(i) $\Rightarrow \begin{cases} \Phi_{C^{\Phi}} \circ \Phi_C(\mathcal{O}_x) = \mathcal{O}_x & {}^{\forall} x \in C \quad ① \\ \Phi_{C^{\Phi}} \circ \Phi_C(\mathcal{O}_C) = \mathcal{O}_C & \quad ② \end{cases}$

① $\Rightarrow$ $\Phi_{C^{\Phi}} \circ \Phi$ の kernel … line bundle on $\Delta_C \subset C\times C$

$\Phi_{C^{\Phi}} \circ \Phi \simeq \otimes \mathcal{L}$ for some line bundle

② $\Rightarrow$ $\mathcal{L} \simeq \mathcal{O}_C$ $\quad \therefore \ \Phi_C$: equivalence

§4. Fourier-Mukai partners of 3-fold with $\kappa > 0$

Thm A $\Rightarrow$ Xが高次元でも $\kappa > 0$ なら

$Y \in FM(X)$ に対して, XとYの関係をうまく記述できる.

① $\dim X = \dim Y = 3 \quad \kappa(X) \geq 2$

② $\dim X = \dim Y = 3 \quad X$ … minimal $\quad \kappa(X) = 1$ の場合記述できる.

例えば, 次の場合を考える

$\dim X = 3$, $\kappa(X) = 1$, X … minimal

$\pi_X : X \to Z_X := \mathrm{Proj} \bigoplus_{m\geq 0} H^0(X, mK_X)$

π_Xの generic fiber … K3 surface と仮定 (Iitaka fibration)

$Y \in FM(X)$ $\quad \Phi$は $Z_X \xrightarrow{\sim} Z_Y$ を induce
$\qquad\qquad\qquad (=: Z)$

$$\begin{array}{ccc} X & & Y \\ & {}_{\pi_X}\searrow \quad \swarrow_{\pi_Y} & \\ & Z & \end{array}$$

$p \in Z$ generic point

Thm A $\Rightarrow$ ${}^{\exists}\Phi_p : D^b(X_p) \xrightarrow{\sim} D^b(Y_p)$ s.t.

$$\begin{array}{ccccc} D^b(X) & \to & D^b(X_p) & \to & D^b(X) \\ \downarrow\wr\,\Phi & \circlearrowleft & \downarrow\wr\,\Phi_p & \circlearrowleft & \downarrow\wr\,\Phi \\ D^b(Y) & \to & D^b(Y_p) & \to & D^b(Y) \end{array}$$

Def. $H \in \mathrm{Pic}(X)$ polarization

$M^H(X/Z)$ … rel. moduli space of stable sheaves

Theorem B $Y \in FM(X)$ if and only if one of the following holds.

(i) $\exists H_X \in \mathrm{Pic}(X)$ polarization

$\exists M_X \subset M^{H_X}(X/Z)$ irred. compo. fine … $\left\{\begin{array}{l}\text{universal sheaf 存在} \\ \text{rel. compact}\end{array}\right.$

and rel. dim $/Z = 2$

(ii) $\exists H_Y \in \mathrm{Pic}(Y)$: $\exists M_Y \subset M^{H_Y}(Y/Z)$

: $X \overset{\text{flops}}{\longleftrightarrow} M_Y$

(Outline) (if part is proved by Bridgeland - Maciocia)

$$\begin{array}{ccc} D^b(X) & \overset{\Phi}{\simeq} & D^b(Y) \\ {\scriptstyle ch\sqrt{td_X}}\downarrow & \circlearrowleft & \downarrow{\scriptstyle ch\sqrt{td_Y}} \\ H^*(X,\mathbb{Q}) & \underset{\phi}{\simeq} & H^*(Y,\mathbb{Q}) \end{array}$$

ϕ : given by $p^*\sqrt{td_X}\, ch(P)\, q^*\sqrt{td_Y}$

$\Phi \mapsto \phi$ is functorial

$$\begin{array}{ccccc} H^*(X,\mathbb{Q}) & \to & H^*(X_p,\mathbb{Q}) & \to & H^*(X,\mathbb{Q}) \\ \phi\downarrow{\scriptstyle s} & \circlearrowleft & \phi_p\downarrow{\scriptstyle s} & \circlearrowleft & \downarrow{\scriptstyle s}\,\phi \\ H^*(Y,\mathbb{Q}) & \to & H^*(Y_p,\mathbb{Q}) & \to & H^*(Y,\mathbb{Q}) \end{array}$$

defined $/Z$

H^0, H^2, H^4

$(0,0,1)$

$(r_p, l_p, s_p) : \phi_p^{-1}(0,0,1)$

$(r_p', l_p', s_p') : \phi_p(0,0,1)$

Lemma うまく Φ をとりかえると、次のいずれかが成立

(i) $\exists H_X \in \mathrm{Pic}(X)$ polarization s.t. $M_X \subset M^{H_X}(X/Z)$

irred. compo. 同伴 vector (r_p, l_p, s_p) の ある stable sheaf 全て

$\Rightarrow M_X$ is fine

(ii)' $\exists H_Y \in \mathrm{Pic}(Y)$: $M_Y \subset M^{H_Y}(Y/Z)$

(r_p', l_p', s_p') : M_Y fine

$D^b(X) \simeq D^b(M_X)$. ($D^b(Y) \simeq D^b(M_Y)$)

X を M_X でおきかえる (Y を M_Y でおきかえる)

$(r_p, l_p, s_p) = (0,0,1)$ のときに、$X \underset{\text{birat.}}{\longleftrightarrow} Y$ を示せばよい。

$\phi_p : (0,0,1)^\perp / (0,0,1) \xrightarrow{\sim} (0,0,1)^\perp / (0,0,1)$

$\qquad\quad \| \qquad\qquad\qquad\qquad \|$

$H^2(X_p, \mathbb{Z}) \xrightarrow{\sim} H^2(Y_p, \mathbb{Z})$

$\exists Z^0 \subset Z$ Zariski open

and iso local system

$\{\psi_p\}_{p \in Z^0} : R^2_{\pi_{X*}}\mathbb{Z}|_{Z^0} \xrightarrow{\sim} R^2_{\pi_{Y*}}\mathbb{Z}|_{Z^0}$

実は $\{\psi_p\}_{p \in Z^0} : R^2_{\pi_{X*}}\mathbb{Z}|_{Z^0} \xrightarrow{\sim} R^2_{\pi_{Y*}}\mathbb{Z}|_{Z^0}$ をうまくえらんで

$\psi_p \cdot \Phi_p$ effective Hodge isometry にできる.

$\Rightarrow \exists \psi_p \cdot \Phi_p \simeq f_p^* \longleftarrow \quad f_p : Y_p \cong X_p$

Lemma. うまく Φ をとりかえると、次のいずれかが成立

Y: nonminimal 3-fold of $\kappa = 2$

$\Rightarrow$ $Y \in FM(X)$ と、次のいずれかが同値

- $X \underset{\text{flops}}{\longleftrightarrow} Y$ $\qquad H^*(X,D) \cong H^*(Y,D)$
- $X \underset{\text{flops}}{\longleftrightarrow} Y^+ \searrow S \swarrow J^H(b) \overset{\text{flops}}{\longleftrightarrow} Y^+$

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きのうの訂正

Thm $(B, B-0)$

I ⓐ ______ ;

ⓑ $\mathrm{Hom}(F_{k_y}, F_{k_x}) = 0$ unless $z = y$ & $0 \leq i \leq \dim Y$

きのうの補足

X: K3 or abel surface/$\mathbb{C}$

Y: $M_H(v)$, $\langle v^2 \rangle = 0$

$\exists P$: univ family

$\Rightarrow P$: flat/X

- $P_{Y \times \{x\}}$: simple

(F: equiv)

- $NS(X) = \mathbb{Z}H$ のとき, $(\Rightarrow NS(Y) \cong \mathbb{Z})$

simple $\langle v^2 \rangle \leqq 0 \Longrightarrow$ stable

$P|_{Y \times \{x\}}$: stable $\Rightarrow$ $X = M_{\hat{H}}(w)$ Mukai

$\hat{H}$: $NS(Y)$の生成元 $\exists w \in H^{ev}(Y, \mathbb{Z})$

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今 $P|_{y\times X}$: μ-stable w.r.t $\exists H$

$\Rightarrow \hat{H}$: det line bundle on Y is ample

$\hat{H} := c_1 \mathcal{E}nd_{X\times Y}(P)|_{(H)} \in H^2(Y, \mathbb{Z})$

<u>Question</u> $P|_{y\times X}$ is μ-stable w.r.t H for all $y \in Y$

$\Rightarrow P|_{Y\times x}$ is μ-stable 〃 $\hat{H}$ for all $x \in X$

<u>Rem</u>
- $NS(X) \cong \mathbb{Z} \Rightarrow$ OK
- X : abel $\Rightarrow$ OK

Twisted version

$X = \bigcup_i U_i$ open covering of X

$\alpha = \{\alpha_{ijk} \in H^0(U_i \cap U_j \cap U_k, \mathcal{O}_X^{\times})\}$: $\mathbb{Z}$-cocycle

α-twisted sheaf E とは

- Coh sheaf $E_i \in Coh(U_i)$
- hom $\phi_{ji} : E_i|_{U_i \cap U_j} \to E_j|_{U_i \cap U_j}$ s.t

$\phi_{ik}\phi_{kj}\phi_{ji} = \alpha_{ijk} \cdot id$

- $Coh^{\alpha}(X)$: Cat of α-twisted coh sheaves

$D^{\alpha}(X) = \mathbb{D}(Coh^{\alpha}(X))$

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Universal family が存在しないとき

$\exists \alpha\ \exists P \in D^{\alpha}(Y\times X)$ univ obj

$\Rightarrow \Phi^{P}_{Y\to X} : D^{\alpha}(Y) \to D(X)$: equiv

<u>一般</u>

$M_H(v)$: moduli of α-twisted stable sheaves

ただし α : torsion class

$Y = M_H^{\alpha}(v)$: 2-dim

$\Rightarrow D^{\beta}(Y) \xrightarrow{\sim} D^{\alpha}(X)$: equiv

<u>Preservation of the stability</u>

Conj E : H-stable sheaf in X

$\Rightarrow \Phi^{P^{\vee}}_{X\to Y}(E \otimes \mathcal{O}(nH))$: $\hat{H}$-stable $n \gg 0$

<u>Thm</u> rank $E \le 2$ or $NS(X) \cong \mathbb{Z}$

- $P|_{Y\times x}$: μ-stable
- $P|_{y\times X}$: μ-stable

$\Rightarrow$ Conj holds

X: abel

P: Poincaré line bundle, $NS(X) = \mathbb{Z}.H \quad (H^2)=2n$) のとき考える

・$\Phi_P : D(X) \longrightarrow D(\hat{X})^{op}$

$$x \longmapsto R\mathcal{H}om_{\pi_{\hat{X}}}(P \otimes \pi_X^*(x), \mathcal{O}_{X\times\hat{X}})$$

$\hat{\Phi}_P : D(\hat{X})^{op} \longrightarrow D(X)$

$$y \longmapsto R\mathcal{H}om_{\pi_X}(P \otimes \pi_{\hat{X}}^*(y), \mathcal{O}_{X\times\hat{X}})$$

・$\hat{\Phi}_P \circ \Phi_P \cong 1, \ \Phi_P \circ \hat{\Phi}_P \cong 1$

$$\begin{tikzcd} & X\times\hat{X} \arrow[dl, "\pi_X"'] \arrow[dr, "\pi_{\hat{X}}"] & \\ X & & \hat{X} \end{tikzcd}$$

$$\begin{tikzcd} D(X) \arrow[r, "\Phi_P"] \arrow[d, "ch"'] & D(\hat{X})^{op} \arrow[d, "ch"] \\ H^{ev}(X,\mathbb{Z}) \arrow[r] & H^{ev}(\hat{X},\mathbb{Z}) \end{tikzcd} \qquad \hat{D} = D \circ D$$

$$(r, D, a) \longmapsto (a, \hat{D}, r)$$

・$\Phi_P^i(E) := \mathcal{H}^i(\Phi_P(E))$

・Spectral seq

$$E_2^{p,q} = \hat{\Phi}_P^p(\Phi_P^{-q}(E)) \Rightarrow \begin{cases} E & p+q=0 \\ 0 & 他 \end{cases}$$

<u>lemma</u> E: torsion free (stable) or purely 1-dim, $\deg E > 0$

・WIT_2 : holds for E w.r.t Φ_P

$(\Phi_P^i(E)=0 \quad i \neq 2)$

$\Rightarrow \min\{\deg G \mid \Phi_P^2(E) \twoheadrightarrow G\} > 0$

[proof]

$$0 \to F_1 \to \Phi_P^2(E) \to F_2 \to 0$$

$$\Rightarrow 0 \to \hat{\Phi}_P^0(F_2) \to 0 \to \hat{\Phi}_P^0(F_1)$$
$$\to \hat{\Phi}_P^1(F_2) \to 0 \to \hat{\Phi}_P^1(F_1)$$
$$\to \hat{\Phi}_P^2(F_2) \to E \to \hat{\Phi}_P^2(F_1) \to 0$$

