Directed Line Segments: How Are They Connected?

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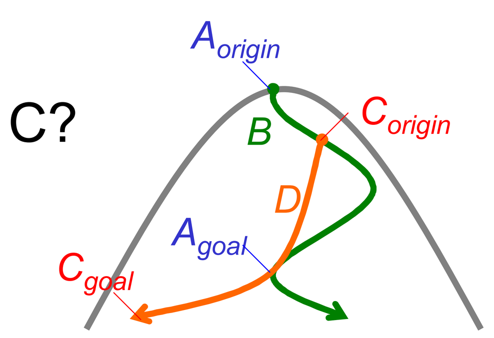
Motivating Question

Imagine a snow mountain with four skiing trails *A-D*, and two skiers reported:

> I went down trail *B*. I noticed *A* and B start at the same point, *C* starts at B's midpoint, and *A* ends at *B*'s midpoint.

> I took trail *D*. I found that *C* shares the same origin and goal with *D*, and *A* ends at *D*'s midpoint.

Now, what is the relation between A and C?



Research Goal

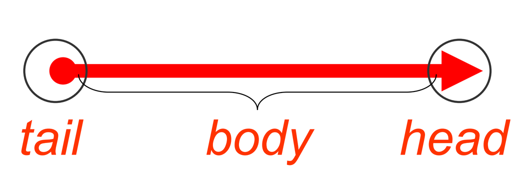
To develop a model and a reasoning method of spatial relations between directed line segments.

Directed line segments (*DL segments*) represent a large variety of linear geographic features, such as roads, watercourses, and skiing trails (as seen in the previous example), as well as movements and actions of entities. **These DL segments often be connected to each other, thereby represents their relations and interactions**, such as split, merge, encounter, and, interference, which are also meaningful for people's conceptualization of geographic environment and phenomena.

Spatial databases, therefore, should be equipped with the ability to deal with spatial relations between DL segments, such that people can naturally interact with the databases based on their familiar concept of space.

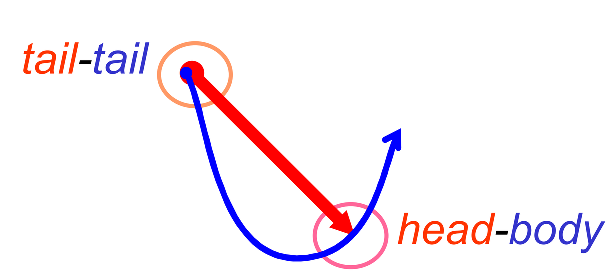
Model of Spatial Relations

DL segments are decomposed into three parts:



Connections between two DL segments are classified into 3×3 = 9 types:

tail-tail, *tail-body*, *tail-head*
body-tail, *body-body*, *body-head*
head-tail, *head-body*, *head-head*



The presence/absence of these nine types of connections are represented by a 3×3 matrix with empty or non-empty entries (ϕ or $\neg\phi$).

	tail	body	head
tail	$\neg\phi$	ϕ	ϕ
body	ϕ	ϕ	ϕ
head	ϕ	$\neg\phi$	ϕ

This 3×3 matrix captures the spatial relation between two DL segments.

Identification of Spatial Relations

Since the tail and head of a DL segment are points, they cannot be connected to more than one part of another non-looped DL segment.

Accordingly, the first column, third column, first row, and third row in the matrix can contain at most one non-empty entry ($\neg\phi$).

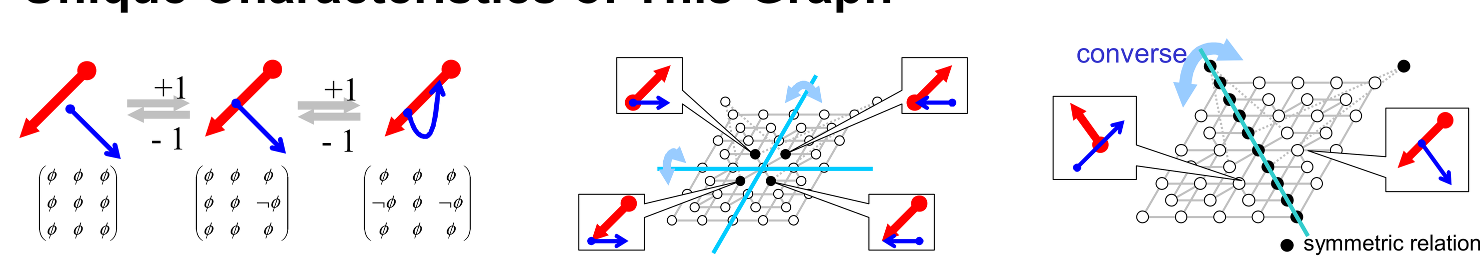
	tail	body	head
tail	$\neg\phi$	ϕ	ϕ
body	ϕ	ϕ	ϕ
head	ϕ	$\neg\phi$	ϕ

Based on this constraint, we identified **68 spatial relations** between two non-looped DL segments, which are schematized in the following two-layered graph:

* All nodes are also linked vertically

* To simplify the graph, columns and rows at the fringe of each layer is repeated.

Unique Characteristics of This Graph



Links indicate the possibility of continuous transformation by gaining/losing an intersection.

Each layer is decomposed into four parts, which are derived from the adjacent part by flipping one DL-segment's direction.

Most of symmetric relations are located on a diagonal line of each layer, while converse relation pairs are located symmetrically across this line.

