Toward Heterogeneous Cardinal Direction Calculus

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Abstract. Cardinal direction relations are binary spatial relations determined under an extrinsically-defined direction system (e.g., *north of*). We already have point-based and region-based cardinal direction calculi, but for the relations between other combinations of objects we have only a model. We are, therefore, developing *heterogeneous cardinal direction calculus*, which allows reasoning on cardinal direction relations without regard to object types. In this initial report, we reformulate the definition of cardinal direction relations, identify the sets of relations between various pairs of objects, and develop the methods for deriving upper approximation of converse and composition.

Keywords: cardinal direction relations, heterogeneous spatial calculi, converse, composition

1. Introduction

Imagine that you are at the peak of *Mt. Fuji*. You can see *Fuji-gawa river* flowing from the northwest to the southwest, and also the city of *Tokyo* spreading out in the north-east. Then, what can you say about the relation between *Fuji-gawa* and *Tokyo*? This question concerns *cardinal direction relations* [1], which are determined under an extrinsically-defined direction system. In addition to compass directions like *north* and *south*, these relations are applicable to capture such relations as *above* and *left-of* in shelves and storages. *Cardinal direction calculi* are the mechanisms that realize spatial reasoning on such cardinal direction relations. So far cardinal direction calculi have been developed for the relations between points [1, 2] and those between regions [3-6]. In our example question, however, *Mt. Fuji*'s peak, *Fuji-gawa*, and *Tokyo* are modeled as a point, a line, and a region, respectively. Accordingly, their arrangement may not be deduced *properly* in the previous calculi, even though we can treat them as regions and conduct spatial reasoning at the cost of reliability.

Actually, most spatial calculi have targeted spatial relations between single-type objects. Kurata [7], therefore, insisted on the necessity of *heterogeneous spatial calculi*, which allow spatial reasoning beyond the difference of object types. As a first step, he developed *9-intersection calculi* that realize topology-based reasoning on multi-type objects [7]. Following this effort, we are currently developing *heterogeneous cardinal relation calculus*, since cardinal direction relations are one of the most basic sets of spatial relations. This article is the first report of this work.

The remainder of this paper is structured as follows: Section 2 reviews existing work on cardinal direction relations. Section 3 reformulates the model of cardinal direction relations, under which Section 4 identifies the sets of cardinal direction relations between various pairs of objects. Section 5 introduces the methods for deriving upper approximation of converse and composition of these directional relations. Finally, Section 6 concludes the discussion.

In this paper, we consider a 2D Euclidean space $\mathbb{R}^2$ with a coordinate system in which the *x*- and *y*-axes are aligned with the north and east, respectively. Simple regions and non-branching line segments are called *regions* and *lines* for short, and lines parallel to *x*-axis, those parallel to *y*-axis, and other lines are called *HLines*, *VLines*, and *GLines* (or *horizontal, vertical, and generic lines*), respectively.

2. Related Work

In general, directional relations concern the orientation of the target object (*referent*) with respect to the reference object (*relatum*). Frank [1] introduced *cone-based* and *projection-based* frames for distinguishing directional relations between two points. The frames are placed such that the relatum comes to the center of the frames. In a spatio-linguistic viewpoint, these frames are sorts of *extrinsic frames of spatial reference* whose direction is fixed in the environment [8]. Frank [1] assessed the converse and composition of his directional relations and identified the presence of some indeterminate compositions (for instance, the composition of *north-of* and *south-of* yields *north-of*, *south-of*, and *equal*). Together with the converse and composition operations, Frank's model forms (*Point-Based*) *Cardinal Direction Calculus*. Ligozat [2] applied this calculus to constraint-based reasoning, identifying maximum tractable subsets of the relations. *Star Calculus* [9] generalizes cardinal direction calculus, such that it captures directional relations in arbitrary granularity.