・F_1: 0-dim $\Rightarrow \hat{\Phi}_P^i(F_1) = 0 \quad i=0,1$

$\hat{\Phi}_P^2(F_1)$: deg=0 の vect bundle

$\Rightarrow E \to (\deg 0) \to 0$ 　矛盾//

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$\cdot F_1 : \mu_{min}(F_1) > 0$

$F_2 : \mu_{max}(F_2) \le 0$

$\Rightarrow \widehat{\mathcal{G}_p^0}(F_1) = 0$ & $\widehat{\mathcal{G}_p^2}(F_2)$: 0-dim

$\Rightarrow \widehat{\mathcal{G}_p^1}(F_1) \xrightarrow{\sim} \widehat{\mathcal{G}_p^2}(F_2)$

$\mathcal{G}_p^0(\widehat{\mathcal{G}_p^0}(F_1)) \twoheadrightarrow \widehat{\mathcal{G}_p^2}(\mathcal{G}_p^1(F_1)) \cong F_2$ //

Cor $v(E) = (r, H, a)$, $\overline{\mathrm{WIT}_2}$ holds for E, E : stable

$\Rightarrow \widehat{\mathcal{G}_p^2}(E)$: stable

∵ $v(\widehat{\mathcal{G}_p^2}(E)) = (a, \hat{H}, r)$

Remark $a > 0 \Rightarrow \overline{\mathrm{WIT}_2}$ holds

Technical lemma

$r, d, a > 0$, $v = (r, d_H, a)$. $d_H > \frac{r}{2}\langle \mu^2 \rangle$

このとき次が成立

(1) $\forall \mu$-semi-stable sheaf F_1 with $v(F_1) = (a_1, \pm d_1\hat{H}, r_1)$

$0 < d_1 < d$

$\Rightarrow r_1 \le rd_1/d$

$d_1/a_1 \le d/a$

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(2) $\forall \mu$-semi sheaf E_1 with $v(E) = (r_1, d_1 H, a_1)$

$\Rightarrow a_1 < ad_1/d$ $\quad 0 < d_1 < d$ $\quad d_1/r_1 \le d/r$

Cor 同じ仮定の下

F : μ-semi-stable $v(F) = (d, d\hat{H}, r)$

$\Rightarrow F$: semi-stable

更に F : stable $\Rightarrow \mu$-stable

∵ $F \sim \bigoplus_i \underset{\substack{\uparrow \\ \mu_0\text{-stable}}}{F_i}$ S-equiv

$v(F_i) := (a_i, d_i\hat{H}, r_i)$

$\Rightarrow \frac{d_i}{a_i} = \frac{d}{a}$ $\forall i$ $\underset{\text{lemma}}{\Rightarrow}$ $r_i \le rd_i/d$ $\forall i$

$r = \sum_i r_i \le r \sum_i d_i/d = r$

$\therefore r_i = rd_i/d$ $\forall i \to \frac{r_i}{a_i} = \frac{r}{a}$ $\forall i$

(lemma の proof)

i) $r_i \le 0 \Rightarrow$ clearly

$r_i > 0$ とする。

$$0 \le \langle v(F_1)^2 \rangle = \langle \ \rangle \frac{d_i}{dr}\underbrace{(rd(rd_1 - r_1 d) + r_1 s)}_{*} \qquad \frac{\langle v^2 \rangle}{2} = s$$

↑ Bogomolov's ineq

If $rd_1 - rd < 0 \Rightarrow (*) \le -rd + r_1 s < 0$ //

$r_1 > r$ のとき

$rd(rd_1 - rd_1) + r_1 s < 0$

$rd(rdr - r_1 d) + r_1 s$

$< rd(rd_1 - r_1 d) + r_1 s$

$= r_1(rd(d_1 - d) + s < r_1(-rd + s) < 0$

WIT$_2$が成立する条件

Prop $w = (a, d\hat{H}, r)$ F: stable $v(F) = w$, $r \ge 0$

$dn > \frac{r}{2}\langle w^2 \rangle \Rightarrow$ WIT$_2$ holds for F

[proof] F: μ-stable としてよい。

· $Hom(P|_{x \times \hat{X}} \otimes F, \mathcal{O}_{\hat{X}}) = 0 \quad \forall x \in X$

$Ext^1(\qquad)$

$\hat{\mathcal{G}}^2(F) \leftrightarrow Ext^2(\ ;\ )$

$Ext^1(P|_{x \times \hat{X}} \otimes F, \mathcal{O}_{\hat{X}}) = 0$

on $x \in X \setminus \{x_1, \dots, x_n\}$ とおす.

loc. free

$$V_0 \xrightarrow{\varphi} V_1$$

$Ker\,\varphi = \hat{\mathcal{G}}^1$

$Coker\,\varphi = \hat{\mathcal{G}}^2$

$Ext^1(P|_{x_i \times \hat{X}} \otimes F, \mathcal{O}_{\hat{X}}) \ne 0$ at $x_1, \dots, x_n$

$\quad\quad \wr\wr$

$Ext^1(F, P^\vee|_{x_i \times X}) \ni \phi_i \ne 0$

$\exists\ 0 \to \oplus P|_{x_i \times \hat{X}} \to I \to F \to 0 \leftrightarrow (\phi_1, \dots, \phi_n)$

$0 \subset F_1 \subset F_2 \subset \cdots \subset F_s = I$ HNF

$F_i / F_{i-1} = I_i$, $v(I_i) = (a_i, d_i\hat{H}, r_i)$

Claim If $s > 1 \Rightarrow d/a > d_1/a_1 \ge \cdots \ge d_s/a_s > 0$

$\Rightarrow 0 < d_i < d \quad \forall i$

[proof] $d_\xi/a_\xi \le 0$ とする.

$$\begin{array}{ccccc} I & \longrightarrow & I_\xi & \twoheadrightarrow & I'_\xi \\ \uparrow & & & & \uparrow \\ \bigoplus P|_{x_i\times\hat{X}} & & & & \mu\text{-stable quot} \end{array}$$

$\Rightarrow \exists\, P|_{x_i\times\hat{X}} \xrightarrow{\varphi} I'_\xi$ non-zero

$\Rightarrow \deg(I'_\xi)=0$ $\therefore \varphi$: isom 矛盾

• $d/a \le d_1/a_1$ とする.

$$\begin{tikzcd} I_1' \arrow[r, hook] \arrow[rrr, bend right, "\phi"'] & I_1 \arrow[r, hook] & I \arrow[r] & F \end{tikzcd}$$

μ-stable

isom up to codim 1

$$\begin{tikzcd}
 & & 0 & 0 & \\
 & & \mathrm{Coker}\,\phi \arrow[u] \arrow[r, phantom, "="] & \mathrm{Coker}\,\phi \arrow[u] & \\
0 \arrow[r] & \bigoplus_i P|_{x_i\times\hat{X}} \arrow[r] & I \arrow[u] \arrow[r] & F \arrow[u] \arrow[r] & 0 \\
0 \arrow[r] & \bigoplus P|_{x_i\times\hat{X}} \arrow[r] \arrow[u, equal] & \tilde{I} \arrow[u] \arrow[r] \arrow[l, bend right] & \mathrm{im}\,\phi \arrow[u] \arrow[r] \arrow[l, bend right] & 0 \\
 & & 0 \arrow[u] & 0 \arrow[u] &
\end{tikzcd}$$

$\mathrm{Hom}(\tilde{I}, \bigoplus P) \to \mathrm{Ext}^1(\mathrm{Coker}\,\phi, \bigoplus P)=0$

これより矛盾が出る.

lemma(1)より, $r_i/r \le d_i/d$ $\forall i$

$\Rightarrow 1 = \sum_i r_i/r \le \sum_i d_i/d = 1$ より $r_i/r = d_i/d$ $\forall i$

$\therefore d_i/r_i = d/r$

Bogomolov

$$0 \le \sum_i \frac{\langle v(I_i)^2\rangle}{r_i} = \sum_i \frac{d_i^2 2n - 2r_i d_i}{r_i}$$

$$v(I_i) = (a_i, d_i\hat{H}, r_i) = \sum_i \left[\left(\frac{d_i}{r_i}\right) r_i 2n - 2a_i\right]$$

$$= 2n\frac{d}{r}d - 2a$$

$$= \frac{\langle v(I)^2\rangle}{r}$$

$$v(I) = w + \sum_i v(\underbrace{P_{x\times\hat{X}}}_{=(1,0,0)})$$

$$= w + (n,0,0)$$

$$= (a+n, d\hat{H}, r)$$

$\langle v(I)^2\rangle = \langle w^2\rangle - 2nr \Rightarrow n$: bdd

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Prop $v=(r,dH,a)$

E : μ-semi stable, $v(E)=v$

$dn > \frac{r}{2}\langle v^2\rangle \Rightarrow WIT_2$ holds & $\mathcal{G}_P^2(E)$: torsion free

Thm $r,a>0$, $dn > r\frac{\langle v^2\rangle}{2}$ $v=(r,dH,a)$

$\Rightarrow M_H(r,d_H,a)^{ss} \simeq \hat{M}_{\hat{H}}(a,d\hat{H},\mu)^{ss}$

Rem $r=1$ $d\geq 2$のとき

IT_0 holds for $\Phi^{P}_{X\to \hat{X}}$

$\Longleftrightarrow 2(d-1)n > s-dn-a$

Terakawa

$r=1,2$, $d\equiv 1 \bmod r$

$\mathcal{G}_P^2(E)$: stable $\Longleftrightarrow nd > s = d^2n-ra$

特に $r=1$のとき $\frac{s}{n}\geq d > \frac{s}{2n}+1$

のときはIT_0が成立するが、stableでない。

Rem $d=1$のとき、"$dn > \frac{r}{2}\langle v^2\rangle$"は不要

Thmのproofの半分

$F\in M_{\hat{H}}(a,d\hat{H},r) \Rightarrow \hat{\mathcal{G}}_P^2(F)$: semi-stableをいう

・WIT_2 : OK ・$E=\hat{\mathcal{G}}_P^2(F)$: not semi-stableとする。

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$0\subset F_1\subset F_2\subset\cdots\subset F_s=E$ HNF

$E_i=F_i/F_{i-1}$ $v(E_i)=(r_i,d_H,a_i)$

$\underbrace{d_1/r_1 \geq d_2/r_2 \geq \cdots \geq d_t/r_t}_{\geq d/r} \geq \underbrace{\cdots \geq d_s/r_s}_{<d/r} \geq 0$

Clame $\langle v(E_i)^2\rangle \leq \langle v(E)^2\rangle$ $\forall i$

$\because$ $\sum_{i=1}^{s} \dim \mathrm{Ext}^1(E_i,E_i) \leq \dim \mathrm{Ext}^1(E,E) = \langle w^2\rangle+2$

$\vee|$

$\dim \mathrm{Ext}^1(E_i,E_i)$

$\|$

$\langle v(E_i)^2\rangle + 2\dim \mathrm{Hom}(E_i,E_i)$

$\vee|$

$\langle v(E_i)^2\rangle + 2$

$0<i\leq t$についてWIT_2 holds for E_i

$\Rightarrow F_t$についてもWIT_2 holds

$0\to \mathcal{G}_P^0(E/F_t)\to 0\to \mathcal{G}_P(F_t)=0$

$\to \mathcal{G}_P^1(E/F_t)\to 0\to \mathcal{G}_P^1(F_t)=0$

$\to \mathcal{G}_P^2(E/F_t)\to F\to \mathcal{G}_P^2(F_t)\to 0$

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$d_i/r_i < d/r \quad i > t$

$\Rightarrow_{\text{lemma}} a_i < a d_i / d \Rightarrow \left(\sum_{i>t} a_i\right) < \frac{a}{d}\left(\sum_{i>t} d_i\right)$

$\Rightarrow \dfrac{\sum_{i>t} d_i}{\sum_{i>t} a_i} > \dfrac{d}{a}$

Fのstabilityに反する。

∴ s=t つまり $d_i/r_i \geq d/r \ \forall i \Rightarrow d_i/r_i = d/r \ \forall i$

$\Rightarrow a_1/r_1 > a/r$ (∵HNF)

$\Rightarrow r_1/a_1 < r/a \Rightarrow F \to \mathcal{G}_P^2(\underset{F_1}{\overset{\shortparallel}{E_1}}) \to 0$ 矛盾//

<u>反対方向</u>

$E \in M_H(r, d_H, a)^{ss}$

$\mathcal{G}_P^2(E)$: not semi-stable とする。

$0 \to G_1 \to \mathcal{G}_P^2(E) \to G_2 \to 0$

・G_2: stable $v(G_2) = (a_2, d_2H, r_2)$

$0 < d_2/a_2 < d/a \cap d_2/a_2 = d/a \cap r_2/a_2 < r/a$

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・$\mu_{min}(G_1) > 0$

・$\hat{\mathcal{G}}_P^0(G_1) = 0$

$\hat{\mathcal{G}}_P^1(G_1)$: loc free $\quad \ker\varphi \quad (V_0 \xrightarrow{\varphi} V_1)$

$\hat{\mathcal{G}}_P^1(G_2) = 0 \Rightarrow$ WIT$_2$ holds

$0 \to \hat{\mathcal{G}}_P^1(G_1) \to \hat{\mathcal{G}}_P^2(G_2) \to E \to \hat{\mathcal{G}}_P(G_1) \to 0$

$v(\hat{\mathcal{G}}_P^2(G_2)) = (r_2, d_2H, a_2) \Rightarrow r_2 \geq 0$

$d_2 \leq d a_2/a < d$

$d_2/r_2 \geq d/r > \langle v(E)^2 \rangle \geq \langle v(G_2)^2 \rangle$

Theoremの前半より、$\hat{\mathcal{G}}_P^2(G_2)$: stable

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14:00- 宮地先生 Derived categories in Representation Theory

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16:00- 佐藤さん (藤野さん) F-equivalence after Fujino

(F: flop)

We will work over $\mathbb{C}$ throughout this talk.

