RfDL: Models for Capturing Directional and Topological Characteristics of Path-Landmark Arrangements

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Abstract. This paper develops a series of projection-based spatial models, called RfDL (*Region-in-the-frame-of-Directed-Line*), which distinguishes the patterns of spatial arrangements between a directed line (DLine) and a simple region. RfDL considers eight types of intrinsic frames based on the combination of *left-right*, *front-side-back*, and *entry-interior-exit* distinctions with respect to the DLine, which naturally yield eight models with different levels of granularities. The use of intrinsic frames allows the RfDL models to capture the directional characteristics of DLine-region arrangements. In addition, we reveal that the finest RfDL model, called $RfDL_{3\text{-}12}$, also captures their topological characteristics, especially when the region is convex. Thanks to these characteristics, $RfDL_{3\text{-}12}$ is highly useful for the qualitative characterization of path-landmark arrangements standing on a moving agent's viewpoint.

1 Introduction

When people describe the movement of an agent, they often use landmarks [1]. Thus, spatial patterns of path-landmark arrangements are crucial for modeling human conceptualization of movement. Even though landmarks may be modeled as points [2], the studies on human route instructions to mobile robots [3, 4] observe the subjects' use of expressions that presume landmarks' spatial extent, such as "*follow it around*" and "*hit the end of …*". This finding motivated us to explore the models of spatial arrangements between a directed line (*DLine*) and a simple region, which correspond to path-landmark patterns, for future applications to dialogue-based man-machine interfaces that concern spatial tasks.

There are already some spatial models that support DLine-region arrangements. For instance, Kurata and Egenhofer [5] developed a model of topological DLine-region relations. These topological relations highlight where the DLine starts, passes, and ends with respect to the region's inside, outside, and border. Accordingly, the relations are useful for modeling some fundamental concepts of motions, such as "*go into*" and "*go across*". In this model, however, all movement patterns whose route does not intersect with the landmark are mapped to a single topological relation, namely *disjoint*, whereas people can distinguish these patterns with such expressions as "*go toward …*," "*pass by … on the left*," and "*go until … comes to the left*". These expressions usually highlight the landmark's direction as seen from the path.

Therefore, a model of DLine-region arrangements, which features their directional characteristics, will be also valuable for characterizing agents' movement patterns, as well as for capturing some human concepts of motions.

In this paper, we develop a series of spatial models, called RfDL (*Region-in-the-frame-of-Directed-Line*), which distinguishes the patterns of DLine-region arrangements. In order to capture the directional characteristics of DLine-region arrangements, RfDL adopts a similar framework to Double Cross [6, 7]. In addition, we reveal that the finest RfDL model, called $RfDL_{3\text{-}12}$, also captures the topological characteristics of the DLine-region arrangements, even though the model stands on a direction-featured framework. As a result, spatial patterns distinguished by $RfDL_{3\text{-}12}$ can be used for characterizing movement patterns based on both their directional and topological characteristics. Such versatile models will fit nicely with real world applications, where multiple aspects of the space are involved together [8].

The remainder of this paper is structured as follows: Section 2 develops RfDL models. Section 3 assesses the topological information conveyed by the patterns of $RfDL_{3\text{-}12}$. Finally, Section 4 concludes with the discussion of future work.