Papadias and Sellis [3] studied the cardinal direction relations between two regions making use of their minimum bounding rectangles (*MBRs*). Regions are mapped to intervals on each axis. Thus, by applying MBRs, 13×13 directional relations were distinguished, as well as the converse and composition of those relations were determined based on Allen's [10] interval calculus,.

Goyal and Egenhofer [4] introduced a *directional-relation matrix*, whose 3×3 elements correspond to the intersections between the referent and the 3×3 *tiles*. The tiles are determined by four lines l_1: $x = \inf_x(B)$, l_2: $x = \sup_x(B)$, l_3: $y = \inf_y(B)$, and l_4: $y = \sup_y(B)$, where $\inf_{x/y}(B)$ and $\sup_{x/y}(B)$ are the greatest lower bound and the least upper bound of the projection of the relatum *B* on *x/y*-axis. This approach is applicable to any pair of objects, but depending on the relatum's shape some tiles may be collapsed to a line or a point. In the simplest approach, directional relations are distinguished by the presence or absence of the 3×3 referent-tile intersections. Based on this framework, calculus aspects of region-based cardinal directions were studied in [5, 6]. Cicerone and Felice [6] identified all realizable pairs of the cardinal directional relation between *A* and *B* and that between *B* and *A*. Skiadopoulos and Koubrakis [5] identified a method for deriving (weak) compositions of cardinal direction relations between two regions.

3. Our Model of Cardinal Direction Relations

We introduce four iconic representations of spatial arrangements of two objects, namely *CD pattern*, *CD^+ pattern*, *PCD pattern*, and *PCD^+ pattern* (Fig. 1a). The CD pattern captures how the referent A intersects with the 3×3 tiles determined by the relatum B, namely NW_B, NC_B, NE_B, MW_B, MC_B, ME_B, SW_B, SC_B, and SE_B. The CD pattern has 3×3 black-and-white cells, each of which is marked if the corresponding element in $\begin{bmatrix} A \cap NW_B & A \cap NC_B & A \cap NE_B \\ A \cap MW_B & A \cap MC_B & A \cap ME_B \\ A \cap SW_B & A \cap SC_B & A \cap SE_B \end{bmatrix}$ is non-empty. This follows the direction matrix in [4], but in our model the tiles are defined as a set of jointly-exclusive and pairwise-disjoint partitions of the space (by this we avoid strange situations where a point-like referent intersects with two tiles at the same time). First, two sets of JEPD partitions, $\{W_B, C_B, E_B\}$ and $\{S_B, M_B, N_B\}$, are defined as Eqs. 1-2. Then, the tiles are defined as the intersections of these two partition sets, like $NW_B = N_B \cap W_B$. Note that NC_B, MW_B, MC_B, ME_B, and SC_B may be collapsed depending on the type of the relatum (Fig. 2).

$$S_B = \{(x,y) \in \mathbb{R}^2 | y < \inf_y(B)\} \;\; N_B = \{(x,y) \in \mathbb{R}^2 | y > \sup_y(B)\} \;\; M_B = \mathbb{R}^2 \setminus (S_B \cup N_B) \qquad \textbf{(1)}$$

$$W_B = \{(x,y) \in \mathbb{R}^2 | y < \inf_y(B)\} \;\; E_B = \{(x,y) \in \mathbb{R}^2 | x > \sup_x(B)\} \;\; C_B = \mathbb{R}^2 \setminus (W \cup E_B) \qquad \textbf{(2)}$$

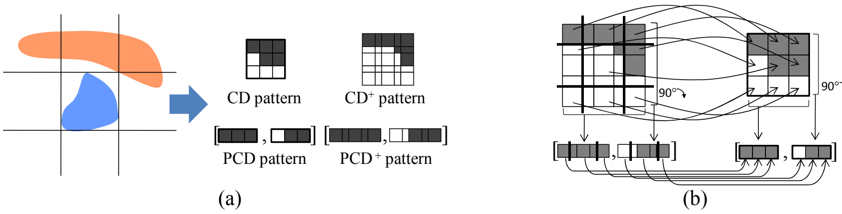


Fig. 1. (a) Four types of iconic representations of a spatial arrangement featuring cardinal directions and (b) converse between these four representations