<u>Def 1</u>. X sm proj var $/\mathbb{C}$

$D(X) = D^b(\mathrm{Coh}(X))$

<u>Def 2</u> X, Y sm proj var

$X \underset{D}{\sim} Y \underset{\text{def}}{\iff} D(X)$ is equivalent to $D(Y)$ as triangulated category.

D-equivalence $(Y \in FM(X))$

<u>Def 3</u> X, Y sm proj var

$X \underset{K}{\sim} Y \underset{\text{def}}{\iff} \exists$
$$\begin{array}{ccc} & Z & \\ {}^{f}\swarrow & & \searrow^{g} \\ X & & Y \end{array}$$
Z : sm proj, f, g birat morph

K-equivalence

s.t $f^* K_X = g^* K_Y$

From now on, we mainly treat toric var.

<u>Thm 4</u> X, Y sm proj ~~var~~ toric var

$X \underset{D}{\sim} Y \Rightarrow X \underset{F}{\sim} Y$

<u>Thm 5</u> (Kawamata) X, Y sm proj

$X \underset{D}{\sim} Y$, $-K_X$ big $\Rightarrow X \underset{K}{\sim} Y$

<u>Lem 6</u> X sm proj toric $\Rightarrow -K_X$ big

$\because$) $-K_X = \sum_{i \in I} D_i$ in $\mathrm{Pic}(X)$ where D_i toric prime div

$\{D_i\}_{i \in I}$ generates $\mathrm{Pic}(X)$

$\Rightarrow {}^{\exists} m > 0$ s.t. $-mK_X \geq {}^{\exists}$(ample) $\Rightarrow -K_X$ big //

· Thm 5 + Lem 6 $\Rightarrow$ Thm 4

Rmk 7 If $\dim X \leq 3 \Rightarrow$ D-equiv $\Leftrightarrow$ K-equiv.
X: toric

Conj 8 $D \Leftrightarrow K$ for sm proj toric

$K \Rightarrow D$?

§ F-equivalence

Def. X, Y sm proj toric

$X \underset{F}{\sim} Y$ F-equivalence $\Leftrightarrow$ $\exists$ a sequence of flops

$X = X_0 \dashrightarrow X_1 \dashrightarrow \cdots \dashrightarrow X_{p-1} \dashrightarrow X_p \cong Y$

s.t. X_i : smooth.

以下 X, Y toric

Rmk 10

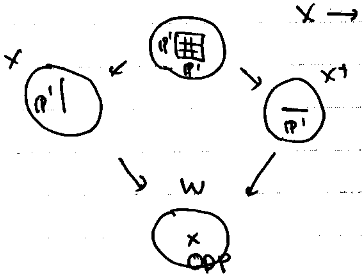
$$\begin{tikzcd} X \arrow[rr, dashed] \arrow[dr] & & X^+ \arrow[dl] \\ & W & \end{tikzcd}$$

toric flop

X : smooth 3-fold

$\Rightarrow$ W has only one ODP.

$X \to W$, $X^+ \to W$ small resol



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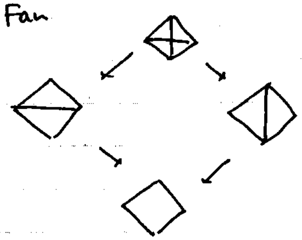


(Atiyah's flop)

特に X^+ smooth

Rmk 11

$$\begin{tikzcd} X \arrow[rr, dashed] \arrow[dr, "\varphi"] & & X^+ \arrow[dl, "\varphi'"'] \\ A \arrow[u, hook] \arrow[dr] & W & A^+ \arrow[u, hook] \arrow[dl] \\ & B \arrow[u, hook] & \end{tikzcd}$$

toric flop X : sm 4-fold

$$\begin{cases} \mathrm{Exc}(\varphi) = A \\ \mathrm{Exc}(\varphi^+) = A^+ \\ \varphi(A) = B \end{cases}$$

$\exists$ 2 types of flops

① X^+ smooth.

$B = \mathbb{P}^1$. $\left.\begin{array}{l} A \to B \\ A^+ \to B \end{array}\right\}$ $\mathbb{P}^1$-bundle   family of Atiyah's flop.

② X^+ singular

$A = \mathbb{P}^2$, $A^+ = \mathbb{P}^1$

$B = pt$

Prop 12

$$\begin{tikzcd} X \arrow[rr, dashed] \arrow[dr] & & X^+ \arrow[dl] \\ A \arrow[u, hook] \arrow[dr] & W & A^+ \arrow[u, hook] \arrow[dl] \\ & B \arrow[u, hook] & \end{tikzcd}$$

toric flop X^+ smooth
$\dim X = n$

$\Rightarrow$ $2 \leq \alpha \leq \beta \leq n-1$

$\operatorname{codim} A = \operatorname{codim} A^+ = \alpha$

$\dim B = \beta - \alpha$ $\alpha = n+1-\beta$

$\left.\begin{array}{l} A \to B \\ A^+ \to B \end{array}\right\}$ $\mathbb{P}^{n-\beta}$-bundle s.t.

$N_{A/X|F} = \mathcal{O}_{\mathbb{P}^{n-\beta}}(-1)^{\oplus \alpha}$

fiber

Therefore this is a family of ~~...~~ higher dimensional generalizations of Atiyah's flop

"Standard flop"

Thm (Orlov) $X \dashrightarrow X^+$ toric flop X, X^+ smooth

$$\Rightarrow X \underset{D}{\sim} X^+$$

∵) prop 12 Orlov: Russ. Math Surveys 58 (2003), 511~ p.544

Cor 14 X, Y sm proj toric

$$X \underset{F}{\sim} Y \Rightarrow X \underset{D}{\sim} Y$$

まとめ **Thm 15** X, Y sm proj toric

$$X \underset{F}{\sim} Y \Rightarrow X \underset{D}{\sim} Y \Rightarrow X \underset{K}{\sim} Y$$

Therefore **Conj 16** X, Y as above

$$X \underset{K}{\sim} Y \Rightarrow X \underset{F}{\sim} Y$$

Rmk 17 $\dim X = 3$ $X \underset{K}{\sim} Y \Rightarrow X \underset{F}{\sim} Y$

Lem 18 X, Y sm proj toric

If $X \underset{K}{\sim} Y$ then $\exists$ a sequence of ~~flips~~ flops and inverse flips

$$X = X_0 \dashrightarrow X_1 \dashrightarrow \cdots \dashrightarrow X_\ell = Y$$

∵) X, Y: isom in codim 1 ($\Leftarrow$ X, Y terminal) well-known

D very ample on Y

D' strict trans on X. D': nef $\Rightarrow X \cong Y$

D' not nef

$\Rightarrow$ D'-negative extr ray $R \subset NE(X)$

$X \dashrightarrow X'$ flip flop. inverse flip

$\varphi_R \searrow W \swarrow$

contraction

repeat

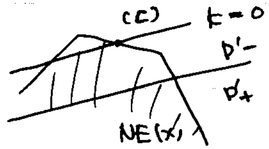
Conj 19 X, Y sm proj toric

$X \underset{K}{\sim} Y \overset{?}{\Rightarrow} \exists$ a sequence of flops

$$X := X_0 \dashrightarrow \cdots \dashrightarrow X_m := Y \quad ??$$

Rmk 20 We can check that $\exists$ curve C on X s.t. $K_X \cdot C = 0$ & $D' \cdot C < 0$

However ? $\exists R$ extr ray s.t. $K_X \cdot R = 0$, $D' \cdot R < 0$



Conj ~~20~~ 21 X, Y sm proj toric

$\exists X = X_0 \dashrightarrow \cdots \dashrightarrow X_m$ s.t. a seq of flops

$$\Rightarrow X \underset{F}{\sim} Y \quad ??$$

Claim 22

Conj 19, 21 imply

$K \Rightarrow F \quad \Rightarrow \quad F \Leftrightarrow K$, $\Leftrightarrow D$

§ **Comments**

Prop 23 $X \underset{K}{\sim} Y$ as above

$\Rightarrow d_k(X) = d_k(Y) \quad \forall k$

where $d_k(X) = \#$ of k-dim cones in Δ_X

∵) $X \underset{K}{\sim} Y \Rightarrow h^{p,q}(X) = h^{p,q}(Y) \quad \forall p,q$

(e.g. Motivic integration)

$\Rightarrow d_k(X) = d_k(Y)$

↑ コバンスキー

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Rmk 24 $\dim X = 4$ X smooth

$$\begin{tikzcd} X \arrow[rr, dashed] \arrow[dr] & & X^+ \arrow[dl] \\ & W & \end{tikzcd}$$

toric flop

X^+ singular $\Rightarrow$ $d_4(X) > d_4(X^+)$ (local)

$$\underset{3}{\overset{\shortparallel}{}} \qquad \underset{2}{\overset{\shortparallel}{}}$$

$X \dashrightarrow X_1 \dashrightarrow \cdots \dashrightarrow Y$

$\underset{\text{smooth}}{X} \qquad \underset{\text{smooth}}{Y}$

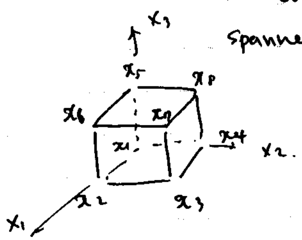
$d_k(X) = d_k(Y)$

途中は smooth の方が自然なのでは？

§ Example

Example 25 $\mathbb{R}^3 \times \{1\} \subset \mathbb{R}^3 \times \mathbb{R} = \mathbb{R}^4$

We consider a cone in $\mathbb{R}^4$ spanned by  $\times \{1\}$



$x_1 = (0,0,0,1)$ $x_2 = (1,0,0,1)$

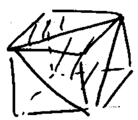
$x_3 = (1,1,0,1)$ $x_4 = (0,1,0,1)$

$x_5 = (0,0,1,1)$ $x_6 = (1,0,1,1)$

$x_7 = (1,1,1,1)$ $x_8 = (0,1,1,1)$

Remove 2 cones $\langle x_5, x_6, x_8, x_1 \rangle$

$\langle x_6, x_7, x_8, x_3 \rangle$



$\Delta_W \longleftrightarrow W$ affine Gorenstein terminal 4-fold

Divide Δ_W into

$$\left.\begin{array}{l} \langle x_1, x_2, x_3, x_6 \rangle \\ \langle x_1, x_3, x_4, x_8 \rangle \\ \langle x_1, x_3, x_6, x_8 \rangle \end{array}\right\} \Delta_Y$$

$$\Delta_Y \updownarrow Y$$

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$Y \to W$ small projective $\rho(Y/W) = 2$

Y : Gor terminal (singular)

$NE(Y/W)$ has 2 rays. R_1, R_2

$\updownarrow$

$\therefore$ 2 walls $\langle x_1, x_3, x_6 \rangle$

$\langle x_1, x_3, x_8 \rangle$

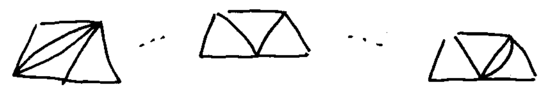
R_1 : $2x_2 + x_8 = x_1 + x_3 + x_6$

R_2 : $2x_4 + x_6 = x_1 + x_3 + x_8$

Reid の関係式

$\varphi_{R_1}, \varphi_{R_2}$: flopping contraction

$$\begin{tikzcd} \underset{\text{smooth}}{X'} \arrow[dr] & & \underset{\text{singular}}{Y} \arrow[ll, dashed] \arrow[rr, dashed] \arrow[dl, "\varphi_{R_2}"'] \arrow[dr, "\varphi_{R_1}"] & & \underset{\text{smooth}}{X} \arrow[dl] \\ & Z' & & Z & \end{tikzcd}$$



However!! Apply 2-ray game

$Y \dashrightarrow X \overset{\text{flop}}{\dashrightarrow} \underset{\text{sm}}{X_1} \overset{\text{flop}}{\dashrightarrow} \underset{\text{sm}}{X_2} \overset{\text{flop}}{\dashrightarrow} \underset{\text{sm}}{X_3} = X'$

$\therefore X \underset{F}{\sim} X'$

Conclusion 26 It seems very hard to construct a counter-example to conj 21 !

Rmk 27 2-ray game above is very complicated!!