2 RfDL: Models of DLine-Region Arrangement Patterns

RfDL is a newly developed series of projection-based spatial models [9] that distinguish the patterns of spatial arrangements between a straight DLine and a simple region embedded in a two-dimensional Euclidean space $\mathbf{R}^2$. The DLine is a simple line with two ordered endpoints [10]. The simple line is a one-to-one mapping from [0, 1] to $\mathbf{R}^2$ [11]. A simple region is a bounded, regular closed set that is homeomorphic to a closed neighborhood in $\mathbf{R}^2$ [11]. Following Orientation Calculi [2], we consider eight intrinsic frames of spatial reference [12], which partition the space into *fields* on/around the DLine based on the combination of *left-right*, *front-side-back*, and *entry-interior-exit* distinctions (Fig 1a). These fields include zero- and one-dimensional ones that fill the gap between two-dimensional fields. The pattern of a DLine-region arrangement is defined as the set of fields over which the region extends. Naturally, the eight frames yield eight RfDL models with different levels of granularities. Each RfDL model is called $RfDL_{m\text{-}n}$ where m-n indicates the number of the fields on/around the DLine. For instance, $RfDL_{3\text{-}12}$ adopts a ‡-shaped frame, which defines three fields on the DLine—*En* (*entry*), *I* (*interior*), and *Ex* (*exit*)—and twelve fields around it—*LF* (*left front*), *SF* (*straight front*), *RF* (*right front*), *LB* (*left back*), *SB* (*straight back*), *RB* (*right back*), *LEn* (*left at entry*), *REn* (*right at entry*), *LI* (*left of interior*), *RI* (*right of interior*), *LEx* (*left at exit*), and *REx* (*right at exit*) (Fig. 1b). The patterns of DLine-region arrangements distinguished by $RfDL_{3\text{-}12}$ (in short, $RfDL_{3\text{-}12}$ patterns) are represented visually by icons with 3×5 cells (Fig. 1c), which geometrically correspond to the 15 fields defined by the $RfDL_{3\text{-}12}$'s frame. The marked cells indicate the fields over which the region extends. Accordingly, $RfDL_{3\text{-}12}$ patterns are distinguished by the icons' marking patterns.

$RfDL_{3\text{-}12}$ is closely related to Double Cross [6, 7]. Double Cross distinguishes 15 patterns of ternary point arrangements [6], which are also viewed as 15 arrangement patterns of a straight DLine and a point [7]. Both $RfDL_{3\text{-}12}$ and Double Cross adopt the

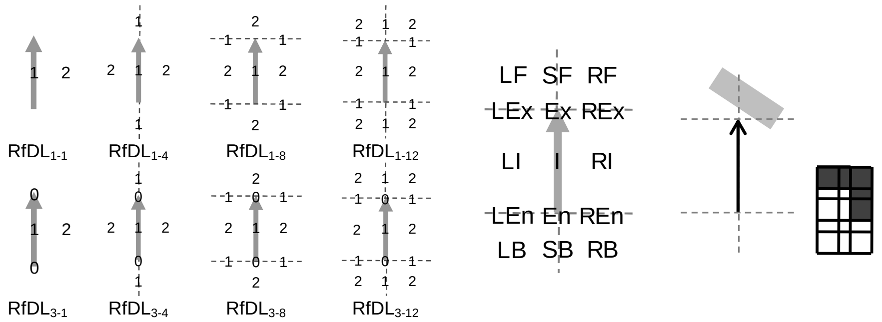


Fig. 1. (a) Eight frames of RfDL models (the numbers show the dimension of each field), (b) fifteen fields that $\text{RfDL}_{3\text{-}12}$'s frame defines, and (c) iconic representation of an $\text{RfDL}_{3\text{-}12}$ pattern.

same ‡-shaped frame that defines 15 fields. Accordingly, the $\text{RfDL}_{3\text{-}12}$ pattern between a DLine *D* and a region *R* is represented as the union of Double Cross patterns between *D* and every point *x* in *R*. This does not mean, however, that the union of arbitrary set of Double Cross patterns forms an $\text{RfDL}_{3\text{-}12}$ pattern, because $\text{RfDL}_{3\text{-}12}$ patterns must satisfy the following two conditions:

- the referent extends over at least one two-dimensional fields; and
- the set of fields over which the referent extends must be connected, even if *En* and *Ex* are removed from this set.

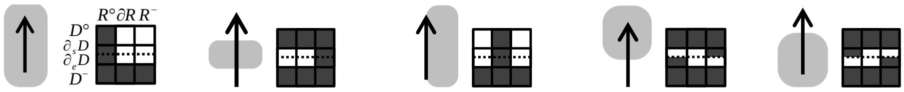
If the second rule is not satisfied, the referent has a spike or a disconnected interior as long as the referent is a single connected set and, accordingly, the referent cannot be a simple region. Due to these two constraints, the number of $\text{RfDL}_{3\text{-}12}$ patterns is limited to 1772, instead of $2^{15} = 32768$.

