Topological Relations of Arrow Symbols in Complex Diagrams

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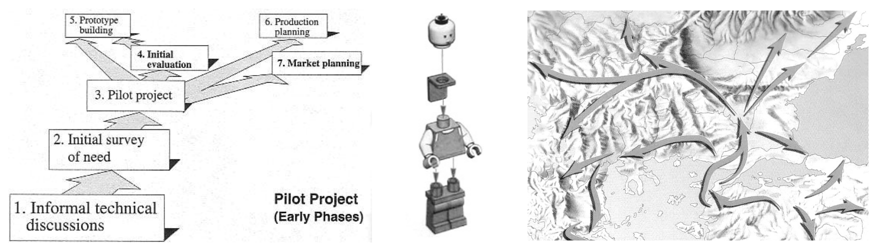
Abstract. Illustrating a dynamic process with an arrow-containing diagram is a widespread convention in people's daily communications. In order to build a basis for capturing the structure and semantics of such diagrams, this paper formalizes the topological relations between two arrow symbols and discusses the influence of these topological relations on the diagram's semantics. Topological relations of arrow symbols are established by two types of links, *intersections* and *common references*, which are further categorized into nine types based on the combination of the linked parts. The topological relations are captured by the existence/non-existence of these nine types of intersections and common references. Then, this paper demonstrates that arrow symbols with different types of intersections typically illustrate two actions with different interrelations, whereas the arrow symbols with common references illustrate a pair of semantics that may be mutually exclusive or synchronized.

1 Introduction

Illustrating a dynamic process with an arrow-containing diagram is a widespread convention in people's daily communications. Fig. 1a-c illustrate examples of diagrams for such dynamic processes as a workflow, an assembling procedure, and geographic propagations. If computers understand such arrow-containing diagrams, people can interact with computers more intuitively, for instance, by sketching a diagram on computer screens (Kurata and Egenhofer 2005a) to instruct the machines about the dynamic processes they will manage, support, or simulate.

Communication through an arrow-containing diagram requires the diagram readers to interpret the meaning of each arrow symbol in the diagram, because arrow symbols have a large variety of meanings (Horn 1998) and are used multi-purposely even in a single diagram without specification (Tversky, et al. in press). Such interpretations are not easy for computers (Kurata and Egenhofer 2005a), and sometimes even difficult for people without well-crafted context (Tversky, et al. in press). Kurata and Egenhofer (2005b) demonstrated that the interpretation of a diagram with a single arrow symbol can be partly derived from its syntactic pattern. People, however, often communicate using more complex diagrams with multiple arrow symbols (Figs. 1a-c). It, therefore, remains a challenging problem to develop a formal method for interpreting such complex arrow-containing diagrams. As a first step toward this goal, this paper analyzes the spatial relations between arrow symbols in such complex diagrams and observes the influence of the spatial relations on the diagram's semantics. Among several types of spatial relations, this paper focuses on topological relations (i.e., spatial relations that are not affected by elastic deformations), because topological information is highly influential in people's conceptualizations of space (Egenhofer and Mark 1995).



(a) (b) (c)

Fig. 1. Examples of complex arrow-containing diagrams that illustrate such dynamic processes as (a) a workflow (Horn 1998), (b) an assembling procedure (from a LEGO® manual), and (c) geographic propagation (Barraclough 2003).

In arrow-containing diagrams, arrow symbols are typically used together with other elements. A unit of an arrow symbol and the elements that the arrow symbol refers to (i.e., originates from, points to, or traverses) is considered a syntactic unit, called an *arrow diagram* (Kurata and Egenhofer 2005a). This paper extends this definition such that an arrow diagram is composed of one or more arrow symbols and the elements to which at least one of these arrow symbols refers. These arrow-related elements are called the *components* of the arrow diagram. Arrow diagrams are a subset of arrow-containing diagrams, since diagrams composed of arrow symbols alone are included in arrow-containing diagrams, but not in arrow diagrams.

An arrow diagram that contains *n* arrow symbols is called an *n-arrow diagram*. If n>1, the *n*-arrow diagram is also called a *multi-arrow diagram* (Fig. 2). The scope of this paper is to capture the meaningful structures embedded in such multi-arrow diagrams. Our premise is that such meaningful structures are sufficiently captured by a set of spatial relations between arrow symbol pairs. For simplifying the discussion, this paper deals with arrow symbols that neither intersect with themselves nor refer to the same component more than once.

The remainder of this paper is structured as follows: Section 2 reviews studies about line-line relations. Based on the formalization of topological line-line relations, Section 3 formalizes the topological relations between two arrow symbols, introducing two types of links that connect the arrow symbols directly or indirectly. Section 4 observes how such topological relations influence the semantics of multi-arrow diagrams. Section 5 demonstrates how this approach captures the structures and semantics of multi-arrow diagrams, using the example in Fig. 2. Section 6 concludes the discussion, pointing out some items for future research.

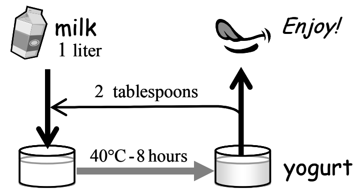


Fig. 2. An example of a multi-arrow diagram (4-arrow diagram), which illustrates the recursive process of producing yogurt from the mixture of milk and yogurt.