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Conclusion 28 In the category of proj sm toric var.

dim 3

$$\begin{tikzcd} K \arrow[r, Leftrightarrow] \arrow[d, Leftrightarrow] & F \arrow[dl, Leftrightarrow] \\ D & \end{tikzcd}$$

dim 4

$$\begin{tikzcd} K \arrow[r, Rightarrow, "?"] & F \arrow[dl, Rightarrow] \\ D \arrow[u, Rightarrow] & \end{tikzcd}$$

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Kaledin

k : field

Def 1

A : Poisson alg. / k

$\Leftrightarrow$ A : commutative k-alg.

$\{,\} : A \otimes_k A \longrightarrow A$: skew - k-linear

such that

$$*) \begin{cases} \{a, bc\} = \{a,b\}c + \{a,c\}b \\ 0 = \{a,\{b,c\}\} + \{b,\{c,a\}\} + \{c,\{a,b\}\} \end{cases}$$

Jacobi identity

$I \subset A$: Poisson ideal $:\Leftrightarrow$
ideal

$\{I, A\} \subseteq I$

Re.

~~A/I~~ I : Poisson $\Rightarrow A/I$: Poisson alg. / k

Def 1'

X/k : Poisson scheme $:\Leftrightarrow$

$(X, \{,\})$

~~Poisson~~ k-scheme

$\{,\} : \mathcal{O}_X \otimes_k \mathcal{O}_X \longrightarrow \mathcal{O}_X$

Def 2 (Hamilton vector field)

$f \in \mathcal{O}_X$

$$\begin{tikzcd} \{f,\cdot\} : \mathcal{O}_X \arrow[r] \arrow[dr, "d"'] & \mathcal{O}_X \\ & \Omega_{X/k} \arrow[u, "H_f"'] \end{tikzcd}$$

k-derivation

$H_f \in \mathrm{Hom}(\Omega_{X/k}, \mathcal{O}_X) = T_{X/k}$

f の Hamilton vect. field

$(X, \{,\})$

$\{f,g\} = \Theta(df \wedge dg)$ $\quad \exists \Theta : \Lambda^2 \Omega_X \longrightarrow \mathcal{O}_X$

Poisson bivector

~~Ex~~

Def

X : smooth alg. var/$\mathbb{C}$

X が symplectic ~~variety~~ manifold :$\Longleftrightarrow$

$\exists \Omega$: regular 2-form s.t.

(1) $d\Omega = 0$

(2) 至るところ非退化

Th

smooth alg. var./$\mathbb{C}$ に対して次は同値

(1) X は symplectic mfd.

(2) X は非退化な Poisson構造$\{,\}$をもつ.

$\hookrightarrow \Theta$ Poisson bivector

proof

(2) $\Rightarrow$ (1)

$$\Omega_X^1 \rightrightarrows T_X$$

$$\Lambda^2\Omega_X^1 \leftleftarrows \Lambda^2 T_X$$

$$\Omega \longleftrightarrow \Theta$$

$d\Omega$ を check する (略)

(1) $\Rightarrow$ (2) non-degenerate

Ω 2-form : given $\qquad f \in \mathcal{O}_X$

$$\Omega_X^1 \overset{\Omega}{\rightleftarrows} T_X \qquad T_X \times T_X \longrightarrow \mathcal{O}_X$$

$$df \longleftrightarrow H_f \qquad \Omega_X \times \Omega_X \xrightarrow{\Theta} \mathcal{O}_X$$

定義

$\{f,g\} := \Omega(H_f, H_g)$ $\qquad \boxed{\Omega(\cdot, H_f) = df(\cdot)}$

Θ を bivector にするような Poisson構造

$\{\ ,\ \}$ が Poisson構造であること:

Lemma1

$L_{H_f}\Omega$

Lie微分

$X[\varepsilon] := X \times_{\mathbb{C}} \operatorname{Spec}\mathbb{C}[\varepsilon]$

$\hat{H_f} : X[\varepsilon] \longrightarrow X[\varepsilon]$

$\Omega \in \Omega^2_{X[\varepsilon]/\mathbb{C}[\varepsilon]}$

$\hat{H_f}^*(\Omega) = \Omega \iff L_{H_f}\Omega = 0$

$$L_{H_f}\Omega = H_f \lrcorner d\Omega + d(H_f \lrcorner \Omega)$$

$$= d(H_f \lrcorner \Omega) = d(-df) = 0$$

Lemma2

$$[H_f, H_g] = H_{\{f,g\}}$$

proof

H, H_1, H_2 : X 上の vector fields

$$(L_H\Omega)(H_1,H_2) + d(H \lrcorner \Omega)(H_1,H_2)$$

$$= H(\Omega(H_1,H_2)) - \Omega([H,H_1],H_2) + \Omega([H,H_2],H_1)$$

$f, g \in \mathcal{O}_X$ $\quad H := H_f \quad H_1 = H_g \quad H_2 = \eta$ をとる

- $H(\Omega(H_1,H_2)) = H_f(\Omega(H_g,\eta)) = H_f(-\Omega(\eta,H_g))$
  $= H_f(-\eta(g)) = -H_f\eta(g)$
- $\Omega([H,H_1],H_2) = \Omega([H_f,H_g],\eta)$
- $\Omega([H,H_2],H_1) = [H_f,\eta]g = H_f\eta(g) - \eta H_f(g)$

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Lemma 1 より

$\Omega(\eta, [H_f, H_g]) = \eta H_f(g) = \eta(\{f,g\}) = \Omega(\eta, H_{\{f,g\}})$

$\left(\{f,g\} := \Omega(H_f, H_g) = dg(H_f) = H_f(g) \right)$ $\quad \Omega(\cdot, H_f) = df$

$\Rightarrow$

η は任意なので $[H_f, H_g] = H_{\{f,g\}}$ $\square$

Jacobi identity:

- $[H_f, H_g]h = H_{\{f,g\}}h = \{\{f,g\},h\}$

  $\quad\ \ \shortparallel$

  $[H_f, H_g]h = H_f H_g(h) - H_g H_f(h)$

  $\qquad = \{f,\{g,h\}\} - \{g,\{f,h\}\}$

- $\{f, gh\} = h\{f,g\} + g\{f,h\}$

Def

X : Poisson scheme $\quad Y \subset X$ closed subscheme

Y : Poisson subscheme $:\Leftrightarrow$

$\quad Y$ の defining ideal が Poisson ideal

Def

X : integral Poisson scheme

$\quad$ X : generically non-degenerate

$\quad \Leftrightarrow$ Poisson bivector Θ が generic point で非退化

Ex) X : holonomic $:\Leftrightarrow$

$\quad {}^{\forall}Y \subset X$ integral Poisson subscheme

$\quad$ は gen. non-degenerate

Def

X = normal var. $/\mathbb{C}$

X : symplectic variety $:\Leftrightarrow$

1) ${}^{\exists}\Omega$: non-degenerate, $d\Omega = 0$ 2-form on X_{reg}

2) $\pi : \tilde{X} \to X$ resolution に対して

$$\begin{array}{ccc} \tilde{X} & \xrightarrow{\pi} & X \\ \cup\ \text{open} & & \cup \\ \pi^{-1}(X_{reg}) & \xrightarrow{\sim} & X_{reg} \end{array}$$

Ω が $\tilde{X}$ 上に regular に延長できる.

Prop.

X : symplectic variety $\Rightarrow$

$\quad X$: poisson ~~schem~~ scheme non-degenerate on X_{reg}

proof

X_{reg} 上 $\{,\}$ が入る $\leftarrow$ Theorem

~~normal~~

$$\begin{array}{ccc} \mathcal{O}_{X_{reg}} \otimes_{\mathbb{C}} \mathcal{O}_{X_{reg}} & \xrightarrow{\{,\}} & \mathcal{O}_{X_{reg}} \\ & \updownarrow \text{normal} & \\ \mathcal{O}_X \otimes \mathcal{O}_X & \xrightarrow[\{,\}]{} & \mathcal{O}_X \end{array}$$

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Th.

X: alg. var. with rational sing./$\mathbb{C}$ とする

次は同値

(1) X は symplectic variety

(2) X: Poisson scheme X^{reg} 上非退化

~~proo~~

Prop.

Poisson scheme の singular locus 自身及び
その既約成分は、Poisson subscheme になる。

$\uparrow$ reduced なもの.

proof

I: Sing(X) の defining ideal

$\{\mathcal{O}_X, I\} \subseteq I$ をいいたい

$f \in \mathcal{O}_X$ に対し $\{f, I\}$ をいえばよい

$\{f, I\} = H_f(I) \subseteq I$

Prop.

X: Poisson Θ: bi-vector

$L_{H_f}\Theta = 0$

目標:

Theorem

X: symplectic variety/$\mathbb{C}$

$\Sigma_0 := \mathrm{Sing}(X)$ $\Sigma_1 := \mathrm{Sing}(\Sigma_0), \cdots, \Sigma_i := \mathrm{Sing}(\Sigma_{i-1})$

このとき、Σ_i の各既約成分の正規化は symplectic variety

Th 1

Symplectic variety X は Poisson scheme として holonomic である.

proof

$Y \subset X$ integral Poisson subscheme

$\{,\}: \mathcal{O}_Y \otimes \mathcal{O}_Y \to \mathcal{O}_Y$ Poisson bracket

$\Theta: \Lambda^2\Omega_Y \to \mathcal{O}_Y$ Poisson bivector

$y \in Y$, $\Theta(y)$: 非退化を示す.

$$\begin{array}{ccc} \pi^{-1}(Y) & \subset & \tilde{X} \\ \downarrow & & \downarrow \pi \\ Y & \subset & X \end{array}$$

π: canonical resolution
(X上の vector field はすべて $\tilde{X}$ 上に lift される)

Key lemma

$\exists U \underset{open}{\subset} Y$, $\exists \omega_U \in H^0(U, \Omega_U^2)$ s.t.

$\Omega|_{\pi^{-1}(U)_{reg}} = \sigma^*\omega_U$

$\forall y \in U$, $\pi^{-1}(y)$ は generically smooth と仮定してよい.

$$\begin{array}{ccccc} \pi^{-1}(U)_{reg} & \subset \ \pi^{-1}(Y) & \subset & \tilde{X} \\ \sigma \downarrow & \downarrow & & \downarrow \\ U & \underset{open}{\subset} \ Y & \subset & X \end{array}$$

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(Θ) $(y) \in \Lambda^2 T_Y(y)$ が退化したとする

$\Leftrightarrow$ $^\exists f \in \mathcal{O}_Y$ s.t.

(1) $(df)(y) \neq 0 \in \Omega^1_Y(y)$

(1') $\Theta(y)(df(y), \cdot) = 0$

$\overset{\parallel}{H_f(y)}$

X_{reg} 上 $df = -H_f \lrcorner \Omega$

$\pi^*(X_{reg})$ 上 $\pi^*(df) = -\widehat{H_f} \lrcorner \Omega$

両方とも $\tilde{X}$ 上で定義されている.

これを $\pi^{-1}(U)_{reg}$ に制限する

$\sigma^*(df) = \widehat{H_f}|_{\pi^{-1}(U)_{reg}} \lrcorner \Omega|_{\pi^{-1}(U)_{reg}}$

$= \widehat{H_f}|_{\pi^{-1}(U)_{reg}} \lrcorner \sigma^*\omega_U$

$\pi^{-1}(U)_{reg} \ni y'$ を $\pi(y') = y$ σ は y' で smooth map になっているようにとる.

$\sigma^*(df)(y) = \sigma^*(H_f(y) \lrcorner \omega_U(y)) = 0$

$\Downarrow$ σ = smooth at y'

$(df)(y) = 0$ 矛盾 □

Th2 X = symplectic variety

$Y \subset X$ Poisson integral subscheme

$\Rightarrow$ Y の normalization は symplectic variety

proof

X : holonomic Poisson subscheme

$\Downarrow$

Y : holonomic Poisson

$\Downarrow$

$Y \supset \mathrm{Sing}(Y)$ $\mathrm{codim}(\mathrm{Sing}(Y) \subset Y) \geq 2$

$\{,\}$ $\{,\}$

~~Lemma~~

Claim:

Y_{reg} の Poisson bivector は至る所非退化

Lemma

Z : smooth holonomic Poisson scheme / $\mathbb{C}$

$\Rightarrow$

$D = (\Theta$ の退化している locus$) \subset Z$

divisor

$L_{H_f}\Theta = 0$ $\rightsquigarrow$ D は Z の Poisson subscheme

$\mathrm{codim}(D \subset Z)$: 偶数 $\rightarrow$ 矛盾

$\Omega^1_{Y_{reg}} \simeq T_{Y_{reg}}$

$\Lambda^2 \Omega^1_{Y_{reg}} \simeq \Lambda^2 T_{Y_{reg}}$

$\underset{\omega}{\cup}$ $\underset{\Theta}{\cup}$

非退化 2-form bivector

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$\tilde{Y}$

$\big\downarrow$ resolution

$Y \supset Y_{reg}$ ω

ω が $\tilde{Y}$ 上 regular にのびることを示せば良い.

<u>Lemma</u>

$\exists$ generically surjective finite map

$$\begin{array}{ccc} f: W & \longrightarrow & Y \\ \cup & & \cup \\ f^{-1}(Y_{reg}) & & Y_{reg} \\ f^*\omega & & \omega \end{array}$$

(W smooth)

に対して $f^*\omega$ が W 上 regular に延長されれば

勝手な resolution $\tilde{Y} \to Y$ に対しても同じことがいえる.

