Structure and Semantics of Arrow Diagrams

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Abstract. Arrows are major components of diagrams, where they are typically used to facilitate the communication of spatial and temporal knowledge. An automated interpretation of arrow diagrams would be highly desirable in pen-based interfaces. This paper develops a method for deducing possible interpretations of arrow diagrams, which is composed of a uni-directional arrow symbol and one or more components. Based on a study of the use of arrow diagrams, we classify their semantics into *properties*, *annotations*, *actions*, and *conjunctions*. Then, we discuss the structural requirements of arrow diagrams for illustrating each class of semantics, as well as the structural rules for adding optional components. Finally, we investigate all possible structures of simple arrow diagrams for each class of semantics and demonstrate that knowledge about the structure of an arrow diagram reduces the ambiguity of its interpretation.

1 Introduction

Diagrams are frequently used in people's daily communications. If computers could understand diagrams as well, then people could operate a computer system more intuitively, for instance, by sketching diagrams. Indeed, a number of pen-based systems that understand human-sketched diagrams have been developed in various fields, and their usefulness has been reported repeatedly (Oviatt 1996, Egenhofer 1997, Landay and Myers 2001, Davis 2002, Ferguson and Forbus 2002). Thus, computational diagram understanding is one of the highly prospective technologies for enriching human-computer interactions.

Arrows are major components of diagrams. Arrows appear in various types of diagrams, such as traffic signs, guideboards, route maps, flowcharts, and illustrations (Horn 1998, Wildbur and Burke 1998). One reason for such popularity is that arrows capture a large variety of semantics with their simple shape (Section 2). Another reason is that the existence of arrows encourages people to interpret causal and functional aspects in a diagram (Tversky *et al.* 2000). For instance, Fig. 1 contains only a few words and some arrow symbols over a background map, but people easily read the mechanism of a spatio-temporal process—the El Niño effect in the Southeastern Pacific Ocean indirectly influences the rise of tofu price in Japan. In this way, arrows are powerful tools that facilitate the communication of spatial and temporal knowledge in a static diagram.

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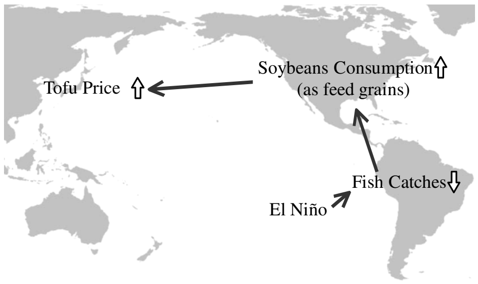


Fig. 1. A diagram with arrows, which illustrates a spatio-temporal process that the El Niño effect (i.e., sea temperature rise in the Southeastern Pacific Ocean) indirectly influence the rise of the tofu price in Japan

People can interpret such diagrams almost intuitively. Interpretation of arrow-containing diagrams is, however, a difficult task for computers due to the polysemy of arrows. For example, in Fig. 1, the arrow symbol departing from *El Niño* could be interpreted as a *spatial movement* (i.e., *El Niño is approaching South America*) or an *annotation* (i.e., *a fish species whose catches are declining is El Niño*), without any knowledge about *El Niño*. To avoid such misinterpretations, the current pen-based systems require their users to specify the meaning of every arrow (Forbus and Usher 2002) or restrict the meaning of arrows to a small set (Alvarado and Davis 2001, Landay and Myers 2001, Kurtoglu and Stahovich 2002). Consequently, people cannot illustrate their knowledge naturally in current pen-based systems. An automated interpretation of arrow-containing diagrams, therefore, remains a challenging problem for developing useful pen-based interfaces.

This paper develops a method for deducing possible interpretations of arrow-containing diagrams. Here, the *interpretation* of a diagram is referred to as (1) a process to determine the semantics of a diagram by reasoning, or (2) the determined semantics itself. The *semantics* of a diagram is referred to as a state or behavior of an entity or entities, which is represented by a diagram. A diagram contains symbols, each of which is assigned to an entity or its feature, which is called the *meaning* of the symbol. The semantics of a diagram is built from the meaning of each symbol in the diagram. We assume that the meaning of each symbol except arrows is already known.

For simplification, we do not consider the appearance of arrow symbols. Although the appearance of an arrow symbols is subject to a variety of visual variables (Bertin 1983), the arrow symbol alone rarely determine any specific meaning. Its semantic role is usually established when one or more arrow symbols depart from, traverse, or point to other elements in the diagram. Thus, we focus on the semantics associated with these related elements.

The combination of arrow symbols and the related elements is considered as a unit of syntax, and called *an arrow diagram*. Then, these arrow-related elements are called the *components* of the arrow diagram. An arrow diagram must have at least one arrow symbol and one component. As a first step, we consider the arrow diagram that contains only a uni-directional arrow symbol and its related elements. Bi-directional arrows are not considered, because they are regarded as a synthesis of two oppositely-directed arrow symbols. Also, independent arrow symbols, such as arrow-shaped traffic signs indicating curving roads and map symbols indicating north direction, are not discussed in this paper, because they have no components.

The remainder of this paper is structured as follows: Section 2 investigates various uses of arrow diagrams, and classifies their semantics into four different classes. Section 3 introduces a framework for the structure of arrow diagrams, based on the alignment of components and their semantic types. Section 4 discusses the structural requirements for illustrating each class of semantics, and Section 5 discusses the structural conditions of optional components. With these two bases, Section 6 investigates all structures of simple arrow diagrams that possibly illustrate each class of semantics, and demonstrates that the possible interpretations of arrow diagrams are determined simply by their structures. Finally, Section 7 presents conclusions and future work.

