9^+-Intersection Calculi for Spatial Reasoning on the Topological Relations between Heterogeneous Objects

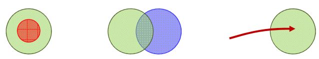
TOKYO METROPOLITAN UNIVERSITY

We developed a series of <u>qualitative spatial calculi</u> that targets <u>topological relations</u> between **mixed-type objects**

Yohei Kurata
Tokyo Metropolitan University
ykurata@tmu.ac.jp

Q. What are topological relations?

A. They are spatial relations that concern how two spatial objects connect and/or overlap



Such topological relations are distinguished systematically by *9^+-intersection* [1, 2]

Q. What are qualitative spatial calculi?

A. They are computational mechanisms for qualitative spatial reasoning

e.g., What relations are possible between A and C?

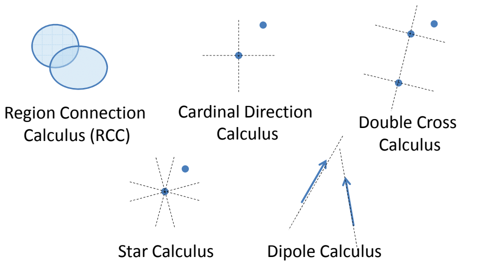
Q. Can we solve such problems if there are <u>many objects</u>?

A. By preparing the following elements, we can solve the problems *computationally*:

- an object domain D
- a set of base relations $\mathcal{B}$ between two objects in D
- a list of converse relations for $\mathcal{B}$
- a composition table for $\mathcal{B}$

Q. Why do we consider mixed-type objects?

A. Existing qualitative spatial calculi target only single-type object relations

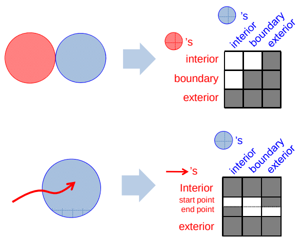


But in geographic contexts, we often deal with <u>mixture</u> of regions, lines, and points

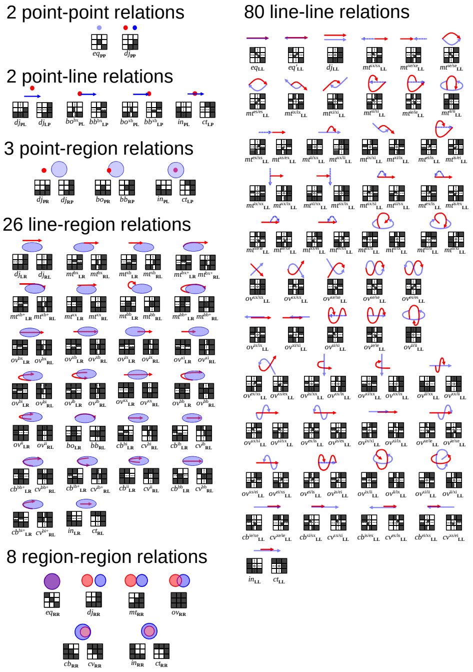
What is 9^+-Intersection?

9^+-intersection [1, 2] is a spatial model by which topological relations between two objects are distinguished by the black-and-white patterns of icons with 3×3 or more cells

This model is a refined version of the famous *9-intersection* model[3]



in a 2D Euclidian space $\mathbf{R}^2$, for instance, we can distinguish[2]:

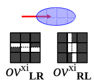


9^+-Intersection Calculus

Let D_1 and D_2 be two object domains (e.g., R: regions, L: Lines, and P: Points) and $\mathcal{T}^+_{D_1D_2\mathcal{S}}$ be a set of topological relations between an object in D_1 and an object in D_2, both embedded in a space $\mathcal{S}$

Converse

Converse of a relation in $\mathcal{T}^+_{D_1D_2\mathcal{S}}$ is a relation in $\mathcal{T}^+_{D_2D_1\mathcal{S}}$ who has a transposed icon pattern.



Let the list of converse of all relations in $\mathcal{T}^+_{D_1D_2\mathcal{S}}$ be $\text{CL-}\mathcal{T}^+_{D_1D_2\mathcal{S}}$

Composition

Under the 9^+-intersection, compositions of two arbitrary relations can be determined <u>systematically</u> by only four constraints (see our paper)

This is an advantage of 9^+-intersection over 9-intersection

Let the composition table of relations in $\mathcal{T}^+_{D_1D_2\mathcal{S}}$ and relations in $\mathcal{T}^+_{D_2D_3\mathcal{S}}$ be $\text{CT-}\mathcal{T}^+_{D_1D_2D_3\mathcal{S}}$

