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Finite Displacement Analysis of Framed Structures with Non-wrapped Parallel Wire Cable Members

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In recent years, effects of secondary stresses, which occur in framed structures stiffened with non-wrapped parallel wire cables during erection, have become a matter of great importance in the field of quality control.

In this paper, more accurate formulae for non-wrapped parallel wire cable members, subjected to secondary stresses due to clamping of bands and local bending of wires, are proposed in order to be directly introduced into a program of an ordinary displacement method for finite displacement analysis. Then, the formulae are extended to enable more efficient analysis without an increase in the number of nodes.

Furthermore, justification of the formulae and efficiency of the program are verified by the results of some numerical applications, and general features of the effects are discussed.

I. INTRODUCTION

As of late, the effects of secondary stresses, which occur in framed structures stiffened with non-wrapped parallel wire cables during erection, have become an important problem encountered in the evaluation of the safety factor of cables and also standard displacements of superstructures. Therefore, more accurate estimation of the effects is nowadays regarded as a matter of major importance from the viewpoint of quality control of erected works.

Since Wyatt¹⁾ performed a basic investigation to systematize such complicated phenomena, studies in this field have been done by many researchers,²⁾ among which a study done by Nishimura *et al.*³⁾ is especially interesting. They presented an analytical method of plane framed structures stiffened with non-wrapped parallel wire cables during the erection, taking into consideration secondary stresses due to clamping of bands and local bending of wires, based on Wyatt's theory. In addition, adequacy of the method was verified experimentally. However, since rotations and elongations were treated as unknowns by neglecting longitudinal displacements their method, which needs a closing circuit condition consisting of member locations depending upon a structural type, is different from an ordinary displacement method and is not always accurate. Moreover, that method is inefficient under certain circumstances. In that method, a parallel wire cable between consecutive points, at which other cables are connected and hanger members are anchored, is divided into two kinds of members, namely a cable member (a freely slipping part) and band members (clamping band parts on both sides), and an increase in the number of nodes is inevitably required.

On the other hand, according to the extensive development of finite element method and finite displacement theory, programs for ordinary displacement methods have come to be applied most generally to geometrical non-linear analyses of ordinary plane framed

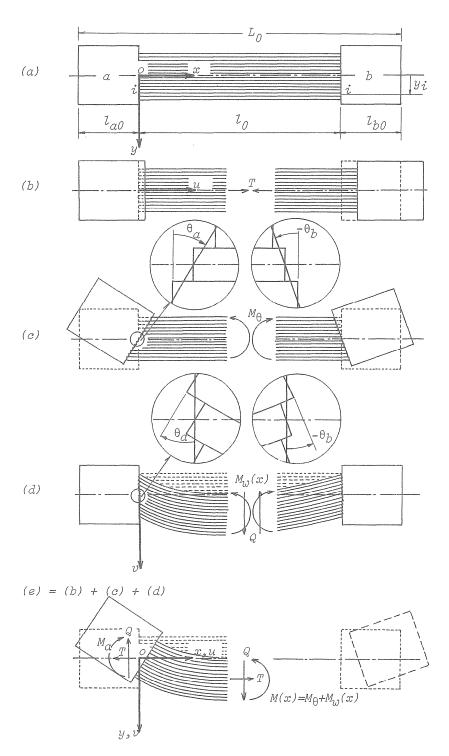


Fig. 1. Non-wrapped parallel wire cable member.

structures. Consequently, the authors⁴⁾ already dealt with several kinds of cables which stiffen plane-framed structures, and studied the effect of sag or sliding support by introducing the proposed formulae into a program for an ordinary displacement method.

This paper presents the following contents.5) First, by using the finite element method

and the finite displacement theory, more accurate formulae for a freely slipping part, subjected to secondary stresses due to clamping of bands and local bending of wires which are based on Wyatt's theory, are proposed in order to be directly introduced into a program for the ordinary displacement method for ordinary plane-framed structures. Then, the formulae are extended to *the ones* for a non-wrapped parallel wire cable member by combining with clamping band parts on the both sides, for the development of more efficient analysis without an increase in the number of nodes. Next, justification of the formulae and efficiency of the program are examined using some numerical examples. Finally, from the results of the finite displacement analysis of an actual framed structure stiffened with non-wrapped parallel wire cable members during erection, the authors attempt to find general features of the effects of the secondary stresses.

II. DEFINITION OF SECONDARY STRESSES

The theory herein is based on the following assumptions in addition to usual ones:

- 1) A non-wrapped parallel wire cable member with a non-stressed length L_0 , illustrated in Fig.1, contains a freely slipping part and clamping band parts on both sides between consecutive points at which cable members are connected and hanger members are anchored.
- 2) In the freely slipping part with the non-stressed length l_0 , wires are not only extensible but also flexible, and frictional resistance between wires is negligible.
- 3) The clamping band parts with non-stressed lengths, l_{a0} and l_{b0} , are extensible but not flexible.
- 4) The shape of any cross section remains constant, and the deformation due to own weight is negligible.