2 Semantics of Arrow Diagrams

What are arrows? Tversky (2001) defined an arrow as a special kind of line, with one end marked, inducing an asymmetry. Linearity and asymmetry are essential features of arrows. Asymmetry makes it possible to represent a direction or an order, whereas linearity makes it possible to represent a length, a path, or a connection. With these two features, arrow diagrams illustrate a large variety of semantics.

2.1 Use of Arrow Diagrams

One of the primary usages of arrow diagrams is to express a *direction*. Gombrich (1990) reported a very early example of an arrow diagram, where an arrow symbol was used to represent the direction of a water stream (Fig. 2a). Arrow diagrams may also refer to metaphorical directions. Upward directions are metaphorically associated with increase or improvement, whereas downwards directions are associated with decrease or debasement (Lakoff and Johnson 1980). Accordingly, upward and downward arrow symbols are used to illustrate those semantics.

An arrow symbol illustrating a direction has a diagrammatic freedom of length. This freedom allows the representation of a directed quantity, which is called a *vector*. A vector is a quantity that is specified by a direction and a magnitude.

Another traditional usage of arrow diagrams is to illustrate a *spatial movement*. Spatial movement is an event where an entity changes its spatial position continuously. Bertin (1983) showed a classic example that illustrates the paths of several expeditions in the Sahara Desert (Fig. 2b) and claimed that an arrow is the most efficient (and often the only) formula for representing a complex movement. The linearity and the asymmetry of an arrow symbol are appropriate features for illustrating the path and the, direction of the spatial movement respectively.

In geography, the flow of people, goods, or services between specific locations is called a *spatial interaction* (Bailey and Gatrell 1995). Since a spatial interaction is an aggregation of spatial movements, an arrow diagram can illustrate a spatial interaction just like a spatial movement, although the detailed route is often abbreviated due to the lack of the designer's concern (Fig. 2c). Monmonier (1990) reports that arrows are particularly useful for showing such spatial interactions as migration streams, spatial diffusion of ideas, migrations of tribes and refugees, and advance of armies. The scale of a spatial interaction is usually illustrated by the width of the arrow symbol, because people typically perceive the width of lines without a bias (Robinson *et al.* 1995). Spatial interactions can be generalized into *interactions*, which refer to the flow of a certain item from one entity to another entity, such that these two entities communicate indirectly. The communicating entities are not limited to locations; they can be other entities, such as people and physical objects (Fig. 2d).

Arrow diagrams are used not only in a spatial context, but also in a temporal context. For instance, timetables often contain arrow diagrams, each of which illustrates that something continues over a certain period (Fig. 2e). This semantics is called *temporal continuity*. People understand the concept of time with the aid of spatial metaphors (Lakoff and Johnson 1980). Temporal continuity is understood with the metaphor of travel, in the sense that something travels continuously over time instead of space. Therefore, arrow diagrams that originally describe travel in space (i.e., spatial movement) are naturally extended to describe a travel in time (i.e., temporal continuity).

Similarly, flowcharts often contain arrow diagrams, each of which captures a *temporal order* of two components (Fig. 2f). The order is distinctively expressed by the directionality of an arrow symbol. The connected two components in an arrow diagram refer to (1) two different entities or (2) two different states of an identical entity. In the former case, the arrow diagram may imply a *conditional relation* or a *causal relation* of the associated components, such that the proceeding component works as a precondition or a cause of the subsequent component. In the latter case, the arrow diagram may imply a *change*. A change is an event where an entity transforms its features, such as identity (Hornsby and Egenhofer 2000), appearance, name, and structure. For instance, a chemical equation, such as $2H_2+O_2 \rightarrow 2H_2O$, illustrates a set of materials transforming its molecular structures.

Arrow diagrams are also used in complicated illustrations, where each arrow symbol connects an element of the illustration with a short description (Fig. 2g). This use is called *labeling*. Arrow symbols or lines are often used for labeling when the illustration becomes messy if descriptions are placed directly on it. Although both an arrow symbol and a line can connect an element and its description, the directionality of the arrow symbol makes it clear that the description is assigned to the element, and accordingly people can distinguish the element and its description more easily.

Finally, the use of arrow diagrams to illustrate *relations* is a widespread convention in sketches (Forbus and Usher 2002). An arrow diagram visualizes the presence of a directed relation between two components. In mathematics, a set of directed binary relations is modeled as a directed graph, which is often visualized using arrow diagrams (Fig. 2h). Directed binary relations are a broad concept that includes temporal orders, conditional relations, causal relations, labeling, and so on.