2 Models of Relations between Line Segments

An arrow symbol is essentially a directed line segment. Thus, the relations between arrow symbols are analogous to those between directed line segments, especially those embedded in a 2-dimensional space. Topological relations between two line segments have been studied extensively by the AI and spatial database communities. Allen (1983) distinguished 13 topological relations between two time intervals, which are essentially uni-directional line segments embedded in a 1-dimensional space. The *4-intersection model* (Egenhofer and Franzosa 1991) captured the topological relations between two spatial objects based on the existence/non-existence of geometric intersections of the objects' interiors and boundaries. The *9-intersection model* (Egenhofer and Herring 1991) extends the 4-intersection model by considering the intersections with respect to the objects' exteriors as well. In the 9-intersection model, the intersections between two spatial objects *A* and *B* are concisely represented by a 3×3 matrix (Eqn. 1), where A°, ∂A, and A^- are *A*'s interior, boundary, and exterior, while B°, ∂B, B^- are *B*'s interior, boundary, and exterior, respectively.

$$M(A,B)=\begin{pmatrix} A^\circ \cap B^\circ & A^\circ \cap \partial B & A^\circ \cap B^- \\ \partial A \cap B^\circ & \partial A \cap \partial B & \partial A \cap B^- \\ A^- \cap B^\circ & A^- \cap \partial B & A^- \cap B^- \end{pmatrix} \qquad \mathbf{(1)}$$

Topological relations between *A* and *B* are captured by the existence/non-existence of these nine types of intersections. Thus, the matrices with different empty/non-empty entries correspond to different topological relations. Although the matrix distinguishes $2^9 = 512$ configurations, the configurations with geometric realizations are limited by some geometric constraints. Based on this model, Egenhofer (1994a) identified 33 topological relations between non-directed line segments embedded in a 2-dimensional space.

Another variation of the 4-intersection model distinguishes two boundaries (start and end points) of directed line segments (Hornsby, *et al*. 1999). In that model, topological relations between two directed line segments *A* and *B* are represented by a 3×3 matrix (Eqn. 2), where $\partial_s A$, A°, and $\partial_e A$ are *A*'s start point, interior, and end point, while $\partial_s B$, B°, and $\partial_e B$ are *B*'s start point, interior, and end point, respectively. Based on this model, Hornsby *et al*. (1999) identified 16 topological relations between two time intervals in a cyclic time (essentially uni-directional line segments embedded in a cyclic 1-dimensional space).

$$M(A,B)=\begin{pmatrix} \partial_s A \cap \partial_s B & \partial_s A^\circ \cap B^\circ & \partial_s A \cap \partial_e B \\ A^\circ \cap \partial_s B & A^\circ \cap B^\circ & A^\circ \cap \partial_e B \\ \partial_e A \cap \partial_s B & \partial_e A \cap B^\circ & \partial_e A \cap \partial_e B \end{pmatrix} \quad (2)$$

Clementini and di Felice (1998) introduced another model of topological relations between two directed line segments embedded in a 2-dimensional space, called *classifying invariants*. Their model captures more detailed topological relations than the 9-intersection model, yielding such distinctions as relations with different number of interior-interior intersections.

Some researchers explored the relations between two line segments other than topological relations. Schlieder (1995) defined point-set ordering in a 2-dimensional space, with which he identified 63 order relations between two straight directed line segments in a 2-dimensional space. Moraz *et al*. (2000), on the other hand, identified 14, 24, and 69 directional relations between two straight directed line segments in a 2-dimensional space, based on a set of relative positions of one segment's endpoints seen from the other segment at three different granularities. Rentz (2001) distinguished 26 order relations between two directed intervals (essentially two directed line segments) in a 1-dimensional space. Nedas *et al*. (in press) incorporated two metric measures, *splitting ratios* and *closeness measures*, into both the 9-intersection matrix and the classifying invariants as their metric refinements, following the premise "*topology defines, metric refines*" (Egenhofer and Mark 1995).

3 Topological Relations between Two Arrow Symbols

Topological relations between two arrow symbols are established by two types of connections between these arrow symbols. One is *intersections*. Two arrow symbols may intersect with each other (Figs. 3a-b), as two line segments do. Thus, the topological relations between two arrow symbols are partly modeled in the same way as the relations between line segments are modeled based on their intersections. Another type of links is *common references*. A common reference of two arrow symbols is established when the arrow symbols refer to the same component (Figs. 3c-d). A common reference connects two arrow symbols indirectly through an intermediate component, while an intersection directly connects the arrow symbols. This analogy motivates us to model the topological relations between arrow symbols from a viewpoint of common references as well as that of intersections. In addition, since two arrow symbols are sometimes connected by both intersections and common references (Figs. 3e-f), the model for capturing both types of connections in a unified way is potentially useful. Sections 3.1 and 3.2, therefore, partly model the topological relations between arrow symbols based on intersections and common references, respectively, and Section 3.3 integrates the two models into a single hybrid model.