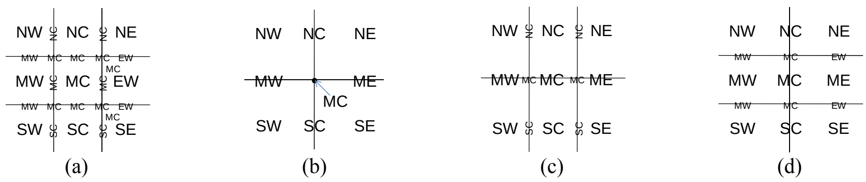


Fig. 2. Tiles determined by the relatum, which is (a) a simple region or a GLine, (b) a point, (c) a HLine, and (d) a VLine, respectively

The CD^+ pattern is a refinement of the CD pattern, which captures how the referent intersects with *n*×*m fields*. The CD^+ pattern has *n*×*m* black-and-white cells, among which the i^{th} left and j^{th} bottom cell is marked if A intersects with the i^{th} left and j^{th} bottom field. The fields are determined by four lines l_1: $x = \inf_x(B)$, l_2: $x = \sup_x(B)$, l_3: $y = \inf_y(B)$, and l_4: $y = \sup_y(B)$, such that not only 2D blocks separated by these lines, but also 1D boundaries between two blocks and 0D boundary points between four blocks are all considered independent fields. Naturally, 5×5, 3×3, 5×3, and 3×5 fields are identified when the relatum is a region or a GLine, a point, a HLine, and a VLine, respectively.

The PCD pattern consists of two sub-patterns, called PCD_x and PCD_y *patterns*. They have three black-and-white cells, each marked if the corresponding element in $[A\cap W_B \ A\cap C_B \ A\cap E_B]$ and $[A\cap S_B \ A\cap M_B \ A\cap N_B]$ is non-empty, respectively. Essentially, $PCD_{x/y}$ patterns are the projection of the CD pattern onto *x*-/*y*-axis. Finally, the PCD^+ pattern is a counterpart of PCD pattern for the CD^+ pattern.

In this paper, *directional relations* refer to the spatial arrangements distinguished by CD patterns, while *fine-grained directional relations* refer to the spatial arrangements distinguished by the CD^+ patterns. We primarily use CD patterns because they allow the unified representation of directional relations by 3×3 binary patterns without regard to object types. Another practical reason is that CD^+ patterns may distinguish an overwhelming number of spatial arrangements (up to $2^{5\times5}$).

4. Identification of Directional Relations

Even though the CD pattern icon distinguishes $2^{3\times3}$ patterns, not all of them are effective as the representation of directional relations (i.e., some patterns have no geometric instance). We, therefore, identify the sets of all effective CD patterns when the referent and relatum are points, H/V/GLines, simple regions, and their combinations (in total 5×5 cases). Suppose *A* be the referent and *B* be the relatum that defines $n\times m$ fields. First, we consider $2^{n\times m}$ CD^+ patterns, from which we remove geometrically-impossible CD^+ patterns by the following constraints:

- if *A* is a point, it intersects with exactly one field;
- if *A* is a H/V/GLine, it intersects with at least one non-0D field, and all fields with which *A* intersects must be connected; in addition,
  - if *A* is an HLine/VLine, all fields with which *A* intersects must be horizontally/vertically aligned;
  - if *A* is a GLine, letting ***C*** be the set of connected components formed by non-0D fields with which *A* intersects, and ***P*** be the set of 0D fields with which *A* intersects and which connect to more than one connected component in ***C***, there are at most two connected components in ***C*** that are connected to exactly one field in ***P*** (otherwise *A* is branching; compare Figs. 3a-3a'); and
- if *A* is a simple region, it intersects with at least one 2D field, and all fields with which *A* intersects must be connected, even if all 0D fields are excluded (otherwise *A* has a spike or disconnected interior; compare Figs. 3b-3b').