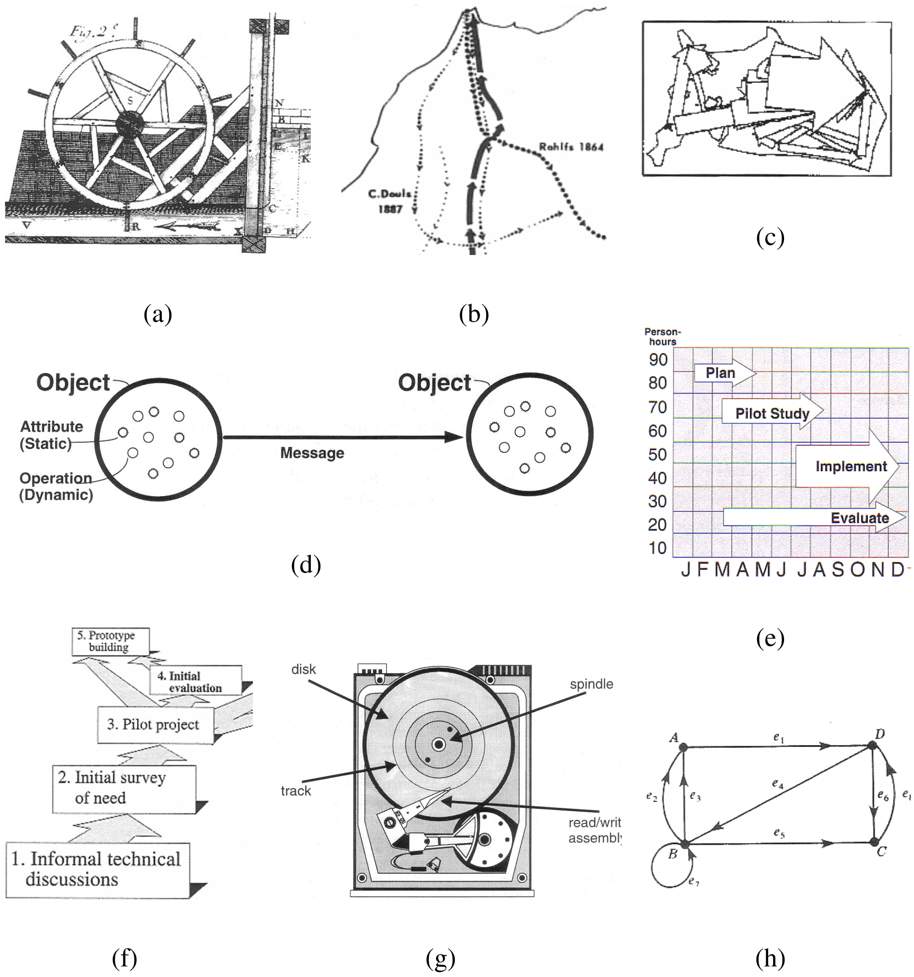


Fig. 2. Examples of arrow diagrams illustrating various semantics: (a) a direction (Gombrich 1990), (b) spatial movements (Bertin 1983), (c) spatial interactions (Tobler 1987), (d) an interaction (Worboys and Duckham 2004), (e) temporal continuities (Horn 1998), (f) temporal orders (Horn 1998), (g) labeling (Worboys and Duckham 2004), and (h) ordered relations (Lipschutz and Lipson 1997)

2.2 Classification of Arrow Diagram Semantics

For the following discussions, the semantics of arrow diagrams are classified into four different classes (Table 1).

First, the arrow-related semantics are divided into semantics that require only one component and semantics that require two or more components (Fig. 3). In the former

Table 1. Classification of the semantics of arrow diagrams

Example of semantics	Class	Definition
Direction, vector	Property	modification of a component by attaching an arrow symbol to the component
labeling	Annotation	modification of a component by connecting a description to the component
Spatial movement, temporal continuity, interaction	Action	a motion of one component that may be caused or cause an interaction with another component.
temporal order, conditional relation, causal relation, change, ordered relation	conjunction	association of components, where an arrow symbol does not express a motion

case, the arrow symbol is attached to a component and modifies it directly. Such semantics is called a *property*. Directions and vectors are examples of properties. Although an arrow diagram illustrating a property sometimes contains two components, only one of them is essential for illustrating that property (Section 5.2).

Among the arrow-related semantics that require two or more components, *labeling* is exceptional, because other semantics associate at least two independent components that do not modify each other, while labeling introduces one component only for modifying another component. Thus, labels are distinguished from the other semantics (Fig. 3) and referred to as an independent class of semantics called *annotation*. An annotation is defined as a modification of a component by connecting the component and its description. Annotations and properties are similar in the sense that both of them modify one component, although their representation styles are different.

The remaining arrow-related semantics are divided into *actions* and *conjunctions* (Fig. 3). Actions are the semantics where an arrow symbol illustrates a motion of one component that may be caused or cause an interaction with another component. Spatial movements, temporal continuities, and interactions are classified into actions, since these semantics refer to a spatial or temporal movement. On the other hand, a conjunction is an association of components, where an arrow symbol does not express a motion. Accordingly, the shape of an arrow symbol is often meaningful for an action, while it is not for a conjunction.

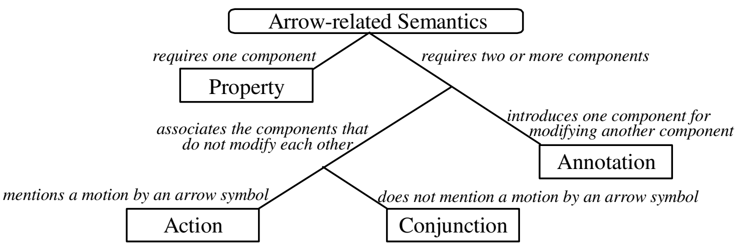


Fig. 3. Classification of arrow-related semantics

3 Structures of Arrow Diagrams

Arrow diagrams illustrate a large variety of semantics, which increases the difficulty of their interpretation. To tackle this problem, we first develop a method for making rough interpretation of an arrow diagram from its structural pattern alone. As the foundation of this method, this section summarizes the formal structures of arrow diagrams developed by Kurata and Egenhofer (2005).