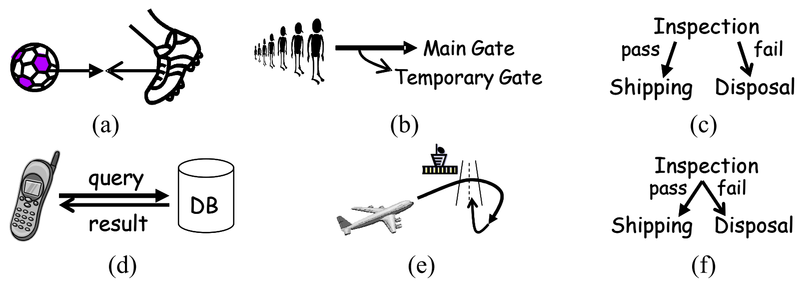


Fig. 3. Examples of 2-arrow diagrams where arrow symbols are connected by (a) a head-head intersection, (b) a body-tail intersection, (c) a common reference to the label "*Inspection*", (d) common references to the cell phone and database icons, (e) both a head-tail intersection and a common reference to the landing strip icon, and (f) both a tail-tail intersection and a common reference to the label "*Inspection*".

3.1 Topological Relations Established by Intersections

An arrow symbol consists of three different parts: *tail*, *body*, and *head*. The tail, body, and head are the rearmost point, interior, and headmost point of the arrow symbol, respectively. An arrow symbols corresponds to a time interval, since both are a kind of line segments with two qualitatively-different boundary points. Thus, following the 9-intersection model for time intervals (Hornsby, *et al*. 1999), this paper captures the topological relations between two arrow symbols A and B based on 3×3 = 9 types of their intersections: *tail-tail*, *tail-body*, *tail-head*, *body-tail*, *body-body*, *body-head*, *head-tail*, *head-body*, and *head-head intersections*. These nine types of intersections are concisely represented by a 3×3 matrix (Eqn. 3), where $\partial_{tail}A$, A°, and $\partial_{head}A$ are A's tail, body, and head, while $\partial_{tail}B$, B°, and $\partial_{head}B$ are B's tail, body, and head, respectively.

$$M_I(A,B)=\begin{pmatrix} \partial_{tail}A\cap\partial_{tail}B & \partial_{tail}A^\circ\cap B^\circ & \partial_{tail}A\cap\partial_{head}B \\ A^\circ\cap\partial_{tail}B & A^\circ\cap B^\circ & A^\circ\cap\partial_{head}B \\ \partial_{head}A\cap\partial_{tail}B & \partial_{head}A\cap B^\circ & \partial_{head}A\cap\partial_{head}B \end{pmatrix} \quad (\mathbf{3})$$

This matrix is called the *9-intersection matrix for arrow symbols*. The first, second, and third row correspond to A's tail, body, and head, while the first, second, and third column correspond to B's tail, body, and head, respectively. For example, since the two arrow symbols in Fig. 3b intersect only at one's body and another's tail, their 9-intersection matrix has only one non-empty element at $A^\circ\cap\partial_{tail}B$.

We first capture the topological relations between two arrow symbols by the existence/non-existence of these nine types of intersections alone. The existence/non-existence of each type of intersections is characterized by respective empty/non-empty entries in the 9-intersection matrix (Fig. 4).

$$
\begin{pmatrix} \phi & \phi & \phi \\ \phi & \phi & \phi \\ \phi & \phi & \neg\phi \end{pmatrix}
\quad
\begin{pmatrix} \phi & \phi & \phi \\ \neg\phi & \phi & \phi \\ \phi & \phi & \phi \end{pmatrix}
\quad
\begin{pmatrix} \phi & \phi & \phi \\ \phi & \phi & \phi \\ \phi & \phi & \phi \end{pmatrix}
\quad
\begin{pmatrix} \phi & \phi & \phi \\ \phi & \phi & \phi \\ \phi & \phi & \phi \end{pmatrix}
\quad
\begin{pmatrix} \phi & \phi & \neg\phi \\ \phi & \phi & \phi \\ \phi & \phi & \phi \end{pmatrix}
\quad
\begin{pmatrix} \neg\phi & \phi & \phi \\ \phi & \phi & \phi \\ \phi & \phi & \phi \end{pmatrix}
$$

(a) (b) (c) (d) (e) (f)

Fig. 4. The 9-intersection matrices that capture the topological relations between two arrow symbols in Figs. 3a-f only in terms of intersections.

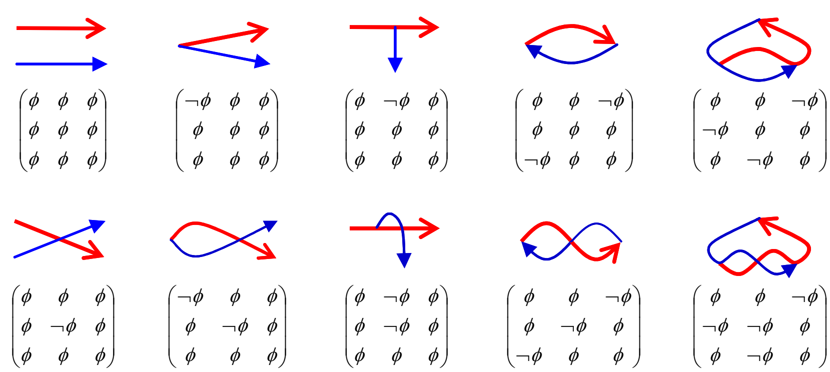
Although the 9-intersection matrix distinguishes $2^9 = 512$ configurations with different empty/non-empty entries, not all configurations have geometric realizations. The head or tail of an arrow symbol is a point, which cannot intersect with more than one part of another arrow symbol that does not intersect with itself. Since we assumed that no arrow symbol intersect with itself, this condition leads to the following constraint on the 9-intersection matrix for two arrow symbols:

- *The first column, third column, first row, and third row have at most one non-empty element.*

On the other hand, the center cell (i.e., $A^\circ \cap B^\circ$) can freely be empty or non-empty. Among the 512 potential configurations of matrices, only 68 configurations satisfy this constraint (Table 1). Each of the 68 configurations corresponds to a different topological relation between arrow symbols in terms of intersections, some of which are shown in Fig. 5.

Table 1. Number of configurations of the 9-intersection matrices with geometric realizations.

		Number of non-empty cells except $A^\circ \cap B^\circ$					
		0	1	2	3	4	
$A^\circ \cap B^\circ$	Empty	1	8	16	8	1	34
	non-empty	1	8	16	8	1	34
		2	16	32	16	2	68



$$
\begin{pmatrix} \phi & \phi & \phi \\ \phi & \phi & \phi \\ \phi & \phi & \phi \end{pmatrix}
\quad
\begin{pmatrix} \neg\phi & \phi & \phi \\ \phi & \phi & \phi \\ \phi & \phi & \phi \end{pmatrix}
\quad
\begin{pmatrix} \phi & \neg\phi & \phi \\ \phi & \phi & \phi \\ \phi & \phi & \phi \end{pmatrix}
\quad
\begin{pmatrix} \phi & \phi & \neg\phi \\ \phi & \phi & \phi \\ \neg\phi & \phi & \phi \end{pmatrix}
\quad
\begin{pmatrix} \phi & \phi & \neg\phi \\ \neg\phi & \phi & \phi \\ \phi & \neg\phi & \phi \end{pmatrix}
$$

$$
\begin{pmatrix} \phi & \phi & \phi \\ \phi & \neg\phi & \phi \\ \phi & \phi & \phi \end{pmatrix}
\quad
\begin{pmatrix} \neg\phi & \phi & \phi \\ \phi & \neg\phi & \phi \\ \phi & \phi & \phi \end{pmatrix}
\quad
\begin{pmatrix} \phi & \neg\phi & \phi \\ \phi & \neg\phi & \phi \\ \phi & \phi & \phi \end{pmatrix}
\quad
\begin{pmatrix} \phi & \phi & \neg\phi \\ \phi & \neg\phi & \phi \\ \neg\phi & \phi & \phi \end{pmatrix}
\quad
\begin{pmatrix} \phi & \phi & \neg\phi \\ \neg\phi & \neg\phi & \phi \\ \phi & \neg\phi & \phi \end{pmatrix}
$$

Fig. 5. Examples of 9-intersection matrices that represent the topological relations between two arrow symbols in terms of intersections. Components referred by arrow symbols are not shown in this diagram.

Among the 68 topological relations, 20 relations are symmetric, while 48 relations have a converse relation. Depending on the existence/non-existence of a body-body intersection, these 68 relations are divided into two halves with one-to-one correspondences between them. For example, in Fig. 5, the upper five relations, each without a body-body intersection, have one-to-one correspondences to the lower five relations, each with a body-body intersection.

3.2 Topological Relations Established by Common References

In a 1-arrow diagram, components are located in front of, behind, or along the arrow symbol; therefore, an arrow symbol defines three different areas where its components can be located, called the *head slot*, *tail slot*, and *body slot* of the arrow diagram (Kurata and Egenhofer 2005a) (Fig. 6). This structure is extended for multi-arrow diagrams, such that each arrow symbol in a multi-arrow diagram individually defines its three slots. Consequently, an n-arrow diagram has $3n$ slots. Since we assumed that an arrow symbol does not refer to the same component more than once, the three slots of one arrow symbol cannot overlap with each other, whereas the slots of different arrow symbols may overlap and contain the same component.

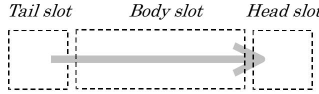


Fig. 6. Three component slots associated with each arrow symbol.

A *common reference* is established when two arrow symbols refer to the same component, called the *common component*. If two arrow symbols, A and B, have a common reference, their common component is contained in A's tail, body, or head slot, as well as in B's tail, body, or head slot. Accordingly, based on the combinations of the slots that contain the common component, common references are classfied into $3 \times 3 = 9$ types: *tail-tail*, *tail-body*, *tail-head*, *body-tail*, *body-body*, *body-head*, *head-tail*, *head-body*, and *head-head common references*. These nine types of common references between A and B are concisely represented, just like their intersections, by a 3×3 matrix (Eqn. 4), where $c_{tail}(A)$, $c_{body}(A)$, and $c_{head}(A)$ are the respective components in A's tail, body, and head slot, while $c_{tail}(B)$, $c_{body}(B)$, and $c_{head}(B)$ are the respective components in B's tail, body, and head slot.