<u>proof</u>

$K(W)/K(Y)$ Galois G

W: G-action を持つ.

$$\begin{array}{ccccc} & & W & \longrightarrow & W/G & \longleftarrow \widetilde{W/G} \\ & & \downarrow & \swarrow & & \\ & & Y & & & \end{array}$$

$f^*\omega$ G-invariant

ω Y

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$W^0 \subset \eta^{-1}(U) \subset \eta^{-1}(Y) \subset \hat{X}$

$$\begin{array}{ccccccc} W^0 & & & & \eta^{-1}(Y) & & \hat{X} \\ & \searrow^{\text{étale}} & & & \downarrow & & \downarrow \eta \\ & & U & \underset{\text{open}}{\subset} & Y & & X \end{array}$$

$\widetilde{W_0} \xleftarrow[\text{resolution}]{} W$:

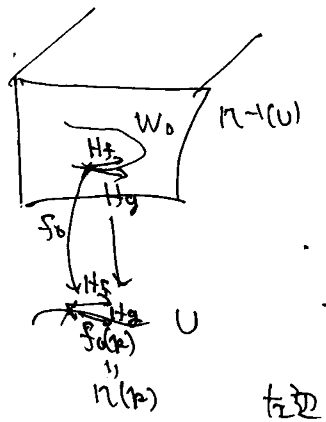
~~$W^0 \subset W \xrightarrow{\tau} \hat{X}$, étale f_0, f~~

$$\begin{array}{ccccc} W^0 & \subset & W & \xrightarrow{\tau} & \hat{X} \\ \downarrow f_0 & & \downarrow f & & \downarrow * \\ U & \subset & Y & \hookrightarrow & X \end{array}$$

ω

Claim:

$f_0^*\omega = \Omega|_{W^0}$



$(TY)(f_0(p))$ は H_g の形の元で生成される.

・ $f_0^*\omega(H_g', H_h') = \Omega|_{W^0}(H_g', H_h')$

を示せばよい.

左辺 $= f_0^*\{g,h\}$

右辺:

$\Omega|_{W_0} = f_0^*\omega'$ (key lemma)

$\uparrow$ $\exists$ 2-form

$\Omega|_{W_0}(H_g', H_h') = \Omega|_{W_0}(\widehat{H_g}, \widehat{H_h})$

右辺 $= \Omega|_{W_0}(\hat{H_g}, \hat{H_h}) = \Omega(\hat{H_g}, \hat{H_h})|_{W_0}$
$= \eta^*\{g,h\}|_{W_0} = f_0^*\{g,h\}$

<u>Ex</u>

$x \in \mathfrak{sl}(n,\mathbb{C}) = \{A : n\times n ;\ \mathrm{tr} A = 0\} \circlearrowleft SL(n,\mathbb{C}) \ni B$
nilpotent

x の Jordan type $\begin{pmatrix} \overbrace{\begin{matrix} 0 & 1 \\ 0 & 0 \end{matrix} \quad \begin{matrix} 0 & 1 \\ 0 & 0 \end{matrix} \cdots \begin{matrix} 0 & 1 \\ 0 & 0 \end{matrix}}^{k} & \\ & \underbrace{0 \ \cdots \ 0}_{n-2k} \end{pmatrix}$

$x \longmapsto BxB^{-1}$
Adjoint action

$\mathcal{O} := SL(n)\cdot x$ orbit

$X := \overline{\mathcal{O}} \subset \mathfrak{sl}(n,\mathbb{C})$

$$X = \overline{\mathcal{O}}_{[2^k, 1^{n-2k}]} \supset \underset{\Sigma_0}{\overset{\shortparallel}{\mathcal{O}}}_{[2^{k-1}, 1^{n-2k+2}]} \supset \cdots \supset \overline{\mathcal{O}}_{[2, 1^{n-2}]} \supset \{0\}$$

$\uparrow$ symplectic resolution: $T^*G(k,n)$, $\quad \uparrow T^*G(k-1,n)$, $\quad \uparrow T^*G(1,n)$

14:00 – 上原北斗 Autoequivalence of derived categories
in neighbourhoods of A_n-configurations
(joint work with A. Ishii)

X sm prj var $/\mathbb{C}$
$D(X) := D^b(\mathrm{Coh}(X))$
$\mathrm{Auteq}\, D(X) := \{\Phi : D(X) \xrightarrow{\sim} D(X)\}/\cong$

$\pm K_X$ ample $\overset{\text{Bondal–Orlov}}{\Longrightarrow}$ $\mathrm{Auteq}\, D(X) = (\mathrm{Pic}\, X \rtimes \mathrm{Aut}\, X) \times \underset{\text{shift}}{\underset{\uparrow}{\mathbb{Z}}}$

X : general $\quad \exists$

<u>Twist functor</u>

- $E \in D(X)$ spherical $\overset{\text{def}}{\Longleftrightarrow}$ • $E \otimes \omega_X \simeq E$
  • $\mathrm{Hom}^i_{D(X)}(E, E) = \begin{cases} \mathbb{C} & (i = 0, \dim X) \\ 0 & (\text{otherwise}) \end{cases}$

- $E \in D(X)$ spherical given
  Twist functor T_E is defined by Fourier–Mukai transform with Kernel $\mathrm{Cone}(\pi_1^* E^\vee \otimes \pi_2^* E \xrightarrow{ev} \mathcal{O}_\Delta)$

$$\begin{tikzcd}[column sep=small] & \Delta \arrow[d, phantom, "\cap"] & \\ & X \times X \arrow[dl, "\pi_1"'] \arrow[dr, "\pi_2"] & \\ X & & X \end{tikzcd}$$

- $$\begin{tikzcd} \mathbb{R}\mathrm{Hom}(E, \alpha) \otimes E \arrow[rr] & & \alpha \arrow[dl] \\ & T_E(\alpha) \arrow[ul, "{[1]}"] & \end{tikzcd}$$
  triangle
  for $\forall \alpha \in D(X)$)

- $T_E \in \mathrm{Auteq}\, D(X)$ (Seidel–Thomas)

<u>Example</u> X : K3 $\quad \mathcal{L} \in \mathrm{Pic}\, X \Rightarrow \mathcal{L}$: spherical
$Z = C_1 \cup \cdots \cup C_n$ chain of (-2) curves on a surface X
$\mathcal{O}_Z(a_1, \cdots, a_n) \in D(X)$ spherical

Our problem $Z = C_1 \cup \cdots \cup C_n \subset X$ (surface) ADE configuration

$$D_Z(X) = D^b(\mathrm{coh}_Z(X)) = \{E \in D(X) \mid \mathrm{supp}\, H^i(E) \subset Z\}$$

What is $\mathrm{Auteq}\, D_Z(X)$? generated by twists and trivial ones?

Rem $D_Z(X) \simeq D^G_{\{0\}}(\mathbb{C}^2)$

- $\mathrm{Auteq}^{FM} D_Z(X) \subseteq \mathrm{Auteq}\, D_Z(X)$, where $\mathrm{Auteq}^{FM} = \{\text{FM transforms}\}$

Rem X sm proj $\overset{\text{Orlov}}{\Longrightarrow}$ $\mathrm{Auteq}^{FM} D(X) = \mathrm{Auteq}\, D(X)$

Thm $Y = \mathrm{Spec}\, \mathbb{C}[[x,y,z]]/(x^2+y^2+z^{n+1})$. A_n-singularity

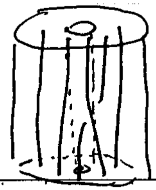
$f : X \longrightarrow Y$ min resol

$Z = f^{-1}(0) = C_1 \cup \cdots \cup C_n$

Then $\mathrm{Auteq}^{FM} D_Z(X) = \langle B, \mathrm{Pic}\, X \rangle \rtimes \mathrm{Aut}\, Y \times \mathbb{Z}$

where $B := \langle T_{\mathcal{O}_{C_1}(-1)}, \cdots, T_{\mathcal{O}_{C_n}(-1)}, Tw_Z \rangle$

Rem
- $B \cap \mathrm{Pic}\, X = \langle \mathcal{O}_X(C_1), \cdots, \mathcal{O}_X(C_n) \rangle \subset \mathrm{Pic}\, X$
- $\langle B, \mathrm{Pic}\, X \rangle = B \rtimes \mathbb{Z}/(n+1)\mathbb{Z}$ (index $n+1$)
- $B \underset{\text{easy}}{=} \langle T_{\mathcal{O}_{C_i}(a)} \mid 1 \le i \le n,\ a \in \mathbb{Z} \rangle = \langle T_\alpha \mid \alpha \in D_Z(X) \text{ spherical} \rangle$
- Conjecturally B is the Artin group of type $\tilde{A}$ (affine braid)



Strategy $\alpha \in D_Z(X)$

$$l(\alpha) := \sum_{i,p} \mathrm{length}_{\mathcal{O}_{X,\eta_i}} \mathcal{H}^p(\alpha)_{\eta_i} \qquad \eta_i = \text{gen. pt of } C_i$$

> 0 if $\dim \mathrm{supp}(\alpha) > 0$

Step 1 $\alpha \in D_Z(X)$ spherical

$l(\alpha) > 1 \Rightarrow \exists \Phi \in B$ s.t. $l(\Phi(\alpha)) < l(\alpha)$

Step 2 $\Phi \in \mathrm{Auteq}\, D_Z(X)$ given

$\alpha = \Phi(\mathcal{O}_{C_1})$ $\beta = \Phi(\mathcal{O}_{C_1}(-1))$

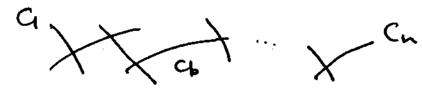
Assume $l(\alpha) = 1$ and $l(\beta) > 1$

$\Rightarrow \exists \Psi \in B$ s.t. $l(\Psi(\alpha)) = 1$, $l(\Psi(\beta)) < l(\beta)$

$\rightsquigarrow$ May assume $l(\Phi(\alpha)) = l(\Phi(\beta))$

$\rightsquigarrow \exists a, b, i \in \mathbb{Z}$ $\quad \Phi(\alpha) = \mathcal{O}_{C_b}(a)[i]$

$\Phi(\beta) = \mathcal{O}_{C_b}(a-1)[i]$



$\rightsquigarrow b = 1$ or n

By induction on n

Prop. $\Phi \in \mathrm{Auteq}\, D_Z(X)$ given

$\Rightarrow \exists \Psi \in B\ \exists i$

$\forall x \in Z\ \exists y \in Z \quad \Psi \circ \Phi(\mathcal{O}_x) = \mathcal{O}_y[i]$

$$\mathcal{O}_{C_1}(-1) \to \mathcal{O}_{C_1} \to \mathcal{O}_x$$

$$\begin{array}{ccccc} \Psi\Phi\mathcal{O}_{C_1}(-1) & \to & \Psi\Phi\mathcal{O}_{C_1} & \to & \Psi\Phi\mathcal{O}_x \\ \wr\!\mid & & \wr\!\mid & & \wr\!\mid \\ \mathcal{O}_{C_b}(a-1)[i] & & \mathcal{O}_{C_b}(a)[i] & & \mathcal{O}_y[i] \end{array}$$

$\Rightarrow$ Thm

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Tool of computation

$\alpha \in D(X)$ $\dim X = 1 \Rightarrow \alpha = \bigoplus_i \mathcal{H}^i(\alpha)[-i]$

X non-sing.

$\dim X = 2 \Rightarrow \alpha$ is determined by $\mathcal{H}^i(\alpha)$. $e^i(\alpha) \in \mathrm{Ext}^2_X(\mathcal{H}^i(\alpha), \mathcal{H}^{i-1}(\alpha))$

(idea) $\alpha = \alpha^{\bullet}$

$$\tau_{\le p}\alpha = (\to \alpha^{p-2} \to \alpha^{p-1} \xrightarrow{d^{p-1}} \ker d_p \to 0 \to \cdots)$$

$$\begin{array}{ccc} \tau_{\le p}\alpha & \longrightarrow & \alpha \\ & {}_{[1]}\nwarrow \quad \swarrow & \\ & \tau_{>p}\alpha & \end{array}$$

triangle

$e^p(\alpha)$

$$\mathrm{Hom}(\tau_{>p}\alpha, \tau_{\le p}\alpha[1]) \underset{2\dim}{\simeq} \mathrm{Ext}^2_X(\mathcal{H}^p(\alpha), \mathcal{H}^{p-1}(\alpha))$$

$\{e^p(\alpha)\}_p$ determine d_2-map of spectral seq's

① $E_2^{pq} = \bigoplus_i \mathrm{Ext}^p_X(\mathcal{H}^i(\alpha), \mathcal{H}^{i+q}(\alpha)) \Rightarrow \mathrm{Hom}^{p+q}(\alpha, \beta)$

② $E_2^{pq} = F^p(\mathcal{H}^q(\alpha)) \Rightarrow F^{p+q}(\alpha)$ $F: D(X) \to \mathrm{Coh}(X)$

① より

Lem $\alpha \in D_Z(X)$ spherical $Z = C_1 \cup \cdots \cup C_n$ ADE fundamental cycle

$\Rightarrow$ 1) $\mathcal{H}^i(\alpha)$: $\mathcal{O}_Z$-mod. pure of 1-dim

2) $\mathrm{Ext}^1_X(\mathcal{H}^i(\alpha), \mathcal{H}^j(\alpha)) = 0$ for $\forall i,j$

An-case 1) $\Rightarrow \mathcal{H}^i(\alpha)$ is a direct sum of sheaves $\mathcal{O}_{C_s \cup \cdots \cup C_t}(a_s, \cdots, a_t)$

$1 \le s \le t \le n$

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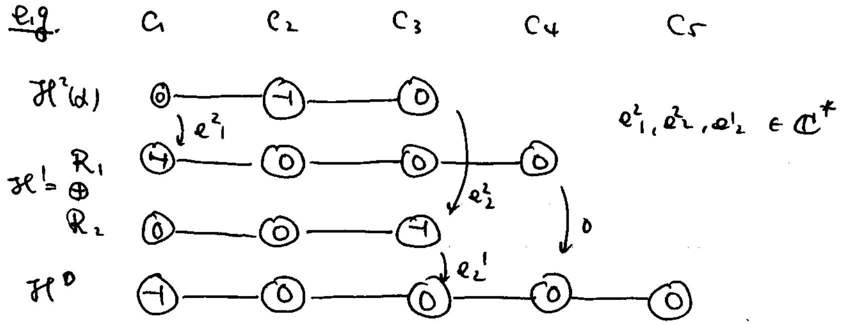
(Step 1) α spherical $l(\alpha) > 1 \Rightarrow \exists \Phi \in B$ $l(\Phi(\alpha)) < l(\alpha)$

$(n \ge 2)$ $\Downarrow$ $\Uparrow$②

$\exists \Phi \in B$ s.t. $\sum_q l(\Phi(\mathcal{H}^q(\alpha))) < l(\alpha) = \sum_q l(\mathcal{H}^q(\alpha))$

e.g.