The *components* of an arrow diagram are diagrammatic elements that an arrow symbol originates from, traverse, or points to. We classify the components of arrow diagrams into the following five types:

- An *object* takes an action, either independently (e.g., a person in Fig. 4a) or as a result of interaction (e.g., a bag in Fig. 4a).
- An *event* occurs in time, and is characterized by a set of changes. An event occurs over an interval (e.g., snow in Fig. 4b) or at an instant (e.g., a traffic accident in Fig. 4b).
- A *location* is a position in space. It may be a point (e.g., a place in Maine in Fig. 4c) or a homogeneous area (e.g., Maine).
- A *moment* is a position in time. It may be an instant (e.g., *8:20* in Fig. 5a) or a homogeneous interval (e.g., morning).
- A *note* is a short description that modifies an arrow symbol (e.g., *send* in Fig. 4a) or another component (e.g., *Mr. K* in Fig. 4a and *You are here* in Fig. 4c). A note and the modified component are placed adjacently to each other or connected with each other by an arrow symbol.

For simplification, an object, an event, a moment, a location, and a note are sometimes denoted as *O*, *E*, *M*, *L*, and *N*, respectively.

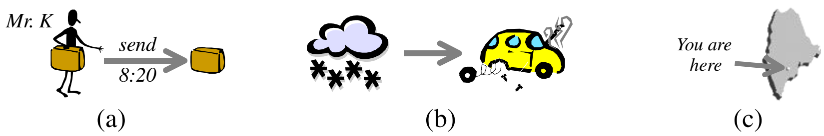


Fig. 4. Arrow diagrams that contain various types of components

Although a component may be mentioned by an icon, a text, or a specific position in a background drawing, this classification is not concerned with such a descriptive style of the component. We assume that automated interpreters of arrow semantics will distinguish these component types based on their knowledge base.

Components of an arrow diagram are located in front of the arrow's head, behind the arrow's tail, or along the arrow's body. We, therefore, consider that an arrow symbol identifies three different areas where the components of the arrow diagram are located (Fig. 5). These areas are called *component slots*, and they are further classified into a *tail slot*, a *head slot*, and a *body slot*. Each component in an arrow diagram is uniquely assigned to one of these three slots, thereby making the distinction of *tail components*, *body components*, and *head components*. The component slots need to be distinguished, because the same symbols, used in different slots, illustrate significantly different semantics (Kurata and Egenhofer 2005).

An arrow diagram has three slots, where five types of components may be located. The combination of the types of components in the three slots composes a certain pattern that is specific to every arrow diagram. These patterns are described as

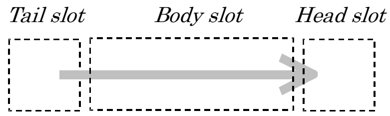


Fig. 5. Three component slots associated with an arrow symbol

([*M*|*E*|*L*|*O*|*N*]*, [*M*|*E*|*L*|*O*|*N*]*, [*M*|*E*|*L*|*O*|*N*]*), where [*x*]* means empty or a sequence of any number of *x*, *x*|*y* means *x* or *y* but not both, and the three elements in parentheses indicate the types of components in tail, body, and head slot, respectively. For example, the structures of arrow diagrams in Fig. 4a-c are described as (*ON*, *MN*, *O*), (*E*, –, *E*), and (*N*, –, *L*), respectively. These patterns capture fundamental structures of arrow diagrams, since they capture the alignment of components while abstracting the individual difference and the absolute location of the components.

4 Structural Requirements of Arrow Diagrams

This section discusses the structures of arrow diagrams that are required for illustrating certain semantics. The discussion follows the classification of the arrow-related semantics in Section 2.

4.1 Structural Requirements for Illustrating a Property

When an arrow diagram illustrates a property, its arrow symbol is tied to only one component. Consequently, all visual variables of the arrow symbol, such as length, width, shape, color, orientation, and pattern (Bertin 1983), can be controlled by its designer. Among these variables, length and orientation are predominant due to the linearity and asymmetry of arrow symbols. Arrow symbols are, therefore, appropriate for representing some feature that is related to a length, an orientation, or both. A length is, however, illustrated more easily by a line or a bar. Consequently, arrow symbols are exclusively used to illustrate (1) properties that are specified by an orientation (i.e., directions) or (2) properties that are specified by both an orientation and a length (i.e., vectors).

The component whose property is described by an arrow symbol is called a *subject*. The subject can be placed in any slot, since nothing conflicts with the subject (Fig. 6a-c). The different positions of the subject, however, yield slightly different semantics:

- if the subject is placed in the tail slot (Fig. 6a) the illustrated property may be related to an outgoing action,
- if the subject is placed in the body slot (Fig. 6b) the illustrated property may be related to a passing-through action, and
- if the subject is placed in the head slot (Fig. 6c) the illustrated property may be related to an incoming action.

Thus, a moving direction of a car, a wind direction, and a direction of an external force are distinctively illustrated by arrow diagrams with these different structures (Fig. 6a'-c').

The arrow diagrams in Fig. 6a'-c' indicate that an object, a location, and an event may represent a subject of a property (i.e., they can have a direction or a vector). A moment and a note cannot have these properties, since the moment is a one-dimensional concept and the note is a subsidiary component.