Then, the remaining CD^+ patterns are converted to CD patterns as shown in Fig. 1b. We confirmed that these CD patterns are all effective (i.e., we succeeded to draw an instance for every CD pattern). Thus, these CD patterns represent the complete set of directional relations. For instance, we identified 222 directional region-region relations (four more than the relations listed in [6], due to the slightly different definition of relations). Table 1 shows the number of the directional relations we have identified for 5×5 cases, while Table 2 summarizes their patterns.

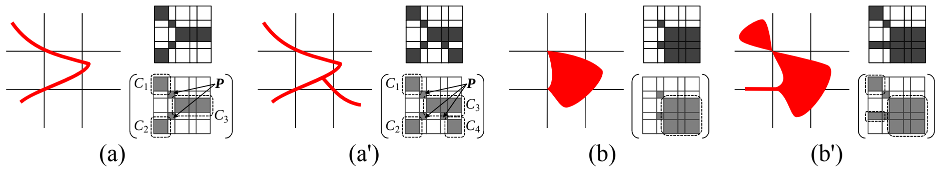
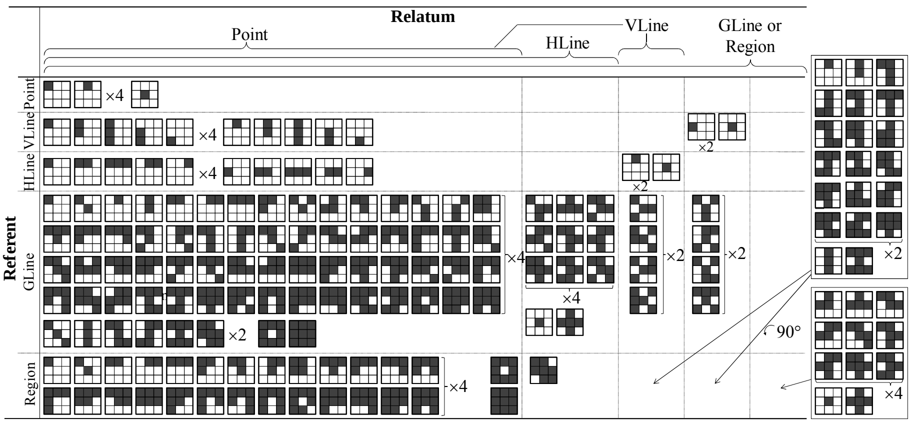


Fig. 3. Examples of CD^+ patterns when the referent is a simple or non-simple object

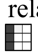
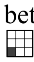
Table 1. Numbers of cardinal direction relations distinguished by our framework

		Relatum				
		Point	HLine	VLine	GLine	Region
Referent	Point	9	9	9	9	9
	HLine	15	18	15	18	18
	VLine	15	15	18	18	18
	GLine	254	300	300	308	308
	Region	106	146	146	222	222

Table 2. Cardinal direction relations distinguished by our framework (icons with "×4"/"×2" also represent the CD patterns derived by rotating {90,180,270}/180 degree)



5. Projection-Based Spatial Inference

Given the directional relation between two regions *A* and *B* as , what are possible relations between *B* and *A*? Similarly, given the relation between a GLine *C* and a point *D* and that between *D* and a region *E* as  and , respectively, what are possible relations between *C* and *E*? Figs. 4a-b show one possible solution for each question, but what else? The answers to these questions are derived by *converse* and *composition* operations. In general, given the relation r_1 between the objects O_1 and O_2 and r_2 between O_2 and O_3, the converse of r_1 returns the relation(s) that may hold between O_2 and O_1, while the (weak) composition of r_1 and r_2 returns the relation(s) that may hold between O_1 and O_3. The converse and composition operations are fundamentals of qualitative spatial calculi [11].