In a similar way, we identified that $\text{RfDL}_{1\text{-}1}$, $\text{RfDL}_{1\text{-}4}$, $\text{RfDL}_{1\text{-}8}$, $\text{RfDL}_{1\text{-}12}$, $\text{RfDL}_{3\text{-}1}$, $\text{RfDL}_{3\text{-}4}$, and $\text{RfDL}_{3\text{-}8}$ distinguish 2, 23, 142, 479, 8, 92, and 520 patterns of DLine-region arrangements, respectively. Providing such a series of models is useful to prevent the model's overspecification. Suppose a robot that moves in a room avoiding pieces of furniture. In this scenario, the robot's path never overlaps with the landmarks (furniture). This means that $\text{RfDL}_{3\text{-}n}$ is overspecific for the characterization of this robot's movement. The adoption of $\text{RfDL}_{1\text{-}n}$ instead of $\text{RfDL}_{3\text{-}n}$ drastically reduces the number of possible patterns and enables efficient computation processing.

3 Topological Characteristics Captured by $\text{RfDL}_{3\text{-}12}$

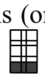
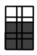
The patterns of $\text{RfDL}_{3\text{-}n}$ models describe whether each endpoint and interior of the DLine intersects with the region's or not. This is a crucial topological property of DLine-region arrangements. If a single model may capture both directional and topological characteristics, we can avoid mixed use of two spatial models for describing the same spatial scene. This idea motivated us to assess the topological information that the patterns of each RfDL model conveys. Here we focus on the finest model $\text{RfDL}_{3\text{-}12}$ and consider the mapping from its 1772 patterns to 26 topological

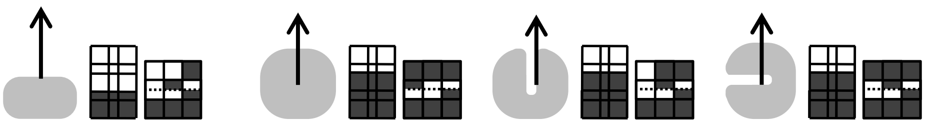
DLine-region relation distinguished by the 9^+-*intersection* [5]. In the 9^+-intersection, topological relations between a DLine D and a simple region R are distinguished by the presence or absence of intersections between D's four parts (interior $D°$, start point $\partial_s D$, end point $\partial_e D$, and exterior D^-) and R's three parts (interior $R°$, boundary ∂R, and exterior R^-). This information is represented by a bitmap-like icon (Figs. 2a), in which each marked block indicates the presence of the corresponding intersection. Accordingly, topological DLine-region relations are distinguished by the icon's marking patterns (Figs. 2a-e). These topological relations capture some fundamental concepts of motions [5]. Therefore, the mapping from the 1772 $RfDL_{3\text{-}12}$ patterns to the 26 topological relations will be useful for making qualitative interpretation of the motion patterns represented by $RfDL_{3\text{-}12}$ patterns.



(a) *move within* (b) *go across* (c) *go on the edge of* (d) *go into* (e) *go out of*

Fig. 2. DLine-region configurations and their topological relations, together with the corresponding concepts of motions [5].

The actual mapping from 1772 $RfDL_{3\text{-}12}$ patterns to the 26 topological DLine-region relations was derived computationally by a set of rules listed in Appendix. Among the 1772 $RfDL_{3\text{-}12}$ patterns, 1351 patterns (76.2%) are mapped to a single topological relation, while the others are mapped to multiple relations (on average 1.49 relations per $RfDL_{3\text{-}12}$ pattern). For instance, an $RfDL_{3\text{-}12}$ pattern  is mapped to a single topological relation  (Fig. 3a), while  is mapped to three topological relations , , and  (Figs. 3b-d). Such ambiguity occurs because if a certain part of the DLine intersects with the region, $RfDL_{3\text{-}12}$ patterns may not specify whether this part intersects with the region's interior or boundary, while topological relations always do (Fig. 3b-c). Nevertheless, many $RfDL_{3\text{-}12}$ patterns (76.2%) successfully capture the topological characteristics of DLine-region arrangements without ambiguity.