これで定まる α は spherical

$$\left(\begin{array}{c} \mathcal{O}_{C_1 \cup \cdots \cup C_n}(a_1, \cdots, a_n) \\ \big\} T_{\mathcal{O}_{C_1}(a_1-1)} \\ \mathcal{O}_{C_2 \cup \cdots \cup C_n}(a_2, \cdots, a_1) \end{array} \right. \quad (n \ge 2)$$

$T_{\mathcal{O}_{C_2}(-2)}$ $l(\alpha) = l(T_{\mathcal{O}_{C_2}(-2)}\alpha)$

$T_{\mathcal{O}_{C_1}(-1)}$

(0) — (−1) — (0)

(0) — (−2) — (1) — (0)

+1 (1) — (−2) — (0)

(0) — (−2) — (1) — (0) — (0)

$l(\alpha) < l(T_{\mathcal{O}_{C_1}(-1)}, T_{\mathcal{O}_{C_2}(-2)}\alpha)$

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16:00 ~ Ambro F.

Recent developements on $\exists$ flips
(Shokurov "Prelimiting flips")

New ideas (1) saturation of algebras / linear systems

(2) linear systems with

(3) Diophantine approx'n of dim

§ Flips via canonical rings

X normal proj/$\mathbb{C}$ $\quad D$ $\mathbb{R}$-Cartier div on X

$H^0(X, D) = \{\varphi \in k(X)^\times \mid (\varphi) + D \geq 0\} \cup \{0\}$

Question Is $R_X(D) = \bigoplus_{m=0}^{\infty} H^0(X, mD)$

finitely generated.

$\dim X = 1$. YES

$= 2$ (Zariski) YES unless $\left\{ \begin{array}{l} D = P + N \\ P \text{ nef . big} \\ Bs|jP| \neq \emptyset \\ \forall j \geq 1 \end{array} \right.$

≥ 3 ? Usually No

Expect YES

(1) X smooth (terminal canonical) + $D = K_X$ $\Leftarrow$ MMP$_d$ + Abundance

(2) (X, B) log variety (klt log can'l) $\quad D = K + B$

Existence of flips is a special case of (1)

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$$\begin{tikzcd} X \arrow[rr, dashed] \arrow[dr, "f"'] & & X^+ = \mathrm{Proj}\left(\bigoplus_{m \geq 0} H^0(X, mK_X)\right) \arrow[dl, "f^+"] \\ & Y & \end{tikzcd}$$

flipping contraction

Mori Classify f $\longrightarrow$ Construct f^+ explicitly

Shokurov Establish the f.g. of the $\mathcal{O}_Y$-algebra

$\bigoplus_{m \geq 0} f_* \mathcal{O}_X(mK_X)$ by induction on dim

§ Inductive idea for f.g. of $R_X(D)$

$Y \in |rD|$ $\quad Y$. normal $\quad Y \not\subset \mathrm{Supp}\, D$

$$\begin{array}{ccc} H^0(X, mD) & \longrightarrow & H^0(Y, mD) \\ a & \longmapsto & a|_Y \\ R_X(D) & \longrightarrow & R_Y(D|_Y) \\ & \searrow & \cup \\ & & R_X(D)|_Y \end{array}$$

Lemma $R_X(D)$ f.g. $\Leftrightarrow$ $R_X(D)|_Y$ f.g.

$\because$ $\exists\, t \in k(X)^\times$ $\quad (t) + rD = Y$

Then $R_X(D)$ is generated by

· $t \in R_X(D)_r$

$R_X(D)_{\leq r-1}$

· lift gen'ls of $R_X(D)|_Y$

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subalgebras of div'l algebras

divisorial algebras

normal subalgs div'l algebras

$\to \mathcal{I} \subset R_X(D)$

$\mathbb{C}$ − v

$\mathcal{I}$ f.g.

$\Leftrightarrow \Sigma$ f.g.

maximal alg

§ Normal algebras $\quad \mathcal{I} = \bigoplus_{m \geq 0} \mathcal{I}_m \subset R_X(D)$

$L_m = \{ (a) + mD \geq 0 \mid a \in \mathcal{I}_m^{\times} \} \subset |mD|$

$$\begin{array}{ccl} X_i & & \mu_i^* L_{\varphi_i} = |M_i| + F_i \\ \mu_i \downarrow & & S_i = H^0(X, M_i) \\ X & & S = \bigoplus_i H^0(X, M_i) \end{array}$$

NB We may replace X by any neighborhood

$\mathcal{I} \longmapsto \{M_i\}_{i \geq 0}$
$\begin{cases} \cdot\, M_i + M_j = M_{i+j} \\ \cdot\, M_0 = \mathbb{C} \\ \cdot\, M_i \in \mu_i^*(iD) \end{cases}$

$D_i = \dfrac{M_i}{i} \leq D$

Thm $\mathcal{I}$. f.g. $\Leftrightarrow$ $\exists I \in \mathcal{J}_p$ s.t. D_i constant $\forall \Sigma | i$

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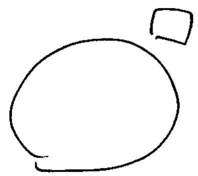
Rem $\quad \mathcal{I} \subset R_X(D)$ f.g. $\Rightarrow$ $\mathcal{I}$ is divisorial but usually not on X

Example. $X = T_N\mathrm{emb}(\Delta)$ proper toric var.
(1) D: $\mathbb{Q}$-Cartier div $\Rightarrow$ $R_X(D)$ f.g. $M = N^*$
(2) ~~Let~~ $\{\square_i\}_{i \geq 0}$ Lattice polytopes in $M_{\mathbb{R}}$ $(= N_{\mathbb{R}}^*)$
- $\square_0 = \{0\}$
- $\square_i + \square_j \subseteq \square_{i+j}$
- $\exists\, \square$ bounded set s.t. $\square_i \subseteq i\square$

$$\mathcal{I} = \bigoplus_{i \geq 0} \Big(\bigoplus_{m \in M \cap \square_i} \mathbb{C} x^m \Big)$$

$\mathcal{I}$. f.g. $\Leftrightarrow$ $\dfrac{\square_i}{i}$ = constant



$\square$ = convex hull of

§ (X, B) log pairs
(1) X normal variety, B. $\mathbb{Q}$-Cartier div.
(2) $K + B$ $\mathbb{Q}$-Cartier.

(X, B) log variety $\quad$ log pair
$B \geq 0$
(X, B) klt

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$$\begin{array}{ll} Y & \mu^*(K+B) = K_Y + B_Y \\ \big\downarrow \mu & \text{klt}: \text{ coeff of } B_Y < 1 \\ X & \quad \ulcorner -B_Y \urcorner \geq 0 \end{array}$$

§ Saturated algebras

DEF S normal algebra of X
(X, B) log pair
$\mathcal{J}$ is (X,B)-saturated if $\exists\, I \in \mathbb{Z}_{>0}$

$$H^0(Y, \ulcorner -B_Y + jD_{i,Y} \urcorner) \subseteq H^0(Y, M_{j,Y}) \qquad \forall\, Y \to X$$

Rem For $\forall\, i,j$ fixed enough to check the
- M_i, M_j defined on Y
- $B_Y \cup M_i$ ane

Rem $\mathcal{J} = R_X(D) \;\not\longrightarrow\; \mathcal{J}$ is $(X\,B)$-sat
$B \geq 0$
D $\mathbb{Q}$-Cartier

Proof

$$\begin{array}{ll} Y & \mu^*(iD) = |M_i| + F_i \\ \mu \big\downarrow & \mu^*(jD) = |M_j| + F_j \\ X & I \cdot D \text{ Cartier} \end{array}$$

$$H^0(\ulcorner -B_Y + jD_i \urcorner) \subseteq H^0(\ulcorner -B_Y + j\mu^*(D) \urcorner)$$
$$= H^0(Y, \ulcorner -B_Y \urcorner + \mu^*(jD)) \subseteq H^0(X, jD)$$
$$\| \; H^0(Y, M_j)$$

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$$\begin{array}{lll} E \subset Y & f^*D & f^{-1}D + E \\ \big\downarrow f & & \\ X & & \end{array}$$

Rem. $(X, B) \dashrightarrow (X', B_{X'})$ log crepant

$$\begin{tikzcd} & Y \arrow[dl, "\mu"'] \arrow[dr, "\mu'"] & \\ (X,B) \arrow[rr, dashed] & & (X', B_{X'}) \end{tikzcd}$$

$\mu^*(K+B) = \mu'^*(K_{X'} + B_{X'})$

$\mathcal{J}$ is (X,B)-sat $\iff (X', B_{X'})$ sat.

Example $(X\, B)$ curve. $\quad \mathcal{J} = \bigoplus_{i \geq 0} \mathcal{O}_X(m_i P)$

$\mathrm{id} \downarrow \qquad B = b \cdot P \quad b < 1$

$p \in X \qquad d_i = \frac{1}{i} m_i$

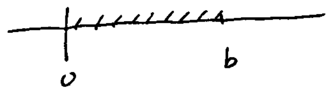
$\ulcorner -b + j d_i \urcorner \leq j d_j \quad \forall\, i,j,\ I \qquad d_i \nearrow d \in \mathbb{R}$

$$\iff \begin{cases} \cdot\ d \in \mathbb{Q} & b > 0 \\ \cdot\ d = d_i & i \gg 1 \\ \cdot\ \mathrm{index}(d) \leq \frac{1}{1-b} & \end{cases}$$

$\ulcorner -b + jd \urcorner_i \leq j d_j \quad \forall\, j,\ I$

$\ulcorner -b + \{jd\} \urcorner \leq 0$

$1 - \frac{1}{q} = \sup_{j,I} \{jd\} \leq b$



$d \in \mathbb{Q}$

$d = \frac{p}{q}$

j sat $\quad j \cdot d \in \mathbb{Z} \quad I \mid j$

$\lceil -b + jd \rceil \leq j d_j \leq jd \qquad \forall b \geq 0$

$\lceil -b' \rceil + jd \leq j d_j \leq jd \qquad \Rightarrow d_j = d$

FGA Conj $\mathcal{I}$. normal alg on X

- $\mathcal{I}$ (X, B) saturated
- (X, B) log Fano $(-(K+B)$ ample$)$

$\Rightarrow \mathcal{I}$ f.g.

Thm $\exists$ flips in dim $d+1$ $\Leftarrow$ FGA_d $\mathrm{LMMP}_{\leq d}$

Thm FGA_1, FGA_2 hold

Rem $\exists$ 4-fold flips but FGA_3 UNKNOWN

X Fano	X general
D nef $\Rightarrow \exists b \geq 1$ $\lvert bD \rvert$ free	D nef, $D - K_X$ nef and big $\not\Rightarrow \lvert bD \rvert$ free $b \geq 1$
$\mathcal{I}$. X-sat $\overset{?}{\Rightarrow} \mathcal{I}$. f.g.	$\mathcal{I}$. X-sat ADJOINT $\not\Rightarrow$ f.g.

