

**Errata and Supplements to
Discrete Convex Analysis (SIAM, 2003)
1st printing (hard cover) (ISBN 0-89871-540-7)**

- Page 94, line 4 from bottom: Delete “is as follows.”
- Page 109, line 12: $4 \min(\lambda_j, \lambda_k) (\geq 4/N) \implies 2 \min(\lambda_j, \lambda_k) (\geq 2/N)$
- Page 113, line 12 from bottom (last paragraph of the proof of Theorem 4.18):
Let p be such that the optimal solutions to (A) with respect to p form a minimal face of the feasible region of (A).
- Page 133, (6.2): $\Delta f(z, v, u) \implies \Delta f(z; v, u)$
- Page 139, Proposition 6.8 (2): Condition (6.26) follows from (6.27), and hence (6.26) is redundant.
- Page 143, three lines above Theorem 6.13:
 $f_{[a,b]}$ for an integer interval $[a, b] \implies$
 $f_{[a,b]}$ for the restriction of f to an integer interval $[a, b]$
- Page 143, Theorem 6.13 (8) [Convolution of M-convex functions]
The proof here makes use of transformation by a network, but an alternative direct proof can be found in:
K. Murota: On infimal convolution of M-convex functions, RIMS Kokyuroku, No.1371 (2004), 20–26, and METR 2004-12, Department of Mathematical Informatics, University of Tokyo, March 2004
<http://www.keisu.t.u-tokyo.ac.jp/research/techrep/data/2004/METR04-12.pdf>
- Page 151, Theorem 6.30, Proof of Claim 1:
The final step reads: “This shows (B-EXC₊[**R**]) for \bar{B} . Therefore, B is an M-convex set.” Before we can argue in this way, we have to verify $\bar{B} \cap \mathbf{Z}^V = B$, which is possible.
- Page 152, Section 6.8: A characterization of gross substitutes property in terms of an exchange property is also found in:
H. Reijnierse, A. van Gallekom, and J. A. M. Potters: Verifying gross substitutability, *Economic Theory*, **20** (2002), 767–776.
- Page 172, Proof of Theorem 6.74:
“Theorem 6.4 can be strengthened to a statement that (M-EXC[**Z**]) and (M-EXC_{loc}[**Z**]) are equivalent if $\text{dom } f$ satisfies (Q-EXC_w). (This can be shown by modifying the proof of Claim 2 in the proof of Theorem 6.4.)”
The detail of the argument can be found in a memorandum of A. Shioura: Level set characterization of M-convex functions (February 1998); see Claim 2 on page 6.

- Page 185, Theorem 7.14 [L-optimality criterion]
The proof here makes use of the optimality criterion for integrally convex functions, but an alternative direct proof can be found in:
K: Murota: A proof of the L-optimality criterion theorem, unpublished note, July 2004,
<http://www.comp.tmu.ac.jp/kzmurota/paper/loptimality04.pdf>
- Page 219, Theorem 8.17 [M-convex intersection theorem]
The proof here makes use of the M-separation theorem, but an alternative direct proof can be found in:
K. Murota: A proof of the M-convex intersection theorem, RIMS Kokyuroku, No.1371 (2004), 13–19, and METR 2004-03, Department of Mathematical Informatics, University of Tokyo, January 2004
<http://www.keisu.t.u-tokyo.ac.jp/research/techrep/data/2004/METR04-03.pdf>
- Page 305, Section 10.3.1:
A detailed analysis of the steepest descent algorithm for L-convex functions can be found in:
K. Murota and A. Shioura: Exact bounds for steepest descent algorithms of L-convex function minimization, *Operations Research Letters*, **42** (2014), 361–366.
- Page 331, ($-M^{\natural}$ -SWGS[Z]): For $x \in \arg \min U[-p] \implies$ For $x \in \arg \max U[-p]$
- Page 333, “(SNC) for $\tilde{U} \implies M^{\natural}$ -concavity for U ”:
The converse of this statement is also true, that is,
$$\text{(SNC) for } \tilde{U} \iff M^{\natural}\text{-concavity for } U$$
holds. See K. Murota: Multiple exchange property for M^{\natural} -concave functions and valuated matroids, arXiv: 1608.07021, <http://arxiv.org/abs/1608.07021>, August, 2016.
- Page 365, [39]: P. G. Doyle and J. L. Snell: *Random Walks and Electrical Networks*, Mathematical Association of America, Washington DC, 1984.
- Page 369, [105] (S. Iwata and M. Shigeno): (2003) \implies (2002)
- Page 372, [143] (K. Murota): (1999) \implies (1998)
- Page 375, [192] (A. Shioura): (2003) \implies (2004)
- Page 376, [202]: D. M. Topkis, \implies D. M. Topkis:

Updates of bib-infor:

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- [156] should be changed to:
K. Murota and A. Shioura: Fundamental properties of M-convex and L-convex functions in continuous variables, *IEICE Transactions on Fundamentals of Electronics, Communications and Computer Sciences*, **E87-A** (2004), 1042–1052.
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