We introduce methods for deriving upper approximations of converse and composition of cardinal directional relations. These methods project spatial objects onto *x*- and *y*-axes, conduct converse/composition on each axis, and synthesize the results to derive the candidates for the converse/composition. The basic idea comes from MBR-based spatial inference in [3], but our target is not limited to regions.

By projection onto an axis each object is mapped to an interval or a point. First, suppose that the referent *A* and the relatum *B* correspond to two intervals i_A and i_B on the *x*-axis. Then, the PCD^+_x-pattern for *A* and *B* corresponds to the interval relation between i_A and i_B. Naturally, the converse of this PCD^+_x-pattern is determined by the converse of this interval relation (Fig. 5a). Similarly, if *A* and *B* correspond to two intervals on the *y*-axis, the converse of their PCD^+_y-pattern is determined by the converse of the corresponding interval relation. This mapping from $PCD^+_{x/y}$ patterns to their converse is coarsened based on the correspondence between $PCD^+_{x/y}$ and $PCD_{x/y}$ patterns in Fig. 6. Fig. 7a summarizes the result. By combining the converse of a PCD_x pattern and that of a PCD_y pattern determined by Fig. 7a, we can determine the converse of a PCD pattern. Making use of this projection-based converse, the candidates for the converse of the given directional relation can be derived. For instance, imagine that *A* and *B* are regions whose directional relation is (Fig. 4a). The PCD pattern for *A* and *B* is . The converse of this PCD pattern is { , , , }, since 's converse is { , } and 's converse is { , } (Fig. 7a). From these four PCD patterns, we can derive ten CD patterns { , , , , , , , , , }, but is removed because it does not represent any directional region-region relation (Table 2). As a result, we obtain nine directional region-region relations as the candidates for the converse of the given relation.

When both the referent and relatum correspond to a point on *x*/*y* axis, we can derive the mapping from $PCD_{x/y}$ patterns to their converse (Fig. 7b), making use of the correspondence between $PCD^+_{x/y}$ patterns and point-order relations (Fig. 5b). Similarly, when the referent and relatum correspond to a point and an interval, we can derive the mapping from $PCD_{x/y}$ patterns to their converse (Figs. 7c-d), making use of the correspondence between $PCD^+_{x/y}$ patterns and point-interval relations (Fig. 5c).

In a similar way, we consider projection-based compositions. Suppose that three objects *A*, *B*, and *C* correspond to the intervals i_A, i_B, and i_C on the *x*-axis. Then, the PCD^+_x pattern representing *A*-*B* relation and that representing *B*-*C* relation correspond to the interval relation between i_A and i_B and that between i_B and i_C, respectively. Naturally, the composition of these two PCD^+_x patterns is determined by the composition of these interval relations. Thus, from Allen's composition table [10], we can derive the composition table of $PCD^+_{x/y}$ patterns. By coarsening this table based on the correspondence between $PCD^+_{x/y}$ and $PCD_{x/y}$ patterns (Fig. 6), the composition table of $PCD_{x/y}$ patterns was developed (Table 3). We skip the detail, but this table can be used even when the referent, relatum, or both correspond to points (instead of regions), taking the realizability of $PCD_{x/y}$ patterns into account (see * in Table 3). Consequently, using Table 3, we can determine the compositions of two PCD patterns, from which the candidates for the composition of the given directional relations can be derived. For instance, the projection-based composition of a GLine-point relation and a point-region relation (Fig. 4b) is derived from the composition of and and that of and —the result is { , , } (this corresponds to the *Fuji-gawa–Tokyo* relation in Section 1).