$$M_C(A,B) = \begin{pmatrix} c_{tail}(A) \cap c_{tail}(B) & c_{tail}(A) \cap c_{body}(B) & c_{tail}(A) \cap c_{head}(B) \\ c_{body}(A) \cap c_{tail}(B) & c_{body}(A) \cap c_{body}(B) & c_{body}(A) \cap c_{head}(B) \\ c_{head}(A) \cap c_{tail}(B) & c_{head}(A) \cap c_{body}(B) & c_{head}(A) \cap c_{head}(B) \end{pmatrix} \quad (4)$$

This matrix is called the *9-CR matrix for arrow symbols*. Each cell shows the set of common components contained in each pair of slots. For instance, since both two arrow symbols in Fig. 3c contain *Inspection* in their tail slots, its 9-CR matrix has a non-empty element at $c_{tail}(A) \cap c_{tail}(B)$.

Since common references connect two arrow symbols in the same way intersections do, the topological relations between arrow symbols are captured in a similar way by the existence/non-existence of the nine types of common references. The existence/non-existence of the nine types of common references is characterized by respective empty/non-empty entries in the 9-CR matrix (Fig. 7). Although the 9-CR matrix distinguishes $2^9 = 512$ configurations with different empty/non-empty entries, not all these configurations have geometric realizations. The head or tail slot of an arrow symbol may contain more than one component (say, an icon and its caption), but it is unrealistic that these components belong to the different slots of another arrow symbol, since these components are located at the same (or undistinguishable) position pointed by the arrow symbol's head or tail. On the other hand, the components in the body slot of one arrow symbol can be contained in two different slots of another arrow symbol, since these components can be located at different positions. Since we assumed that no arrow symbol refers to the same component more than once, the following constraint on the 9-CR matrix is derived:

- *The first column, third column, first row, and third row have at most one non-empty element.*

This constraint is identical to that of the 9-intersection matrix for arrow symbols. Accordingly, among the 512 potential configurations of matrices, 68 configurations satisfy this condition. Each of these 68 configurations corresponds to different topological relation between arrow symbols in terms of common references.

$$\begin{pmatrix}\phi & \phi & \phi\\ \phi & \phi & \phi\\ \phi & \phi & \phi\end{pmatrix} \quad \begin{pmatrix}\phi & \phi & \phi\\ \phi & \phi & \phi\\ \phi & \phi & \phi\end{pmatrix} \quad \begin{pmatrix}\neg\phi & \phi & \phi\\ \phi & \phi & \phi\\ \phi & \phi & \phi\end{pmatrix} \quad \begin{pmatrix}\phi & \phi & \neg\phi\\ \phi & \phi & \phi\\ \neg\phi & \phi & \phi\end{pmatrix} \quad \begin{pmatrix}\phi & \phi & \phi\\ \phi & \phi & \phi\\ \phi & \neg\phi & \phi\end{pmatrix} \quad \begin{pmatrix}\neg\phi & \phi & \phi\\ \phi & \phi & \phi\\ \phi & \phi & \phi\end{pmatrix}$$

(a) (b) (c) (d) (e) (f)

Fig. 7. The 9-CR matrices that capture the topological relations between two arrow symbols in Figs. 3a-f in terms of common references.

3.3 Integration of 9-Intersection Matrix and 9-CR Matrix

Intersections and common references are analogous in the sense that both associate two arrow symbols by connecting the tail, body, or head of one arrow symbol to the tail, body, or head of another arrow symbol. Intersections and common references are, therefore, generically called *links*. Based on the combination of the connected parts of two arrow symbols, links are classified into $3 \times 3 = 9$ types: *tail-tail*, *tail-body*, *tail-head*, *body-tail*, *body-body*, *body-head*, *head-tail*, *head-body*, and *head-head links*. Each type of link is further categorized into *direct links* (= intersections) and *indirect links* (= common references).

The intersections and common references of two arrow symbols A and B are represented by two matrices $M_I(A,B)$ and $M_C(A,B)$, respectively. These two matrices are easily integrated into a single 3×3 matrix (Eqn. 5), where $m_{I\,ij}$ and $m_{C\,ij}$ are the respective elements of $M_I(A,B)$ and $M_C(A,B)$ at (i,j) ($i,j \in \{1,2,3\}$).

$$M_L(A,B)=\left[m_{L\,ij}\right]$$

$$m_{L\,ij}=\begin{cases}\phi & \text{if } m_{I\,ij}=\phi \text{ and } m_{C\,ij}=\phi\\ I & \text{if } m_{I\,ij}=\neg\phi \text{ and } m_{C\,ij}=\phi\\ C & \text{if } m_{I\,ij}=\phi \text{ and } m_{C\,ij}=\neg\phi\\ IC & \text{if } m_{I\,ij}=\neg\phi \text{ and } m_{C\,ij}=\neg\phi\end{cases} \quad \textbf{(5)}$$

$M_L(A,B)$ is called the *9-link matrix for arrow symbols*, because each cell of this matrix indicates the existence of links at each position pair and their types. For instance, the arrow symbol pairs in Figs. 3a-f correspond to the 9-link matrices in Figs. 8a-f. A merit of this hybrid matrix is that the topological relation between two arrow symbols is described by a single matrix even when two arrow symbols have both intersections and common references (Figs. 3e-f).