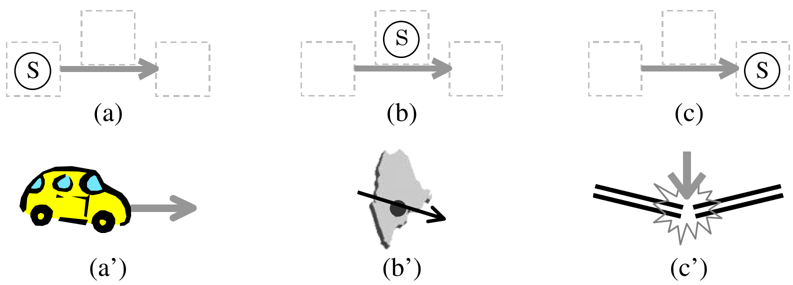


Fig. 6. (a-c) Basic structures of arrow diagrams for illustrating a property of a subject (S). (a'-c') Examples of the corresponding arrow diagrams, which illustrate: (a) a moving direction of a car, (b) a wind direction, and (c) a direction of an external force

4.2 Structural Requirements for Illustrating an Annotations

An arrow symbol may illustrate an annotation by connecting a component and its description (Fig. 7). Annotation and properties are similar in the sense that both modify a component. Thus, the annotated component is called a *subject* by analogy.

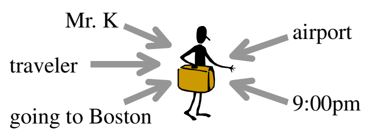


Fig. 7. Examples of arrow diagrams illustrating annotations. Annotations describe various features of a subject, such as name (*Mr. K*), category (*traveler*), status (*going to Boston*), spatial position (*airport*), and temporal position (*9:00am*).

When illustrating an annotation, an arrow diagram takes only one type of structure with regard to a description and a subject in order to specify that the description is assigned to the subject (Fig. 8).

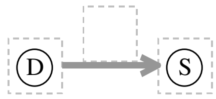


Fig. 8. Basic structure of arrow diagrams for illustrating an annotation (D: description; S: subject)

The subject in annotations is represented by any type of component except for notes, since notes are subsidiary. The description is represented by a note, a location, or a moment. A location and a moment specify the spatial and temporal position of the subject, respectively (e.g., *airport* and *9:00am* in Fig. 7), whereas a note describes other feature of the subject, such as name, category, and status (*Mr. K*, *traveler*, and *going to Boston* in Fig. 7).

4.3 Structural Requirements for Illustrating an Action

In this section, we show how different types of actions can be modeled with arrow diagrams eliciting different types of structures.

In a primitive sense, an arrow is a flying weapon that moves in space. Naturally, an arrow diagram is used to illustrate a *spatial movement* of an entity. In addition, an arrow diagram is analogically used to illustrate a *temporal continuity* of an entity, because an entity travels in time, just like it travels in space.

The movement of an entity may accompany another action. If there is another entity in the way of the moving entity, it implies that the moving entity gets into contact with this entity. This type of semantics is called an *encounter*. An encounter is an action where an entity (*mover*) physically or conceptually moves, and eventually has a contact with another entity (*receiver*). Encounters are described by such verbs as *approach*, *enter*, *join*, *conflict*, *receive*, *consume*, *mount*, and *import*.

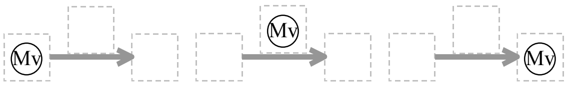
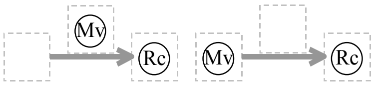
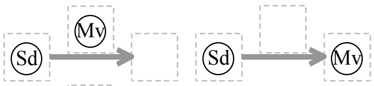
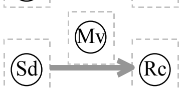
An encounter refers to two entities getting together. Conversely, a *division* occurs when two entities become separated. It is an action where an entity (*mover*) moves away from another entity (*sender*). Divisions are described by such verbs as *exit*, *withdraw from*, *branch off*, *leave*, *produce*, *uninstall*, and *export*.

The third possibility is the combination of a division and an encounter, that is, an action where an entity (*mover*) moves away from another entity (*sender*), and then gets into contact with the third entity (*receiver*). This semantics is called a *ditransitive action*, since it is analogous to a ditransitive verb in linguistics, such as *send, give*, *show*, *tell*, *explain*, *sell*, and *buy*. Ditransitive verbs refer to two types of objects: (1) *direct objects*, which receive the subject's action directly, and (2) *indirect objects*, which receive the subject's action indirectly. The direct objects intermediate the subject and the indirect objects. If this intermediation accompanies physical or conceptual movement, the semantics can be represented diagrammatically with an arrow diagram as a ditransitive action.

Any of the above five types of actions requires a mover. In addition, an encounter requires a receiver, a division requires a sender, and a ditransitive action requires both a sender and a receiver. The sender and the receiver must be placed in the tail and the head slot, respectively, since arrow's tail and head imply the initial and final positions of a motion. Meanwhile, the mover should be placed in any slot that is not occupied by the sender or the receiver, so that the mover can be visually distinguished from them. Thus, this classification of actions yields eight different structures of arrow diagrams with regard to a mover, a sender, and a receiver (Table 2).

A mover is represented by an object or an event, because objects and events are the only components that may move in space or time. A sender is represented by an object, an event, or a location. For example, an industrial plant (object), deforestation (event), and a volcano (location) can send out air pollutants (mover). Similarly, a receiver is represented by an object, an event, or a location. For example, a famous statue (object), a festival (event), and a historic site (location) can receive a tourist (mover). Arrow diagrams can be applied to illustrate all of these scenarios.