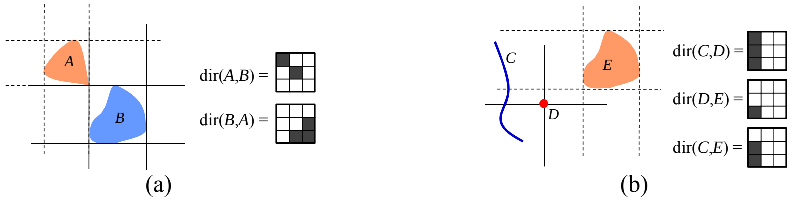


Fig. 4. Examples of projection-based converse and composition

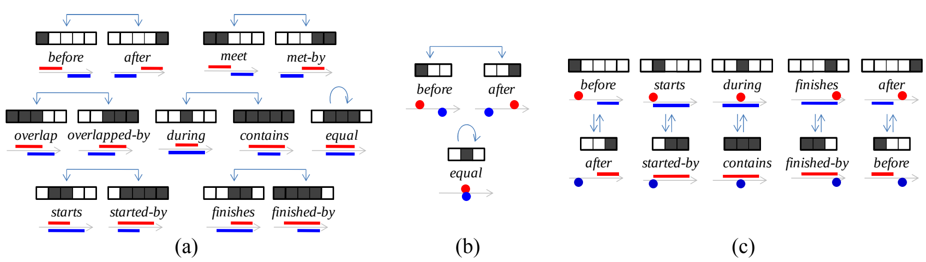


Fig. 5. Mapping from $PCD^{+}_{x/y}$ patterns to their converse, when the referent and relatum correspond to (a) two intervals, (b) two points, and (c) a point and an interval, respectively

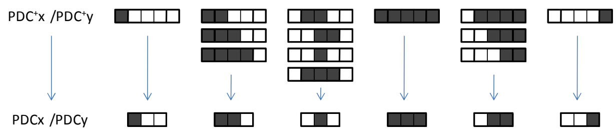


Fig. 6. Correspondence between $PCD^{+}_{x/y}$ and $PCD_{x/y}$ patterns when the $PCD^{+}_{x/y}$ patterns have higher granularity (otherwise $PCD^{+}_{x/y}$ and $PCD_{x/y}$ patterns are equivalent)

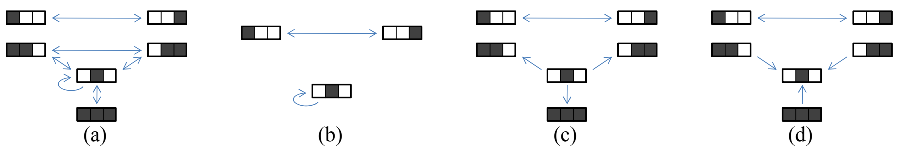
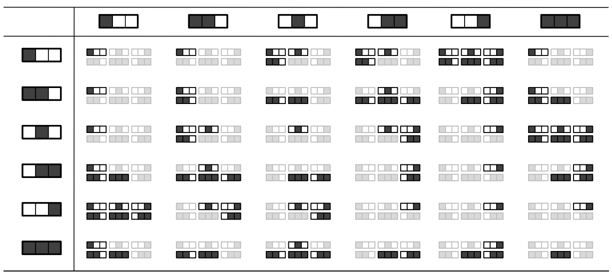


Fig. 7. Converse of $PCD_{x/y}$ patterns when the referent and relatum correspond to (a) two intervals, (b) two points (c) a point and an interval, and (d) an interval and a point on an axis

Table 3. Composition table of $PCD_{x/y}$ patterns



* , , and in the composition results are removed when they correspond to point-point or point-interval relations, and is removed when they correspond to interval-point relations

6. Conclusions and Future Work

In this paper, we developed the basis of *heterogeneous cardinal relation calculus*, which aims at the support of direction-featured spatial reasoning without regard to object types. We captured cardinal direction relations with bitmap-like icons with 3×3 cells and identified sets of relations between points, lines, regions, and their combinations. Then, we developed methods for deriving upper approximation of converse and composition of these relations. Actually, these approximations agree with the precise converse and composition when the source relations are *convex* (i.e., the icon's black cells form a single rectangle); otherwise, the results of projection-based converse or composition may include unnecessary patterns and, therefore, other methods are necessary for deriving more precise converse and composition. These methods, as well as the proof of the above fact, will be reported in our next report.

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