$$\begin{bmatrix}\phi&\phi&\phi\\\phi&\phi&\phi\\\phi&\phi&I\end{bmatrix}\quad\begin{bmatrix}\phi&\phi&\phi\\I&\phi&\phi\\\phi&\phi&\phi\end{bmatrix}\quad\begin{bmatrix}C&\phi&\phi\\\phi&\phi&\phi\\\phi&\phi&\phi\end{bmatrix}\quad\begin{bmatrix}\phi&\phi&C\\\phi&\phi&\phi\\C&\phi&\phi\end{bmatrix}\quad\begin{bmatrix}\phi&\phi&I\\\phi&\phi&\phi\\\phi&C&\phi\end{bmatrix}\quad\begin{bmatrix}IC&\phi&\phi\\\phi&\phi&\phi\\\phi&\phi&\phi\end{bmatrix}$$

(a) (b) (c) (d) (e) (f)

Fig. 8. The 9-link matrices that capture the topological relations between two arrow symbols in Figs. 3a-f in terms of both intersections and common references.

Since each cell is four-valued (ϕ, *I*, *C*, or *IC*), the 9-link matrix distinguishes $4^9 = 262{,}144$ configurations, although not all configurations have geometric realizations. The tail or head of an arrow symbol is a point and, therefore, cannot be linked to two different parts of another arrow symbol that neither intersect with itself nor refer to the same component more than once. The tail or head of an arrow symbol, however, may have both an intersection and a common reference with the same part of another arrow symbol (Fig. 3f). Thus, the following constraint on the 9-link matrix is derived:

- *The first column, third column, first row, and third row have at most one non-empty element.*

This constraint is, again, identical to that of the 9-intersection matrix for arrow symbols. Among the 262,144 potential configurations of matrices, 1,864 configurations satisfy this condition (Table 2), each of which corresponds to a different topological relation. Among the 1,864 topological relations, 184 relations are symmetric, while 1,680 relations have a converse relation.

Table 2. Number of configurations of the 9-link matrices with geometric realizations.

		Number of non-empty cells except $m_{L\,22}$					
		0	1	2	3	4	
$m_{L\,22}$	empty	1	8×3^1	16×3^2	8×3^3	1×3^4	466
	non-empty	1×3^1	8×3^2	16×3^3	8×3^4	1×3^5	1,374
		4	96	576	864	324	1,864

In this way, we developed a model for capturing topological relations between arrow symbols in a unified way. This model is called the *9-link model for arrow symbols*. The 9-link model deals with both direct links (intersections) and indirect links (common references) between spatial objects, whereas the 9-intersection model deals with the direct links.

4 Topological Relations and Semantics

The meaning of multi-arrow diagrams is influenced by the arrow symbols' relations. Our premise is that in a multi-arrow diagram each arrow symbol illustrates *atomic semantics* together with its related components and links of the arrow symbols indicates interrelation between these atomic semantics. This section discusses what kinds of interrelations between atomic semantics are indicated by intersections, common references, and their combinations.

Before starting the discussion, we have to be careful about the *nested structure* in a multi-arrow diagram. An arrow symbol in a multi-arrow diagram sometimes refers to an inner arrow diagram instead of individual components, thereby forming a nested structure. In Fig. 9a, for example, the arrow symbol departing from *El Niño* refers not to *Fish catch*, but to the inner arrow diagram composed of *Fish catch* and a downward arrow symbol, which illustrates the decrease of fish catch. Accordingly, this diagram illustrates a dynamic process that El Niño triggers the decrease of fish catch. The use of such nested structures enriches the representation ability of multi-arrow diagrams. A problem is that the existence of a nested structure is not visually distinctive—in multi-arrow diagrams with a nested structure, arrow symbols apparently have a common reference (Fig. 9a) or an intersection (Fig. 9b). To focus the study on the fundamental aspects of arrow diagrams, the following discussion only deal with multi-arrow diagrams without nested structures.

(a) (b)

Fig. 9. Multi-arrow diagrams with a nested structure, which apparently have (a) a common reference to the label "*Fish catch*" or (b) an head-body intersection.

4.1 Semantic Roles of Intersections

If two arrow symbols have intersections but no common references, each arrow symbol typically represents an *action* (i.e., movement of one component, sometimes triggered by or/and triggering an interaction with another component), and each intersection indicates an interrelation between such actions. In Fig. 3b, for example, each arrow symbol represents the movement to the main gate or temporal gate. Then, the body-tail intersection between these arrow symbols indicates that the movement to the temporal gate takes over a part of the mover to the main gate (i.e., a group of people). Like this example, each type of intersection indicates the following interrelations:

- A head-tail or tail-head intersection typically indicates that the tail-side arrow symbol completely takes over the mover of the head-side arrow symbol (Fig. 3e).
- A body-tail or tail-body intersection typically indicates that the tail-side arrow symbol partially takes over the mover of the body-side arrow symbol (Figs. 3b).
- A head-head intersection indicates that the movers of two arrow symbols meet and probably interact (Fig. 3a).
- A head-body or body-head intersection typically indicates that the mover of the head-side arrow symbol may merge with or influence the mover of the body-side arrow symbol.
- A tail-tail intersection typically indicates that the movers of two arrow symbols move away from each other, probably as a result of a certain event.
- A body-body intersection may indicate the interaction between the movers of two arrow symbols. Otherwise, the arrow symbols happened to cross at the body-body intersection.