Integration

In order to conduct spatial reasoning within the existing framework of QSC, we consider a generalized object domain D* and generalized base relations $\mathcal{B}^*$ between two objects in D*

e.g., in the case of a 2D Euclidean space $\mathbf{R}^2$

$D^* = R \cup L \cup P$

$\mathcal{B}^* = \left(\bigcup_{D_1,D_2\in\{R,L,P\}} \mathcal{T}^+_{D_1D_2\mathbb{R}^2}\right) \cup \{eq^*\}$

Then, the list of converse pairs in $\mathcal{B}^*$ is derived by concatenating all relevant lists of converse relations $\text{CL-}\mathcal{T}^+_{D_1D_2\mathcal{S}}$ $(D_1, D_2 \in\{R, L, P\})$ and adding an item that indicates the converse of eq* is eq*

Similarly, the composition table for $\mathcal{B}^*$ is derived by adjoining all relevant composition tables $\text{CT-}\mathcal{T}^+_{D_1D_2D_3\mathcal{S}}$ $(D_1, D_2, D_3 \in\{R, L, P\})$ and adding a line/column for eq*

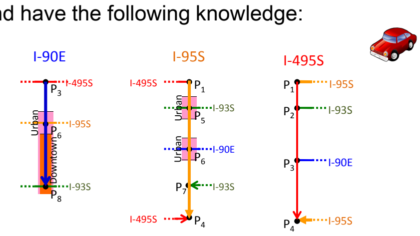
Now that we have prepared all of the above four elements of ordinary QSC, we can conduct qualitative spatial reasoning computationally by an existing QSC solver (say, SparQ[4])

An Example Application

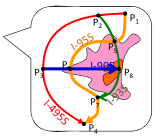
Our spatial knowledge grows by everyday experience. From such imperfect knowledge, what spatial model we can build?

For instance, let's consider Boson's highway network, which consists of I-90, I-95, I-93, and I-495.

Imagine we have driven I-90, I-95, and I-495 and have the following knowledge:



From this knowledge, can we build a <u>unique</u> model of the highway network, just like the actual one?



If we use the 9^+-intersection calculi, we can deduce that the I-93 goes through both Boston's downtown and urban area and there is no other solution

Similarly, when we consider the imaginary drive on three of the four highways, we can deduce the possible relations between unvisited highway and two areas as follows:

Unvisited highway x	Candidates for the relation pair [(x, downtown), (x, urban)]
I-90E	$[ov^{xi}_{LR}, ov^{xi}_{LR}]$
I-93S	$[ov^{xx}_{LR}, ov^{xx}_{LR}]$
I-95S	$[dj_{LR}, ov^{xx}_{LR}]$ $[mt^{xx}_{LR}, ov^{xx}_{LR}]$ $[ov^{xx}_{LR}, ov^{xx}_{LR}]$
I-495S	$[dj_{LR}, dj_{LR}]$ $[dj_{LR}, mt^{xx}_{LR}]$ $[dj_{LR}, ov^{xx}_{LR}]$ $[mt^{xx}_{LR}, mt^{xx}_{LR}]$ $[mt^{xx}_{LR}, ov^{xx}_{LR}]$ $[ov^{xx}_{LR}, ov^{xx}_{LR}]$

Conclusions

- We have identified the composition tables of 9^+-intersection-based relations using simple rules
- We developed a series of qualitative spatial calculi based on the 9^+-intersection, making use of the existing framework of ordinary binary spatial calculi
- The developed calculi allow more finer reasoning than the previous 9-intersection calculi[5]

References

[1] Yohei Kurata and Max J. Egenhofer (2007) The 9+-Intersection for Topological Relations between a Directed Line Segment and a Region. in: 1st Workshop on Behaviour Monitoring and Interpretation (BMI'07), Universität Bremen TZI Technical Report 42, 62-76.

[2] Kurata, Y. (2008) The 9+-Intersection: A Universal Framework for Modeling Topological Relations. In: GIScience 2008, LNCS 5266, 181-198

[3] Max J. Egenhofer and John R. Herring (1991) Categorizing Binary Topological Relationships between Regions, Lines and Points in Geographic Databases. In: NCGIA Technical Reports 91-7

[4] Jan Oliver Wallgrün, Lutz Frommberger, Diedrich Wolter, Frank Dylla, and Christian Freksa (2006) Qualitative Spatial Representation and Reasoning in the SparQ-Toolbox. In: Spatial Cognition V. LNAI 4387.

[5] Yohei Kurata (2009) 9-Intersection Calculi for Spatial Reasoning on the Topological Relations between Multi-Domain Objects. In: IJCAI Workshop on Spatial and Temporal Reasoning.