Some adjoint

Thm $(-)$ (X, B) log pair $\mathcal{I}$ normal (X,B)-alg
Then $\mathcal{I}$ is fg if one of the holds

(1) $\kappa(\mathcal{I}) = 1$

(2) $\mathcal{I} = R_X(D)$: D nef big $\mathbb{R}$-divisor on X
$\delta D - (K+B)$ nef $\exists \delta \in \mathbb{R}$

(3) $\mathcal{I} = \bigoplus_{i \geq 0} H^0(M_i)$ all M_i defined on X
$\delta \underrightarrow{\lim} D_i - (K+B)$ is nef and big

§ Toric saturated algebras

$X = T_N \operatorname{emb}(\Delta)$

D $\mathbb{Q}$-Cartier $\Longleftrightarrow$ $h : N_{\mathbb{R}} \longrightarrow \mathbb{R}$
h : Δ-linear
$h(N) \subseteq \mathbb{Q}$

$$D = \sum_{e \in \Delta(1)} -h(e) V(e)$$

$\square = \square_h = \{ m \in M_{\mathbb{R}} \mid \langle m, e \rangle \geq h(e) \quad \forall e \in N_{\mathbb{R}} \}$

$H^0(X, D) = \bigoplus_{m \in M \cap \square} \mathbb{C} x^m$

$\square \rightsquigarrow h_\square(e) = \min_{m \in \square} \langle m, e \rangle \rightsquigarrow D$

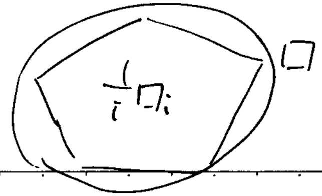
$\mathcal{I} = \bigoplus H^0(M_i)$ toric normal alg

$M_i = \sum -h_i(e) V(e) \qquad h_i = h_{\square_i}$

(X, B) toric log pair $\qquad \psi : N_{\mathbb{R}} \longrightarrow \mathbb{R}$
ψ Δ-linear
$\psi(e) > 0 \quad e \neq 0$

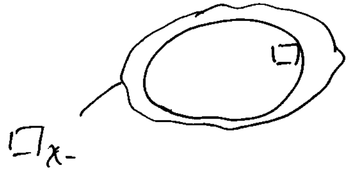
$\frac{1}{i} \square$

$\square = \lim_{i \to \infty} \frac{1}{i} \square_i$ bounded convex set



Prop $\mathcal{J}$ is (X, B) sat $\Leftrightarrow$

$\forall \Sigma | j \quad \frac{1}{j} M \cap \mathrm{Int}(\square_{h - \frac{\psi}{j}}) \subseteq \square \; (= \square_e)$



i) $\chi^{\omega} \in H^0(\ulcorner -B_{ij} + jD_{i\cdot\gamma} \urcorner)$

$\langle m, e \rangle \geq \lfloor \Lambda - \psi(e) + j h_i(e) \rfloor \quad \forall e \neq 0$

$\Updownarrow \; m \in M$

$\langle m, e \rangle \geq (j \frac{e_i}{i} - \psi)(e)$

$\Leftrightarrow m \in \mathrm{Int}(\square_{j \frac{e_i}{i} - \psi})$

only consider ~~$e \in S^{d-1} \subset N_{\mathbb{R}} \Leftrightarrow$ may~~

$e \in S^{d-1} \subset N_{\mathbb{R}}$

$\Leftrightarrow$ may replace $\frac{h_i}{i}$ with h

~~Prop~~ Thm Assume $\exists r > 0$ s.t. $rh - \psi$ upper

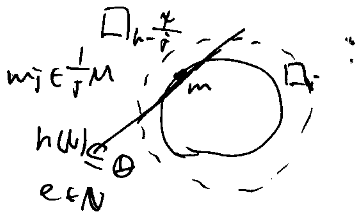
(X, B) sat $\Rightarrow$ f.g.

□ lattice polytope

Proof (1) Supp fun of any face of □ is rat'l

Thm $\alpha \in \mathbb{R}^d$ $\alpha_1 \notin \mathbb{Q}$ Then $\exists \infty$ many j's s.t.

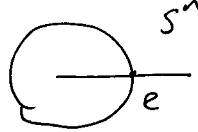
$\mathbb{Z}^d \cap \{ m \in \mathbb{R}^d \mid \| m - j\alpha \| < \frac{1}{j^{\frac{1}{d-1}}}, \; m_1 < j\alpha_1 \} \neq \phi$



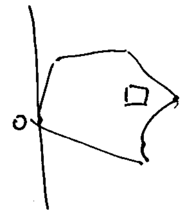
$\Rightarrow h(e) \in \mathbb{Q}$

2) dim = 2

$S^n \subset N_{\mathbb{R}}$

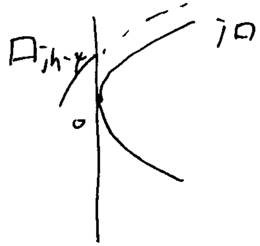


$h = h_{\square}$



$e = e_1$

$h(e_1) \in \mathbb{Q}$



$\mathbb{Z} \cap (0, \inf_{t \in \mathbb{R}} (\psi - jh)(t, -1)) = \phi$

$\forall j \quad \psi(t_j, -1) \leq 1 + jh(t_n, -1) \leq 1$

$K = \{ t \mid \psi(t, -1) \leq 1 \}$

$\rightarrow$ contradict

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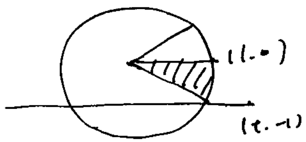
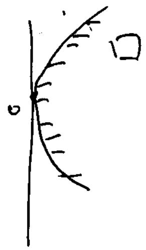
$t_j \longrightarrow t$

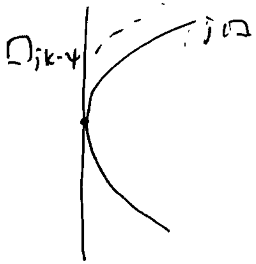
$\dfrac{\psi(t_j, -1) - 1}{j} \le h(t, j-1)$

$\downarrow$

$0 \le h(t, -1) \le 0$







$\forall i, j \quad i = j \quad M = M_i$

$H^0(Y, \ulcorner -B_Y \urcorner + M) \le H^0(M)$

$|\ulcorner -B_Y \urcorner + M| = |M| + \ulcorner -B_Y \urcorner$

(X, B)-saturated

CCS Conj (X, B) log Fano

$\Rightarrow \exists c_b > 0$, $\exists$ finitely many crepant models

$(X, B) \dashrightarrow (X_i, B_{X_i})$

s.t. M free

$\Rightarrow \exists i$ st $\operatorname{lct}(X, B, |M_i|_X) \ge c_0$ ((X,B)-sat big)

- CCS $\Rightarrow$ FGA
- CCS_2 true.

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$(\tilde{X}, B_{\tilde{X}})$

$\downarrow$

(X, B)

$\Rightarrow \forall \ne M$ as above is free on $\tilde{X}$

Example

$X = \mathbb{C}^d \qquad B = b\sum_{i=1}^{d} H_i \qquad 0 \le b < 1$

$a_1, \dots, a_d$ rel prime > 0

$E \subset X_N$ a_d weighted b-up of $\mathbb{C}^d$ with

$\downarrow$

X

$M = -hE$ free$/X$ $\Leftrightarrow \operatorname{lcm}(a_1, \dots, a_d) / h$

$(h > 0)$

M is (X, B) sat $\Leftrightarrow \Delta - b \le \dfrac{\operatorname{Frob}(a_1, \dots, a_d^*, h)}{\sum a_i}$

However CCS holds in

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並河 II 7月7日 9.30〜

Nilpotent orbits and birational geometry (derived category?)

G: complex simple Lie group $\mathfrak{g} := \mathrm{Lie}(G)$

$G \curvearrowright \mathfrak{g}$ adjoint action $\quad G \curvearrowright G$

$g \quad\quad\quad\quad g \quad x \mapsto gxg^{-1}$

$\mathrm{Ad}_g : \mathfrak{g} \to \mathfrak{g}$

$\bar{O}$ = nilpotent orbit closure

$\alpha \in \mathfrak{g}$ nilpotent $\quad O = G \cdot \alpha \subset \mathfrak{g}$ ← nilpotent orbit

$G^\alpha := \{ g \in G \mid \mathrm{Ad}_g(\alpha) = \alpha \}$

$$O \xleftarrow{\sim} G/G^\alpha$$
$$\mathrm{Ad}_g(\alpha) \longleftarrow\!\shortmid \; g$$

$\mathfrak{g}^\alpha := \{ x \in \mathfrak{g} \mid [x, \alpha] = 0 \}$

$$T_\alpha O \xleftarrow{\sim} \mathfrak{g}/\mathfrak{g}^\alpha$$

$$\tilde{\Omega}_\alpha : \mathfrak{g} \times \mathfrak{g} \to \mathbb{C} \quad (x, y) \mapsto K(\alpha, [x, y]) = K([\alpha, x], y) \quad K: \text{killing form}$$

$$\Omega_\alpha : \mathfrak{g}/\mathfrak{g}^\alpha \times \mathfrak{g}/\mathfrak{g}^\alpha \to \mathbb{C}$$

$\Omega = \{\Omega_\alpha\}$: O 上の non-degenerated 2-form, $d\Omega = 0$

Kostant-Kiullor form

Fact (Panyushev) The normalization of $\bar{O}$ is a symplectic variety.

Def. $\nu : Y \xrightarrow[\text{resol.}]{} \bar{O}$ is a symplectic resolution if Ω lifts to a non-degenerate 2-form on Y
(いつも存在するとは限らない)

Remark Crepant resolution $\Leftrightarrow$ symplectic resolution

$P \subset G$ parabolic subgroup $\quad \mathfrak{p} := \mathrm{Lie}(P)$, $n(P)$: nilradical of $\mathfrak{p}$

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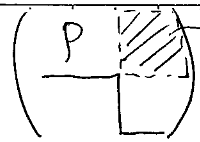
Example 

$\mathfrak{sl}$

Def. $x \in \mathfrak{g}$ nilpotent
P: polarization of $x \overset{\text{def}}{\Leftrightarrow}$ (1) $x \in n(P)$
(2) $\dim O = 2 \dim (G/P)$
(通常 $\dim O \leq 2\dim(G/P)$)

P: xの polarization

$T^*(G/P) \simeq G \times^P n(P) :\underset{\text{def}}{=} G \times n(P)/\sim \quad$ $(g, x) \sim (g', x') \Leftrightarrow g' = gp,\ x' := \mathrm{Ad}_{p^{-1}}(x),\ \exists p \in P.$

$[g, x] \mapsto [g] \in G/P$ ↓ vectorbundle

$$\begin{array}{ccc} G \times^P n(P) & \xrightarrow{\mu} & \bar{O} \subset \mathfrak{g} \\ [g,x] & \longmapsto & \mathrm{Ad}_g(x) \\ \pi \downarrow & & \\ G/P & & \end{array}$$

μ: Springer map (gen. finite, proper, surj.)

$\deg \mu = 1 \Rightarrow \mu$: Springer resolution

$$\begin{array}{ccc} G \times^P n(P) & \xrightarrow{\pi \times \mu} & G/P \times \bar{O} \ni \{(F_\cdot, x)\} \\ & \searrow^{\mu} & \downarrow p_2 \\ & & \bar{O} \ni x \end{array}$$

$\{(F_\cdot, x) \mid xF_i \subseteq F_{i-1}\ \forall i\}$

$0 \subset F_1 \subset F_2 \subset \cdots \subset F_s = V$

G: classical
$SL(V)$
$SO(V)$
$SP(V)$

Theorem (Fu) Any symplectic resolution of $\bar{O}$ is a Springer resolution.