Then, in the freely slipping part with Young's modulus E_c and sectional area A_c , the authors define the tension T and the bending moment M(x) with the following equations expressed by the notations shown in Fig.1, which are based on Wyatt's theory in the same way as in Reference:

$$T = E_c A_c \cdot u'(x),\tag{1}$$

$$M(x) = M_a + Q \cdot x - T \cdot v(x) = M_\theta + M_\nu(x). \tag{2}$$

In Eq.(2), the moments, M_{θ} and $M_{w}(x)$, are very secondary ones due to clamping of bands and local bending of wires, respectively, as follows:

$$M_{\theta} = \frac{E_{c}I_{c}}{I_{0}}(\tan\theta_{a} - \tan\theta_{b}), \qquad M_{w}(x) = -E_{c}I_{n} \cdot v''(x), \tag{3}$$

where

$$I_c = \sum_{i=1}^{n_w} A_w y_i^2, \qquad I_n = n_w \cdot I_w.$$
 (4)

and A_w , I_w , and n_w are the area, the moment of inertia, and the total number of wires, respectively.

Hence, the primary stress σ_t due to T, the maximum secondary stress σ_θ due to M_θ which

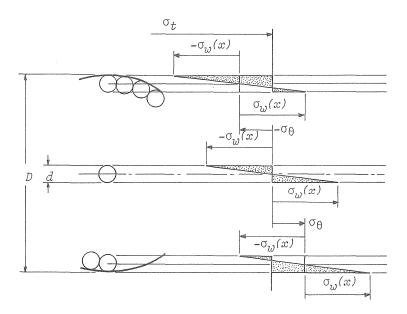


Fig. 2. End-displacements and end-forces.

arises in the extreme wire, and the stress $\sigma_w(x)$ due to $M_w(x)$ which arises in the extreme fiber of each wire, illustrated in Fig.2, are described as follows:

$$\sigma_t = \frac{T}{A_c}, \quad \sigma_\theta = \frac{M_\theta}{I_c} \cdot \frac{D - d}{2}, \quad \sigma_w(x) = \frac{M_w(x)}{I_v} \cdot \frac{d}{2},$$
 (5)

where D and d are the diameters of the cross section and of each wire, respectively.

III. FORMULAE FOR FREELY SLIPPING PART

Consider a non-wrapped parallel wire cable member of which a coordinate axis is expressed as au X^* -axis, illustrated in Fig.3, under a deformed state. Further, let the system coordinates of both ends be (X_{a0}, Y_{a0}) and (X_{b0}, Y_{b0}) under the undeformed state.

In the freely slipping part, by supposing the x-axis paralle to the x'-axis in order to completely exclude a rigid-body displacement, the notations in Fig. 3 exactly agree with the ones in Fig. 1.

Now, on the basis of a well-established principle of the finite element method, it is necessary to introduce shape functions. In this paper, the following forms involving hyperbolic function terms may be adopted:

$$u(x) = c_1 x + c_2, v(x) = C_3 \cosh g_n x + C_4 \sinh g_n x + C_5 x + C_6.$$
 (6)

It is not possible to apply a solution of the equation obtained by expanding the latter equation in a series which will apparently diverge under the following characteristic condition of the non-dimensional parameter $g_n l_0$:

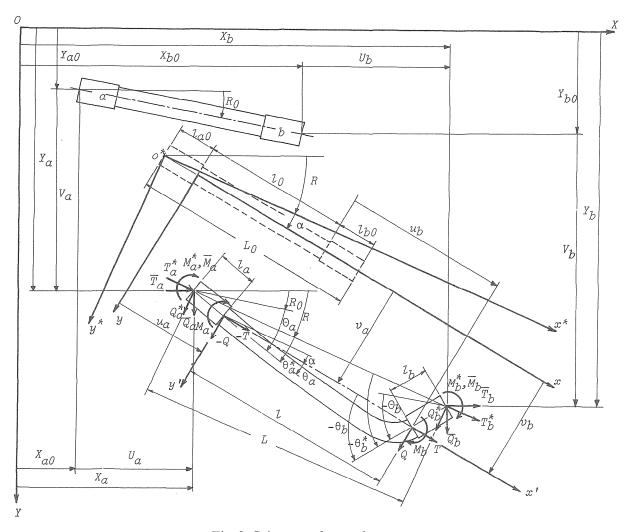


Fig. 3. Primary and secondary stresses.

$$g_n l_0 = \sqrt{\frac{T l_0^2}{E_c I_n}} \gg 1.$$
 (7)

On the other hand, displacements and strains will be related to each other by the curvature $\rho_i(x)$ and the axial strain $\varepsilon_{xi}(x)$ of the *i*-th wire, which may be given by the following equations on the basis of finite displacement theory:

$$\rho_{i}(x) \doteq -v''(x),$$

$$\varepsilon_{xi}(x) \doteq u'(x) + \frac{1}{2}v'(x) + \frac{(\tan \theta_{a} - \tan \theta_{b})y_{i}}{l_{0}}.$$
(8)

where the third term in the latter equation is the characteristic one.