Table 2. Basic structures of arrow diagrams for illustrating actions with regard to a mover (Mv), a sender (Sd), and a receiver (Rc)

Semantics	Structural patterns
spatial movement / temporal continuity	
Encounter	
Division	
Ditransitive relation	

Spatial movement also needs at least one component in an arrow diagram that specifies its origin, route, or destination. Similarly, temporal continuity needs at least one component that specifies its start-time or end-time. The origin, route, and destination are represented by locations in the tail, the body, and the head slot, respectively (Fig. 9a), while the start-time and end-time are represented by moments in the tail and the head slot (Fig. 9 b). These components are referred to as *adverbial components* (Section 5.2).

(a) (b)

Fig. 9. Arrow diagrams illustrating an action contains adverbial components, which specify (a) the origin, the route, and the destination, and (b) the start-time and the end-time of the action

4.4 Structural Requirements for Illustrating a Conjunction

An arrow diagram may illustrate a conjunction, where multiple components are associated without referring to a motion. For example, *Lobster→Maine* is a conjunction where lobster is associated with Maine. Every conjunction has a certain theme that justifies the association (Table 3). In *Lobster→Maine*, lobster and Maine are associated, for instance, under the theme of local specialty. The theme may be specified in a caption, a legend, or an *adverbial component* that modifies the arrow symbol (Section 5.2), or may be infered from the context.

Since an arrow symbol is oriented, the associated components are naturally ordered. Therefore, if a certain rationale is available, the associated components should be aligned in a meaningful way. The underlying theme often provides such a rationale as temporal order (e.g., from old to new), spatial order (e.g., from high to low, from front to back, and from part to whole), logical order (e.g., from presumption to conclusion), or thinking order (e.g., first comes into head, first placed) (Table 3).

Table 3. In conjunctions, components are associated under a certain theme and aligned under a certain rational

Associated Components	Theme	Rationale of Alignment	Representation
Lobster, Maine	local specialty	Thinking order	Lobster→Maine
Plan, Do, See	work process	temporal order	Plan→Do→See
Niagra Falls, Lake Ontario	water flow	spatial order (high to low)	Niagra Falls→Lake Ontario
Maine, New England	geographic attribution	spatial order (part to whole)	Maine→New England
Snow, Delay	causal relation	logical order	Snow→Delay

The associated components are placed in the tail and the head slot of an arrow diagram, such that these components look equally associated, as well as the order among these components are highlighted (Fig. 10). Any type of component, except notes, can be the associated components, as long as people identify the theme that associates these components.

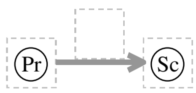


Fig. 10. Basic structure of arrow diagrams for illustrating conjunctions (Pr, Sc: the associated component that proceeds and succeeds in an order, respectively)

5 Optional Components of Arrow Diagrams

The previous section discussed the structural requirements for illustrating each class of semantics. In addition to the components requested by these requirements, arrow diagrams may have the following two types of optional components.

5.1 Adjective Components

An *adjective component* of an arrow diagram modifies an individual component of the arrow diagram. It is analogous to adjectives in linguistics, which modify an individual noun. The adjective component describes a feature of the component, such as its category, name, scale, spatial position, and temporal position—just as descriptions in annotations do.

The component and its adjective component are placed adjacently to each other and, therefore, they are always placed in the same slot (Fig. 11). An adjective component is represented by a location, a moment, or a note, each of which illustrates the place, time, and other features of the modified component.

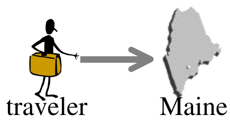


Fig. 11. An arrow diagram that contains adjective components (*traveler* and *Maine*)

5.2 Adverbial Components

An *adverbial component* of an arrow diagram modifies the main semantics that the arrow diagram illustrates. It is analogous to adverbs and adverbial phrases in linguistics, which modify a verb. Properties, actions, and conjunctions may have adverbial components in arrow diagrams, whose functions are as follows:

- An adverbial component of a property describes a feature of the property, such as category, name, or scale (Fig. 12a-c).
- An adverbial component of an action describes a feature of the action, such as (1) origin, route, or destination (Fig. 9a), (2) start-time or end-time (Fig. 9b), or (3) category, measure, overall spatial position, overall temporal position, or scale (Fig. 12d-f).
- An adverbial component of a conjunction describes a feature of the conjunction, such as associating theme, sequential condition, sequential probability, overall spatial position, or overall temporal position (Fig. 12g-i).

Adverbial components are usually optional, but spatial movement requires at least one adverbial component that specifies the origin, the route, or the destination, and temporal continuity requires at least one adverbial component that specifies the start-time or the end-time (Section 4.3).

An adverbial component is represented by a location, a moment, or a note, depending on the feature that it describes:

- a location represents origin, route, destination, or overall spatial position,
- a moment represents start-time, end-time, or overall temporal position, and
- a note represents other features.