(a) (b) (c) (d)

Fig. 3. Four DLine-region configurations, together with two icons representing their $RfDL_{3\text{-}12}$ pattern (left) and their topological relations (right).

We also analyzed the mapping under the assumption that the region is convex, because many landmarks in indoor environments (e.g., tables, carpets, and closets) are represented by convex regions. Under this assumption, the following two rules are added to the rules of mapping listed in Appendix:

- if R extends over D's *LI* and *RI*, $D°$ intersects with $R°$; and
- if R extends over D's *En* and *Ex*, $D°$ does not intersect with R^-.

Meanwhile, the number of $RfDL_{3-12}$ patterns decreases from 1772 to 638 under the assumption, due to the constraint that if *R* extends over two horizontally or vertically remote fields, then *R* also extends over every field between them. Interestingly, at this time every $RfDL_{3-12}$ pattern is mapped successfully to a single topological relation. This indicates that $RfDL_{3-12}$ can be used as a versatile model for capturing both directional and topological characteristics of path-landmark arrangements especially when the landmarks are convex. Similarly, we identified that $RfDL_{3-4}$ and $RfDL_{3-8}$ also captures the topological characteristics of DLine-region arrangements very well.

4 Conclusions and Current Work

This paper developed RfDL models that distinguished the patterns of spatial arrangements between a straight DLine and a simple region in $\mathbf{R}^2$. Making use of intrinsic frames centered at the DLine, RfDL models captured the region's direction as seen from the DLine. In addition, we revealed that $RfDL_{3-12}$ captured the topological characteristics of the arrangements considerably, especially when the region was convex. Such versatile models will be useful for the applications that involve both directional and topological aspects of the space [8], such as the dialogue-based interface of mobile robots [13]. We are now applying $RfDL_{3-12}$ for the interpretation of human route instructions, by associating typical concepts of motions seen in such instructions with the patterns of path-landmark arrangements represented by $RfDL_{3-12}$ patterns [14]. $RfDL_{3-12}$ is highly useful for capturing such concepts as "*go toward*", "*pass … on the right*", "*go into*", and "*go across*", which concerns the direction or spatial extent of the region-like landmark as seen from an agent moving on a path.

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Appendix: Rules for Mapping RfDL$_{3\text{-}12}$ Patterns to Topological Relations

Let D° and $\partial_{s/e} D$ be the interior and start/end point of a DLine D, and R°, ∂R, and R^- be the interior, boundary, and exterior of a region R, respectively.

- $\partial_{s/e} D$ intersects with only one among R°, ∂R, or R^-,
- if R does not extend over D's *En/Ex*, then $\partial_{s/e} D$ intersects with R^-;
- if R extends over D's *En/Ex* and not all fields around it, then $\partial_{s/e} D$ intersects with ∂R (Fig. 4a);
- if R extends over D's *En/Ex* and all fields around it, then $\partial_{s/e} D$ intersects with either R°or ∂R (Fig. 4b);
- if R does not extend over *I*, D° intersects with R^- only; and
- if R extends over D's *I*, then D° intersects with R°, ∂R, or both, and possibly R^- as well, under the following restrictions:
  - if R does not extend over D's *En* or/and *Ex*, then D° intersects with ∂R and R^- (Fig. 4c);
  - if R does not extend over D's *LI* or *RI*, then D° does not intersect with R°;
  - if R extends over both D's *LI* and *RI*, but not all the fields around *En* and those around *Ex*, then D° intersects with R° (compare Fig. 4d that satisfies this condition with Fig. 4e that does not); and
  - if D° intersects with both R° and R^-, then D° also intersects with ∂R.

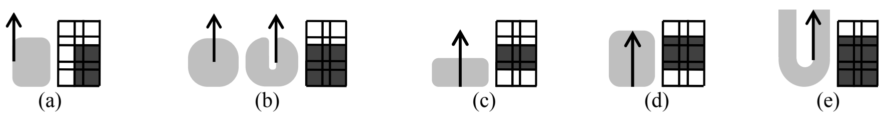

Fig. 4. Seven DLine-region configurations, together with their RfDL$_{3\text{-}12}$ relations.