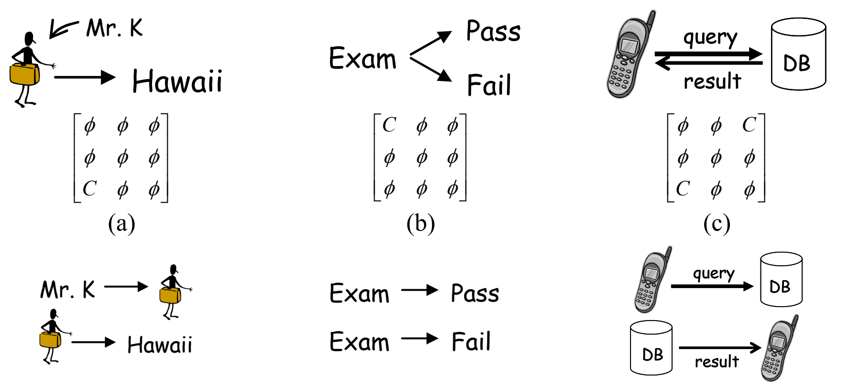
These correspondences between intersections and interrelations of actions are summarized in a 3×3 matrix (Eqn. 9), following the structure of the 9-intersection matrix for arrow symbols. This correspondence, however, does not mean that the existence of each intersection always leads to the corresponding interpretation. For instance, body-tail intersection is occasionally used to indicate alternative scenarios of actions or events, without mentioning *partial takeover*.

$$\begin{pmatrix} \text{separation} & \text{parital takeover} & \text{complete takeover} \\ \text{partial takeover} & \text{(interaction)} & \text{merge or influence} \\ \text{complete takeover} & \text{merge or influence} & \text{meet (+interaction)} \end{pmatrix} \qquad (\mathbf{6})$$

4.2 Semantic Roles of Common References

If two arrow symbols have common references but no intersections, the atomic semantics represented by these arrow symbols are inevitably interrelated in the sense that both atomic semantics refer to the common component. For example, the two atomic semantics of Fig. 10a—*the label "Mr. K" is assigned to the traveler* (which in turn is interpreted as *the traveler is Mr. K*) and *the person goes to Hawaii* (Fig. 10a') —are interrelated in the sense that they refer to the same traveler.

In addition to such weak interrelations, the atomic semantics may be strongly interrelated through mutual interference. For example, Fig. 10b illustrates *an exam results in pass or fail*, where its two atomic semantics, *an exam results in pass* and *an exam results in fail* (Fig. 10b'), are *mutually-exclusive* (i.e., they cannot be true at the same time). On the other hand, Fig. 10c illustrates *a cell phone sends a query to a database, which then returns a search result*, where its two atomic semantics, *a cell phone sends a query to a database* and *a database sends a query to a cell phone* (Fig. 10c'), are *synchronized* (i.e., whenever one is true, the other is also true). In this way, if arrow symbols have a common reference, the atomic semantics may be mutually-exclusive or synchronized. Which type of interference actually holds cannot be determined without background knowledge about, for example, whether the illustrated events may or must occur simultaneously. According to our observation, however, atomic semantics tend to interfere with each other when arrow symbols are symmetrically aligned (Figs. 10b-c). Consequently, symmetry of the 9-CR matrix may be useful for judging the possibility of such semantic interferences.



(a) (b) (c)

(a') (b') (c')

Fig. 10. (a-c) Examples of 2-arrow diagrams with common references and their 9-link matrices, and (a'-c') inner arrow diagrams of Fig. 11a-c, which illustrate the atomic semantics of the original 2-arrow diagrams.

4.3 Semantic Roles of Coexisting Intersections and Common References

If two arrow symbols have both an intersection and a common reference, which connect exactly at the same positions of these arrow symbols, these two types of links work essentially as a single common reference (Fig. 3f). This is because two arrow symbols sometimes intersect unnecessarily when referring to a common component, and accordingly it appears that both an intersection and a common reference connect the same positions of two arrow symbols. Consequently, when the 9-link matrix contains *IC* in any cell except the center, the symbol *IC* can be replaced by *I* without changing the diagram's semantics (Fig. 8f).

On the other hand, if different positions of two arrow symbols are connected by an intersection and a common reference (i.e., the 9-link matrix has *I* and *C* in different cells, or possibly *IC* in the center), these two links should have individual semantic roles. Thus, the intersection of such arrow symbols implies that these arrow symbols illustrate the interrelated actions. For example, Fig. 3e illustrates *an airplane flies over a landing strip and then lands on it*, where its head-tail intersection indicates that the tail-side arrow symbol completely takes over the mover of the head-side arrow symbol (i.e., an airplane). On the other hand, the common reference of the arrow symbols always implies the *synchronization* of two actions, since the actions must occur simultaneously or continuously in order to have an interrelation. For example, the atomic semantics of Fig. 3e, *an airplane flies over a landing strip* and *something lands on a landing strip* (Fig. 11), must be synchronized such that the latter action takes over the mover of the former actions (i.e., the airplane). In this way, the combination of an intersection and a common reference at different positions indicate that the arrow diagram illustrates two interrelated and synchronized actions.