An adverbial component is usually placed in the body slot, especially around the centerof an arrow symbol, so that it appears visually that the adverbial component is devoted to not a part but the whole of the arrow symbol (Fig. 12). Exceptionally,

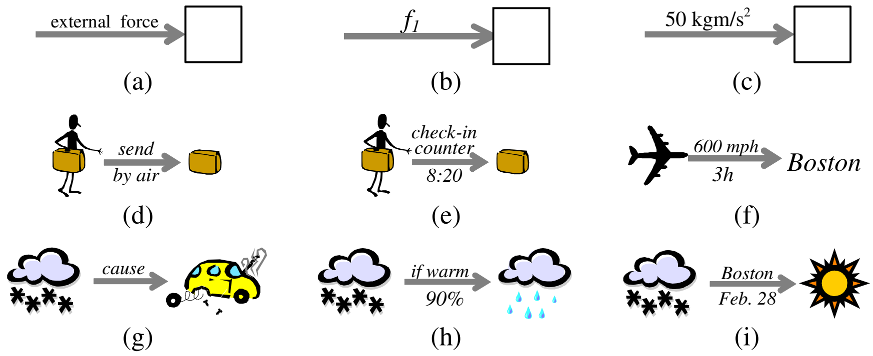


Fig. 12. Arrow diagrams, illustrating (a-c) a property, (d-f) an action, and (g-i) a conjunction. Each arrow diagram contains adverbial components which describe: (a) property's category, (b) property's name, (c) property's scale, (d) action's category and measure, (e) overall spatial and temporal position of the action, (f) action's scales, (g) underlying theme of the conjunction, (h) sequential condition and sequential probability of the conjunction, and (i) overall spatial and temporal position of the conjunction.

locations representing an origin and a destination are placed in the tail and the head slot, respectively, since arrow's tail and head imply the initial and final positions of a motion. Similarly, moments representing a start-time and an end-time are placed in the tail and the head slot, respectively. In any case, an adverbial component must be placed in an empty tail slot, an empty head slot, or an empty part of the body slot; otherwise, the adverbial component is misinterpreted as an adjective component.

6 Deducing Possible Interpretations from Structures

semantics and the structural conditions for adding optional components, we can consider the structures of arrow diagrams that possibly illustrate each class of semantics. One problem is that arrow diagrams take countless structures, since each slot can contain an arbitrary slot Now that we have both the structural requirements for illustrating each class of number of components. For simplification, here we consider only *simple arrow diagrams*, which have at most one component per each. Since each of the three slots has six possible patterns (an object, an event, a location, a moment, a note, and an empty component) but an arrow diagram must have at least one component, simple arrow diagrams may take only $2^3 - 1 = 215$ different patterns of structures. Thus, we can exhaustively investigate all structures of arrow diagrams that possibly illustrate each class of semantics. For example, to illustrate a *property*, one of the three slots must contain a subject, which is represented by an object, an event, or a location. In addition, an optional component (i.e., an adverbial component), which is represented by a note, may be place in the body slot if it is empty. Accordingly, there are 15 structures of simple arrow diagrams that possibly illustrate a property (Table 4). Similarly, we can enumerate 12 structures for annotations, 98 structures for actions, and 64 structures for conjunctions.

Table 4. All structural patterns of arrow diagrams illustrating a property

		Adverbial component	
		–	Note (*N*)
Subject	Object (*O*)	(*O*, –, –) (–, *O*, –) (–, –, *O*)	(*O*, *N*, –) (–, *N*, *O*)
	Event (*E*)	(*E*, –, –) (–, *E*, –) (–, –, *E*)	(*E*, *N*, –) (–, *N*, *E*)
	Location (*L*)	(*L*, –, –) (–, *L*, –) (–, –, *L*)	(*L*, *N*, –) (–, *N*, *L*)

Based on this result, we can judge whether a given arrow diagram has a possibility of illustrating each class of semantics. Among the 215 structures of simple arrow diagrams, 82 structures have no corresponding class, 81 structures correspond to exactly one class, and 52 structures correspond to multiple classes (Fig. 13).

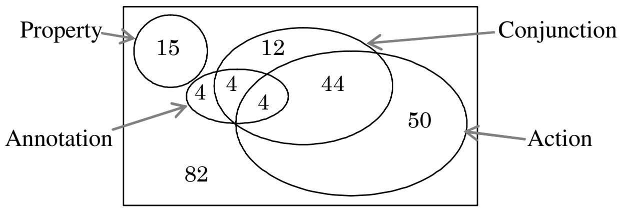


Fig. 13. The correspondence of 215 structures of simple arrow diagrams to the four classes of semantics

An arrow diagram with one of the 82 structures that has no corresponding class is automatically judged as a meaningless diagram (Fig. 14a), and an arrow diagram with one of the 81 structures that correspond to exactly one class leads to a unique class of interpretation (Fig. 14b). Fig. 13 highlights that an arrow diagram illustrating a property is always interpreted uniquely, while an arrow diagram illustrating other semantics (especially an annotation or a conjunction) is often ambiguous.

Fig. 14. (a) An arrow diagram, whose structure (*N*, *O*, *L*) has no corresponding class of semantics, is judged as meaningless, and (b) an arrow diagram, whose structure (*L*, *O*, *L*) corresponds to exactly one class of semantics (action), leads to a unique class of interpretation

Table 5 shows the structures of arrow diagrams that correspond to multiple classes of semantics. The structures S_1 and S_2 indicate that (1) an arrow diagram that annotates a component by a location or a moment (Fig. 15a$_1$) and (2) an arrow diagram that illustrates a conjunction whose proceeding component is a location or a moment (Fig. 15a$_2$) cannot be distinguished by their structures alone. An arrow diagram is, however, uniquely interpreted as a conjunction if the head component cannot be located at the spatial or temporal position that is specified by the tail component (for instance, Fig. 15a$_2$ cannot be interpreted as an annotation because *Lake Ontario* is not located in *Niagara Falls*). Otherwise, we need to judge whether the diagram has a certain theme for associating these components.