Reasoning on Ternary Relations

For three DL segments *A*, *B*, and *C*, if we know the relation between *A* and *B* (R_{AB}) and that between *A* and *C* (R_{AC}), then the possible relations between A and C (R_{AC}) are determined by the following constraints:

known

$$R_{AB}=\begin{pmatrix} I_{AB}^{tt} & I_{AB}^{tb} & I_{AB}^{th}\\ I_{AB}^{bt} & I_{AB}^{bb} & I_{AB}^{bh}\\ I_{AB}^{ht} & I_{AB}^{hb} & I_{AB}^{hh}\end{pmatrix}$$

known

$$R_{BC}=\begin{pmatrix} I_{BC}^{tt} & I_{BC}^{tb} & I_{BC}^{th}\\ I_{BC}^{bt} & I_{BC}^{bb} & I_{BC}^{bh}\\ I_{BC}^{ht} & I_{BC}^{hb} & I_{BC}^{hh}\end{pmatrix}$$

unknown

$$R_{AC}=\begin{pmatrix} I_{AC}^{tt} & I_{AC}^{tb} & I_{AC}^{th}\\ I_{AC}^{bt} & I_{AC}^{bb} & I_{AC}^{bh}\\ I_{AC}^{ht} & I_{AC}^{hb} & I_{AC}^{hh}\end{pmatrix}$$

constraints

For $p_A, p_B, p_C \in \{t,b,h\}$

$$I_{AB}^{p_A t}=\neg\phi \wedge I_{BC}^{t p_C}=\neg\phi \rightarrow I_{AC}^{p_A p_C}=\neg\phi$$
$$I_{AB}^{p_A h}=\neg\phi \wedge I_{BC}^{h p_C}=\neg\phi \rightarrow I_{AC}^{p_A p_C}=\neg\phi$$
$$I_{AB}^{t p_B}=\neg\phi \wedge I_{BC}^{p_B p_C}=\phi \rightarrow I_{AC}^{t p_C}=\phi$$
$$I_{AB}^{h p_B}=\neg\phi \wedge I_{BC}^{p_B p_C}=\phi \rightarrow I_{AC}^{h p_C}=\phi$$
$$I_{AB}^{p_A p_B}=\phi \wedge I_{BC}^{p_B t}=\neg\phi \rightarrow I_{AC}^{p_A t}=\phi$$
$$I_{AB}^{p_A p_B}=\phi \wedge I_{BC}^{p_B h}=\neg\phi \rightarrow I_{AC}^{p_A h}=\phi$$

Deriving Answers to the Initial Question

The left skier reports the relations R_{AB} and R_{BC} as:

$$R_{AB}=\begin{pmatrix}\neg\phi & \phi & \phi\\ \phi & \phi & \phi\\ \phi & \neg\phi & \phi\end{pmatrix} \quad R_{BC}=\begin{pmatrix}\phi & \neg\phi & \phi\\ \phi & \phi & \phi\\ \phi & \phi & \phi\end{pmatrix}$$

Due to the previous constraints, R_{AC} is partially determined as:

$$R_{AC}=\begin{pmatrix}\phi & \phi & \phi\\ \phi & & \\ & \phi & \phi\end{pmatrix}$$

The right skier reports the relations R_{AD} and R_{DC} as:

$$R_{AD}=\begin{pmatrix}\phi & \phi & \phi\\ \phi & \phi & \phi\\ \phi & \neg\phi & \phi\end{pmatrix} \quad R_{DC}=\begin{pmatrix}\neg\phi & \phi & \phi\\ \phi & \phi & \phi\\ \phi & \phi & \neg\phi\end{pmatrix}$$

Due to the previous constraints, R_{AC} is partially determined as:

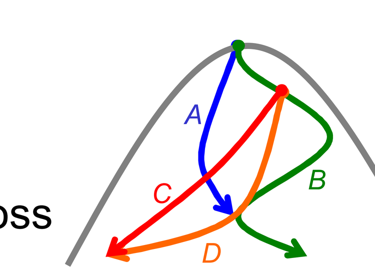
$$R_{AC}=\begin{pmatrix}\phi & & \phi\\ \phi & & \phi\\ \phi & \phi & \phi\end{pmatrix}$$

Thus, R_{AC} must satisfy $\begin{pmatrix}\phi & \phi & \phi\\ \phi & & \phi\\ \phi & \phi & \phi\end{pmatrix}$, which has only two possibilities among the 68 spatial relations:

Possibility 1

$$R_{AC}=\begin{pmatrix}\phi & \phi & \phi\\ \phi & \neg\phi & \phi\\ \phi & \phi & \phi\end{pmatrix}$$

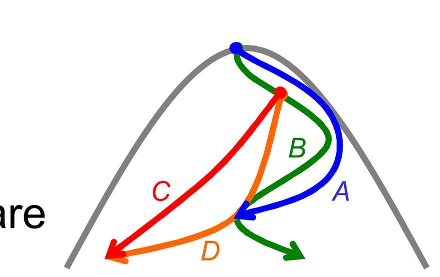
→ A and C cross each other.



Possibility 2

$$R_{AC}=\begin{pmatrix}\phi & \phi & \phi\\ \phi & \phi & \phi\\ \phi & \phi & \phi\end{pmatrix}$$

→ A and C are disjoint.



In this way, knowledge about a network can be enriched from limited information by reasoning.

For More Powerful Reasoning…

In the above example, if we know that *A* departs from *A*'s left side, the possible relation of R_{AB} is limited to *cross*. This implies that such additional knowledge as **left-side/right-side connections** will be useful for narrowing down the possible relations.

We are, therefore, tackling with **the refinement of our current intersection-based model** in order to realize more powerful reasoning on the qualitative relations between DL segments.