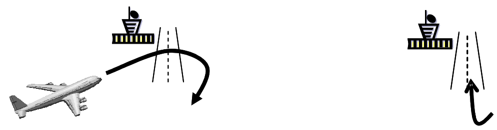
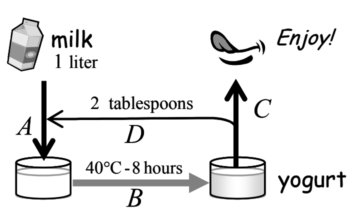


Fig. 11. Inner arrow diagrams of the 2-arrow diagram in Fig. 3e.

5 An Example

Let the four arrow symbols in Fig. 2 be called *A*, *B*, *C*, and *D*, as shown in Fig. 12a. The topological relations between pairs of these arrow symbols are represented by six 9-link matrices (Fig. 12b). Since *D* intersects with *A* and *C*, it is automatically determined that *A*, *C*, and *D* individually illustrate a certain action (i.e., movement of one component sometimes accompanying an interaction). On the other hand, *B* illustrates a *change* of ingredients, which is not an action. The body-head intersection between *A* and *D* indicates that *D*'s mover (*2 tablespoons of yogurt*) merges with or influences A's mover (*milk*). Since *A* and *B* have a head-tail common reference, their semantics are interrelated in the sense that they refer to the same container, although these semantics are not mutually-exclusive or synchronized. Similarly, *B* and *C* have a head-tail common reference, which implies an interrelation between their semantics in terms of subject-sharing. Finally, the body-tail intersection between *C* and *D* indicates that *D* partially takes over *C*'s mover (*yogurt*). This interpretation is supported by the label on *D* (*2 tablespoons*).



(a)

$$M_L(A,B)=\begin{bmatrix}\phi & \phi & \phi\\ \phi & \phi & \phi\\ C & \phi & \phi\end{bmatrix}\quad M_L(A,C)=\begin{bmatrix}\phi & \phi & \phi\\ \phi & \phi & \phi\\ \phi & \phi & \phi\end{bmatrix}$$

$$M_L(A,D)=\begin{bmatrix}\phi & \phi & \phi\\ \phi & \phi & I\\ \phi & \phi & \phi\end{bmatrix}\quad M_L(B,C)=\begin{bmatrix}\phi & \phi & \phi\\ \phi & \phi & \phi\\ C & \phi & \phi\end{bmatrix}$$

$$M_L(B,D)=\begin{bmatrix}\phi & \phi & \phi\\ \phi & \phi & \phi\\ \phi & \phi & \phi\end{bmatrix}\quad M_L(C,D)=\begin{bmatrix}\phi & \phi & \phi\\ I & \phi & \phi\\ \phi & \phi & \phi\end{bmatrix}$$

(b)

Fig. 12. (a) The 4-arrow diagram in Fig. 2 with arrow symbol identifiers *A*-*D* and (b) the 9-link matrices representing the topological relations between pairs of these arrow symbols.

6 Conclusions

People often communicate dynamic information through multi-arrow diagrams. This paper formalized the topological relations of arrow symbols embedded in such multi-arrow diagrams from two viewpoints: intersections and common references. Then, we observed how these topological relations influence the semantics of multi-arrow diagrams. This work forms a basis for future research toward a computational model for interpreting complex arrow-containing diagrams. In order to achieve this goal, further extensions are needed to detect nested structures and to determine the type of semantic interference implied by common references. In addition, the correspondence between topological relations and semantics should be carefully examined with more examples of arrow diagrams and some systematic human subject experiments. Another item for future research is constructing an interpretation framework that may bind the relation between arrow symbols, instead of assigning a fixed semantics to each topological relation based on examples.

Since this paper limited the target to topological relations between arrow symbols, it should be meaningful to study the influence of other spatial relations, such as metric or directional relations, to the semantics.

Our discussions implicitly assumed that an arrow symbol always refers to the entire component. Arrow diagrams, however, sometimes have a *hierarchical common reference*, where one arrow symbol refers to a component while another arrow symbol refers to a part of the same component (Fig. 13). This setting leads to the possibility to extend our 9-link model, which will capture more detailed topological relations between arrow symbols.

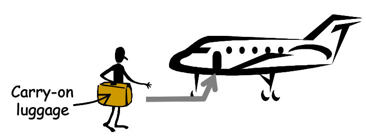


Fig. 13. An arrow diagram with a hierarchical common reference. The thicker arrow symbol refers to a person carrying a bag, while the narrower arrow symbol refers only to the bag.

Another challenging problem is to analyze the conceptual neighborhoods of the topological relations between arrow symbols that we identified and to develop their composition table on these relations as a basis for qualitative spatial reasoning. Such problems are well-studied for regions (Egenhofer 1994b, 2005), but not yet for arrow symbols and even for directed line segments, which are often represented by arrow symbols. The study of these problems would probably leads to the findings of interesting properties of the topological relations between arrow symbols.

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