Classical groupの場合 $\quad G, \mathfrak{g}$

$\mathrm{Nil}(\mathfrak{g}) = \{$nilpotent orbits in $\mathfrak{g}\}$

$\mathrm{Par}(G) := \{$conjugacy classes of par. subgroups in $G\}$

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(A_{n-1}): $G = SL(n)$

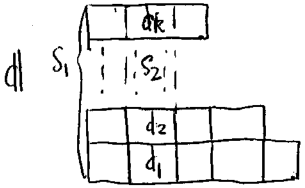
$Nil(sl(n)) \longrightarrow \{\text{partition of } n\}$
$[d_1, \cdots, d_k]$ $d_1 \geq \cdots \geq d_k > 0$, $\sum_i d_i = n$

$\mathcal{O} \longmapsto$ Jordan type

$Par(SL(n)) \longrightarrow \{(s_1, \cdots, s_m) \mid s_i > 0, \sum s_i = n\}$

$\mathbb{C}^n$ $\quad P \longmapsto$ P の flag type $\quad 0 \subset \underbrace{F_1}_{s_1} \subset \underbrace{\cdots}_{s_2} \subset \underbrace{F_m}_{s_m} = \mathbb{C}^n$

Prop. $x \in sl(n)$ nilpotent of type $d = [d_1, \cdots, d_k]$
次が成立
(1) x has a polarization P.
(2) P の flag type は次で与えられる：



${}^t d = (s_1, s_2, \cdots, s_m)$

(3) $\mu_0 : T^*(SL(n)/P_{s(1), \cdots, s(m)}) \longrightarrow \overline{\mathcal{O}}$ は Springer resolution

Example (Mukai flop of type A)

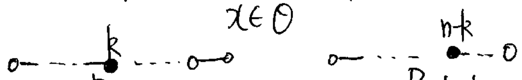
$x \in \mathcal{O} \subset sl(n)$ $\quad k < \frac{n}{2}$
nilpotent orbit
$d = [2^k, 1^{n-2k}]$ $\quad {}^t d = [n-k, k]$

$\underbrace{P_{k,n-k} \quad P_{n-k,k}}_{x \text{ の polarization}}$

$T^*G(k,n) \dashrightarrow T^*G(n-k,n)$, $\quad \mu^+ \searrow \overline{\mathcal{O}} \swarrow \mu^-$

$[(0 \subset \underbrace{Im(x)}_{k\text{次元}} \subset \mathbb{C}^n, x)]$ $\quad [0 \subset \underbrace{Ker(x)}_{n-k\text{次元}} \subset \mathbb{C}^n, x]$

$\mu^+ \searrow x \in \mathcal{O} \swarrow \mu^-$



$P_{k,n-k}$ $\quad P_{n-k,k}$

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— (B_n), (C_n) 省略

(D_n): $SO(2n)$ $\quad d = [d_1, \cdots, d_k]$ partition of $2n$
d : ~~every~~ very even $\Leftrightarrow$ $\forall d_i$: even

$Nil(so(2n)) \longrightarrow \{$ partitions of $2n$ s.t. even parts occur with even multiplicity $\} =: P(2n)$

$\mathcal{O} \longmapsto$ Jordan type

d : not ~~very~~ very even $\Rightarrow \varphi^{-1}(d) = \{\mathcal{O}\}$
d : ~~very~~ very even $\Rightarrow \varphi^{-1}(d) = \{\mathcal{O}_I, \mathcal{O}_{II}\}$ $\leftarrow \mathcal{O}(2n)$

$Par(SO(2n))$
$V = \mathbb{C}^{2n}, \langle , \rangle$

$F_\bullet$ isotropic flag $\Leftrightarrow$ $F_i^\perp = F_{s-i}$
$0 \subset F_1 \subset \cdots \subset F_s = V$ $\quad (1 \leq i \leq s)$

$F_\bullet$: admissible isotropic flag $\Leftrightarrow$ F の stabilizer group P は F より細かい isotropic flag の stabilizer group には成っていない

$s = 2k+1$ isotropic flag の type は $(\underbrace{p_1, \cdots, p_k}_{=\mathbb{P}}, q, p_k, \cdots, p_1)$
$s = 2k$ $\quad - \ - \ - \ - \ - \ (p_1, \cdots, p_k, p_k, \cdots, p_1) = (p_1, \cdots, p_k, 0, p_k, \cdots, p_1)$

Remark. $F_\bullet$: admissible $\Leftrightarrow$ $q \neq 2$

$Par(SO(2n)) \xrightarrow{\psi} \{(\mathbb{P}, q) \mid q \neq 2, q \geq 0, p_i > 0, 2\sum_{i=1}^k p_i + q = 2n\}$

$P \longmapsto$ flag type of P

$q \neq 0$ or $p_k \leq 1$
$\Rightarrow \#(\psi^{-1}(\mathbb{P}, q)) = 1$
$q = 0, p_k \geq 2 \Rightarrow \psi^{-1}((\mathbb{P}, 0)) = \{P^+, P^-\}$

Example $(p, \overset{q}{\bar{o}}, p) = (p, o)$

$\{\text{isotropic flags of type } (p,0)\} = G_{iso}(p,2p)$
$= \underset{\substack{\Downarrow \\ F}}{G^+_{iso}(p,2p)} \sqcup \underset{\substack{\Downarrow \\ F^+}}{G_{iso}(p,2p)}$

$p\geq 1$のとき. F, F^+は conjugate
$p\geq 2$のとき. F, F^+は non-conjugate

Def. $\underset{\text{parabolic}}{P \subset SO(2n)}$ $\nu(P) = (p_1,\cdots,p_k, q, p_k,\cdots,p_1) \leftrightarrow d$

$\underset{=\mathrm{ord}(p_1,\cdots,p_k,q,p_k,\cdots,p_1)}{\pi := {}^t d}$ & PのLevi type.

$q\neq 2,\ q\geq 0$

$\mathrm{Pai}(2n,q) = \{\pi: \text{partition of } 2n \mid \pi_i = \text{odd if } i\leq q,\ \pi_i = \text{even if } i>q\}$

$\mathrm{Par}(SO(2n)) \twoheadrightarrow \{(P,q) \mid \cdots\}$

$\mathrm{Nil}(so(2n)) \twoheadrightarrow \{\cdots\} =: P(2n)$

q Spaltenstein map

$$S_q : \mathrm{Pai}(2n,q) \longrightarrow P(2n), \quad \pi \longmapsto S_q(\pi)$$

$I(\pi) := \{ j\in\mathbb{N} \mid j: \text{odd}, \pi_j: \text{even}, \pi_j \geq \pi_{j+1}+2\}$

$$S_q(\pi)_j = \begin{cases} \pi_j - 1 & (j\in I(\pi)) \\ \pi_j + 1 & (j-1\in I(\pi)) \\ \pi_j & (\text{otherwise}) \end{cases}$$

Thm (Spaltenstein, Hesselink)
$x\in so(2n)$ of type $d\in P(2n)$

(1) x has a polarization P with Levi type $\pi\in \mathrm{Pai}(2n,q) \Leftrightarrow d = S_q(\pi)$.

(2) (1)において、$\pi := \mathrm{ord}(p_1,\cdots,p_k,q,p_k,\cdots,p_1)$ とする. Pのflag typeは $(p_{\sigma(1)},\cdots,p_{\sigma(k)}, q, p_{\sigma(k)},\cdots,p_{\sigma(1)})$, $\sigma\in G_k$で与えられる.

(3) (1)のとき. Springer map $\mu: T^*(SO(2n)/P) \longrightarrow \bar{O}$ の degreeは次で与えられる:

$$\deg(\mu) = \begin{cases} 2^{\#I(\pi)-1} & (q=0, \pi^i: \text{odd } (\exists i)) \\ 2^{\#I(\pi)} & (q\geq 1 \text{ or } q=0, \pi^i: \text{even } (\forall i)) \end{cases}$$
$\pi^i := ({}^t\pi)_i$.

Example (Mukai flop of type D)
$n\geq 3$. odd $x\in so(2n)$ of type $[2^{n-1},1^2]$

$$S_0: \mathrm{Pai}(2n,0) \longrightarrow P(2n), \quad \pi = [2^n] \longmapsto [2^{n-1},1^2]$$

$\pi = [\overbrace{2,\cdots,2}^{n}]$ $\xrightarrow{S_0}$ $[2^{n-1},1^2] = [2,\cdots,2,1,1]$

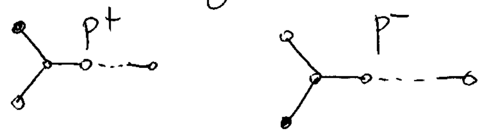
$I(\pi) = \{n\}$

$\underbrace{P^+, P^-}_{x\text{のPolarization}}$ flag type $(n, \overset{q}{\bar{o}}, n)$

$SO(2n)/P^\pm = G^\pm_{iso}(n,2n)$.

$$T^*G^+_{iso}(n,2n) \dashrightarrow T^*G^-_{iso}(n,2n), \quad \mu^+: T^*G^+_{iso}(n,2n)\to \bar{O}, \quad \mu^-: T^*G^-_{iso}(n,2n)\to \bar{O}$$

$[0\subset V\subset \mathbb{C}^{2n}, x] \mapsto x\in O \mapsfrom [0\subset V'\subset \mathbb{C}^{2n}, x]$



$T^*G^+_{iso}(n,2n)$ $\quad G^+_{iso}(n,2n)$ $\quad G^+_{iso}(n-2,2n-4)$ $\cdots$ $G^+_{iso}(3,6)$ $\quad$ 1点

$\downarrow$

$$\bar{O}_{[2^{n-1},1^2]} = \underset{\{0\}}{O_{[1^{2n}]}} \sqcup O_{[2^2,1^{2n-4}]} \sqcup \cdots \sqcup O_{[2^{n-3},1^6]} \sqcup O_{[2^{n-1},1^2]}$$

Def. (locally trivial family of Mukai flops)

$X \xrightarrow{f} Y \xleftarrow{f'} X'$ two resolution of Y.

The diagram is a locally trivial family of Mukai flops of type A (resp. type D) if $\exists \{U_\lambda\}$ partial open cover of Y, $\bigcup_\lambda U_\lambda \supset \mathrm{Sing}(Y)$ s.t.

$f^{-1}(U_\lambda) \longrightarrow U_\lambda \longleftarrow f^{-1}(U_\lambda)$ is the product of Mukai flop of type A (resp. type D) with a disc Δ^m.

Theorem. $\bar{O} \subset \mathfrak{g}$

nilpotent orbit closure ($\bar O$); classical ($\mathfrak{g}$)

Assume that $\bar O$ has a symplectic resolution

$\{X_1, \cdots, X_m\}$

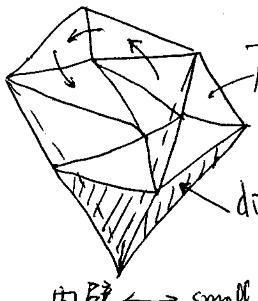
$\uparrow$

$\bar O$ の symplectic resolution

$\overline{\mathrm{Amp}}(X_i/\bar O)$

$\overline{\mathrm{Mov}}(X_i/\bar O) = \overline{\mathrm{Mov}}$ (iによらない)

$\overline{\mathrm{Mov}}$



$\overline{\mathrm{Amp}}(X_i/\bar O)$ simplicial polyhedral

divisorial contraction (外壁)

内壁 ↔ small contraction ← locally trivial family of Mukai flop of type A, type D.

$$\begin{tikzcd} X_i \arrow[rr, dashed] \arrow[dr] \arrow[ddr, bend right] & & X_j \arrow[dl] \arrow[ddl, bend left] \\ & X_{ij} \arrow[d] & \\ & \bar O & \end{tikzcd}$$

Example. $x \in so(10)$ $[4^2, 1^2]$

S_0 : $\mathrm{Pai}(10,0) \longrightarrow P(10)$

$\pi := [4^2, 2] \longmapsto [4^2, 1^2]$ $\quad I(\pi) = \{3\}$

$t_\pi = [3^2, 2^2]$

x has 4 polarizations $P^\pm_{3,2,2,3}$, $P^\pm_{2,3,3,2}$

$Y^\pm_{ijji} := T^*(SO(10)/P^\pm_{ijji})$

$$\begin{tikzcd}[column sep=small] & Y^+_{3,2,2,3} \arrow[r, dashed] \arrow[dl, "\text{div. contr.}"'] \arrow[dr] & Y^+_{2,3,3,2} \arrow[r, dashed] \arrow[d] \arrow[dr] & Y^-_{2,3,3,2} \arrow[r, dashed] \arrow[d] \arrow[dr] & Y^-_{3,2,2,3} \arrow[d] \arrow[dr, "\text{div. contr.}"] & \\ X^+_{3,4,3} & & X^+_{5,5} \quad X_{2,6,2} & X^-_{5,5} & & X^-_{3,4,3} \end{tikzcd}$$

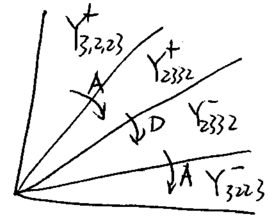
over $\bar O$

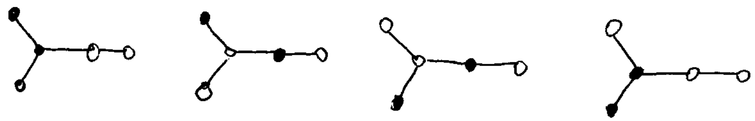
A型

$$\begin{tikzcd} T^*G(3,5) \arrow[r, dashed] \arrow[dr] & T^*G(2,5) \arrow[d] & Y^+_{3,2,2,3} \arrow[l, "\text{locally}"'] \arrow[r, dashed] \arrow[dr] & Y^+_{2,3,3,2} \arrow[d] \\ & \bar O_{[2^2,1]} & & \bar O \end{tikzcd}$$

D型

$$\begin{tikzcd} T^*G^+_{iso}(3,6) \arrow[r, dashed] \arrow[dr] & T^*G^-_{iso}(3,6) \arrow[d] & Y^+_{2,3,3,2} \arrow[l, "\text{loc.}"'] \arrow[r, dashed] \arrow[dr] & Y^-_{2,3,3,2} \arrow[d] \\ & \bar O & & \bar O \end{tikzcd}$$





Exceptional simple Lie alg? (group)

E_6 : G

p^+ $\qquad$ p^-

$$\begin{tikzcd} T^*(G/P^+) \arrow[r, dashed] \arrow[dr] & T^*(G/P^-) \arrow[d] \\ & \bar O_{2A_1} \end{tikzcd}$$

32次元

Mukai flop of type $E_{6,\mathrm{I}}$ (Caylay-Mukai flop)

p^+ $\qquad$ p^-

$\bar O_{A_2+2A_1}$ 50次元

Mukai flop of type $E_{6,\mathrm{II}}$

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<u>Conj</u> 例外型 (G_2, F_4, E_6, E_7, E_8) の場合にも、Mukai flop of type A, D、$E_{6,\mathrm{I}}$, $E_{6,\mathrm{II}}$ で用いることにより古典型と同じことが成立する。

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