Table 5. Structures of arrow diagrams corresponding to multiple classes of semantics

Structures		Number of structures	Semantics
S_1	(*L*|*M*, –, *L*|*M*)	4	annotation and conjunction
S_2	(*L*|*M*, –, *O*|*E*)	4	annotation, action, and conjunction
S_3	(*O*|*E*, –|*L*|*M*|*N*, *O*|*E*|*L*|*M*), (*L*|*M*, *L*|*M*|*N*, *O*|*E*)	44	action and conjunction

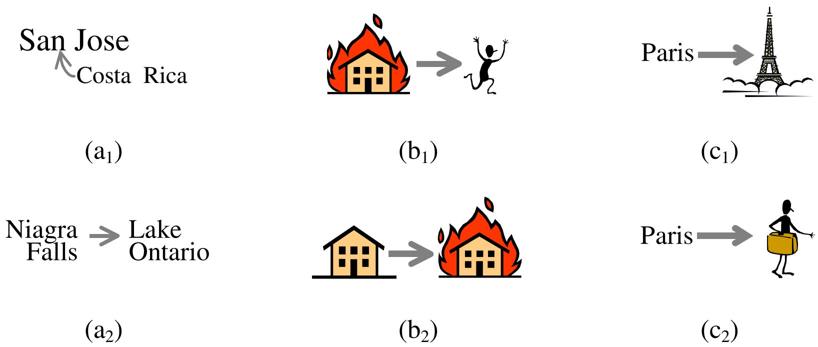


Fig. 15. Three pair of arrow diagram whose semantic classes are different but whose structures are same. The structures of arrow diagrams illustrating (a$_1$) an annotation and (a$_2$) a conjunction are both (*L*, –, *L*), those illustrating (b$_1$) an action and (b$_2$) a conjunction are both (*O*, –, *O*), and those illustrating (c$_1$) an annotation and (c$_2$) an action are both (*L*, –, *O*).

Similarly, arrow diagrams illustrating an action or a conjunction, whose structures are in S_2 or S_3, cannot be distinguished by their structures alone (Fig. 15b$_1$ and 15b$_2$). The large number of these ambiguous structures shows the importance of their distinction. An arrow diagram is uniquely interpreted as a conjunction if neither its tail component nor its head component is supposed to move (for instance, Fig. 15b$_2$ cannot be interpreted as an action because both the house and the burning house are immovable). Otherwise, we need to judge whether the diagram has a certain theme for associating these components and whether the action possibly illustrated in the arrow diagram is a typical scenario of actions.

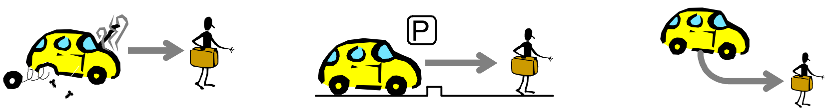
We further need the distinction of arrow diagrams illustrating annotations and actions, whose structures are S_2 (Fig. 15c$_1$ and 15c$_2$). An arrow diagram is uniquely interpreted as an annotation if the tail component is not supposed to move (for instance, Fig. 15c$_1$ does not illustrate an action because the Eiffel Tower is immovable). Otherwise, their distinction may depend on the context.

7 Conclusions and Future Work

This paper presented the classification of arrow-related semantics, the structural requirements for illustrating each class by arrow diagrams, and the structural conditions for adding optional components. Based on these settings, we demonstrate that 82 structures of simple arrow diagrams are automatically judged as meaningless, 81 structures leads to a unique class of semantics, and 52 structures lead to two or three possible classes of semantics. This indicates that knowledge about the structure of an arrow diagram reduces the ambiguity of its possible interpretations.

The knowledge about the structure is still useful for deducing more detailed interpretation of the arrow diagram. For example, Section 4.3 indicated that five types of actions, each of which has different structural requirements. Thus, the structural differences of arrow diagrams are probably useful for distinguishing these different types of actions.

In addition to the structures, various clues lead people to a unique and more detailed interpretation of an arrow diagram. For example, intrinsic mobility, movable space, and movable direction of each component are critical clues for judging whether the component is supposed to move (Fig. 16). This knowledge is, then, useful for judging whether an arrow diagram may illustrate an action, and if so, which type of action it may illustrate (i.e., *encounter* or *division*). Similarly, the caption or the adverbial component of an arrow diagram may specify or imply a theme for association, which is useful for judging whether an arrow diagram may illustrate a conjunction, and if so, which type of conjunction the arrow diagram may illustrate.



(a) (b) (c)

Fig. 16. Thanks to the knowledge about (a) intrinsic mobility, (b) movable space, and (c) movable direction of the car, arrow diagrams that are typically interpreted as *a person leaves a car*, not as *a car approaches a person*

Making use of such available clues comprehensively, we are now challenging to develop more sophisticated method for interpreting arrow diagrams. This method is expected to enhance the usability of pen-based systems, such that its user can communicate with the systems more naturally by drawing an arrow-containing diagram. In addition, since an arrow diagram is popular and often the simplest tool for the representation of spatial and temporal knowledge, to reveal the mechanism of arrow diagrams should lead to further understanding of people's communication about space and time.

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