

**Errata and Supplements to  
Discrete Convex Analysis (SIAM, 2003)  
1st printing (hard cover) (ISBN 0-89871-540-7)**

- Page 94, line 4 from bottom: Delete “is as follows.”
- Page 109, line 12:  $4 \min(\lambda_j, \lambda_k) (\geq 4/N) \implies 2 \min(\lambda_j, \lambda_k) (\geq 2/N)$
- Page 113, line 12 from bottom (last paragraph of the proof of Theorem 4.18):  
Let  $p$  be such that the optimal solutions to (A) with respect to  $p$  form a minimal face of the feasible region of (A).
- Page 133, (6.2):  $\Delta f(z, v, u) \implies \Delta f(z; v, u)$
- Page 139, Proposition 6.8 (2): Condition (6.26) follows from (6.27), and hence (6.26) is redundant.
- Page 143, three lines above Theorem 6.13:  
 $f_{[a,b]}$  for an integer interval  $[a, b] \implies$   
 $f_{[a,b]}$  for the restriction of  $f$  to an integer interval  $[a, b]$
- Page 143, Theorem 6.13 (8) [Convolution of M-convex functions]  
The proof here makes use of transformation by a network, but an alternative direct proof can be found in:  
K. Murota: On infimal convolution of M-convex functions, RIMS Kokyuroku, No.1371 (2004), 20–26, and METR 2004-12, Department of Mathematical Informatics, University of Tokyo, March 2004  
<http://www.keisu.t.u-tokyo.ac.jp/research/techrep/data/2004/METR04-12.pdf>
- Page 151, Theorem 6.30, Proof of Claim 1:  
The final step reads: “This shows (B-EXC $_+$ [ $\mathbf{R}$ ]) for  $\bar{B}$ . Therefore,  $B$  is an M-convex set.” Before we can argue in this way, we have to verify  $\bar{B} \cap \mathbf{Z}^V = B$ , which is possible.
- Page 152, Section 6.8: A characterization of gross substitutes property in terms of an exchange property is also found in:  
H. Reijnierse, A. van Gallekom, and J. A. M. Potters: Verifying gross substitutability, *Economic Theory*, **20** (2002), 767–776.
- Page 172, Proof of Theorem 6.74:  
“Theorem 6.4 can be strengthened to a statement that (M-EXC[ $\mathbf{Z}$ ]) and (M-EXC $_{\text{loc}}$ [ $\mathbf{Z}$ ]) are equivalent if  $\text{dom } f$  satisfies (Q-EXC $_w$ ). (This can be shown by modifying the proof of Claim 2 in the proof of Theorem 6.4.)”  
The detail of the argument can be found in a memorandum of A. Shioura: Level set characterization of M-convex functions (February 1998); see Claim 2 on page 6.

- Page 185, Theorem 7.14 [L-optimality criterion]  
The proof here makes use of the optimality criterion for integrally convex functions, but an alternative direct proof can be found in:  
K. Murota: A proof of the L-optimality criterion theorem, unpublished note, July 2004,  
<http://www.comp.tmu.ac.jp/kzmurota/paper/loptimality04.pdf>
- Page 219, Theorem 8.17 [M-convex intersection theorem]  
The proof here makes use of the M-separation theorem, but an alternative direct proof can be found in:  
K. Murota: A proof of the M-convex intersection theorem, RIMS Kokyuroku, No.1371 (2004), 13–19, and METR 2004-03, Department of Mathematical Informatics, University of Tokyo, January 2004  
<http://www.keisu.t.u-tokyo.ac.jp/research/techrep/data/2004/METR04-03.pdf>
- Page 305, Section 10.3.1:  
A detailed analysis of the steepest descent algorithm for L-convex functions can be found in:  
K. Murota and A. Shioura: Exact bounds for steepest descent algorithms of L-convex function minimization, *Operations Research Letters*, **42** (2014), 361–366.
- Page 331, ( $-M^{\natural}$ -SWGS[ $\mathbf{Z}$ ]): For  $x \in \arg \min U[-p] \implies$  For  $x \in \arg \max U[-p]$
- Page 333, “(SNC) for  $\tilde{U} \implies M^{\natural}$ -concavity for  $U$ ”:  
The converse of this statement is also true, that is,  
$$\text{(SNC) for } \tilde{U} \iff M^{\natural}\text{-concavity for } U$$
holds. See K. Murota: Multiple exchange property for  $M^{\natural}$ -concave functions and valuated matroids, arXiv: 1608.07021, <http://arxiv.org/abs/1608.07021>, August, 2016.
- Page 365, [39]: P. G. Doyle and J. L. Snell: *Random Walks and Electrical Networks*, Mathematical Association of America, Washington DC, 1984.
- Page 372, [143] (K. Murota): (1999)  $\implies$  (1998)
- Page 375, [192] (A. Shioura): (2003)  $\implies$  (2004)
- Page 376, [202]: D. M. Topkis,  $\implies$  D. M. Topkis:

#### Updates of bib-infor:

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- [156] should be changed to:  
K. Murota and A. Shioura: Fundamental properties of M-convex and L-convex functions in continuous variables, *IEICE Transactions on Fundamentals of Electronics, Communications and Computer Sciences*, **E87-A** (2004), 1042–1052.
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