Consequently, omitting the higher order terms, an equation connecting the force vector f to the displacement vector d, at both ends of the freely slipping part, may be derived by the stationary condition of potential energy as

$$f = k(d) \cdot d = (k_0 + k_1(d)) \cdot d,$$
 (9)

where

$$f = \{-T, -Q, M_a, T, Q, M_b\}^T,
 d = \{u_a, v_a, \tan \theta_a, u_b, v_b, \tan \theta_b\}^T,$$
(10)

and the matrices, k_0 , and $k_1(d)$, indicate the linear matrix and the non-linear matrix including the terms of the first degree with d, respectively.

Substituting the following equations into Eq. (9),

$$\begin{cases}
 u_b - u_a = l - l_0, \\
 v_b - v_a = 0,
 \end{cases}$$
(11)

and approximating the rearranged equation by effectively using the following condition,

$$\sinh g_n l_0 \doteq \cosh g_n l_0 \gg g_n l_0 \gg 1,\tag{12}$$

the newly proposed stiffness matrix k(d) for the freely slipping part is simplified and concretely written as follows:

$$k(d) = \begin{pmatrix} E_c A_c / l_0 & 0 & 0 & -E_c A_c / l_0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ & E_c I_c / l_0 & 0 & 0 & -E_c I_c / l_0 \\ & & E_c A_c / l_0 & 0 & 0 \\ & & & 0 & 0 \\ SYMM. & & & E_c I_c / l_0 \end{pmatrix}$$

$$+ \begin{pmatrix} 0 & A & B & 0 & -A & C \\ & D & E & -A & -D & E \\ & & F & -B & -E & G \\ & & 0 & A & -C \\ & & D & -E \\ SYMM. & & & F \end{pmatrix}$$

$$(13)$$

where

$$A \doteq -\frac{E_c A_c}{2l_0} \frac{1}{g_n l_0} (\tan \theta_a + \tan \theta_b),$$

$$B \doteq -\frac{E_c A_c}{2} \frac{1}{g_n l_0} \tan \theta_a, \quad C \doteq -\frac{E_c A_c}{2} \frac{1}{g_n l_0} \tan \theta_b,$$

$$D \doteq \frac{E_c A_c}{2l_0^2} (l - l_0), \quad E \doteq \frac{E_c A_c}{2l_0} \frac{1}{g_n l_0} (l - l_0),$$

$$(14)$$

$$F \doteq \frac{E_c A_c}{2} \frac{1}{g_n l_0} (l - l_0), \quad G \doteq 0.$$

Moreover, denoting a tangential stiffness matrix with $\Delta k(d)$, an incremental form of Eq. (9) is shown as follows:

$$\Delta f = \Delta k(d) \cdot \Delta d = (k_0 + 2k_1(d)) \cdot \Delta d. \tag{15}$$

Then, by setting $E_c I_c$ and $g_n l_0$ equal to zero and infinity respectively, the newly proposed tangential stiffness matrix $\Delta(k)d$ can be checked with the one for ordinary cable members.

IV. COMBINING WITH CLAMPING BAND PARTS

Extending the newly proposed formulae to combine a freely slipping part with clamping band parts on both sides, the end-force vector \vec{F} in the system coordinates and the vector f^* in the member coordinates for a non-wrapped parallel wire cable member, illustrated in Fig.3, may be derived as follows:

$$\bar{F} = \bar{c} \cdot f^* = \bar{c} \cdot e \cdot c \cdot f, \tag{16}$$

where

$$\vec{F} = \{ \vec{T}_a, \ \bar{Q}_a, \ \overline{M}_a, \ \bar{T}_b, \ \bar{Q}_b, \ \overline{M}_b \}^T,
f^* = \{ T_a^*, Q_a^*, M_a^*, T_b^*, Q_b^*, M_b^* \}^T,$$
(17)

and the coordinate transformation matrices, \bar{c} , e, and c, are concretely expressed as

$$\bar{\mathbf{c}} = \begin{bmatrix} \bar{\mathbf{c}}_{ab} & \mathbf{o} \\ \mathbf{o} & \bar{\mathbf{c}}_{ab} \end{bmatrix}, \quad \bar{\mathbf{c}}_{ab} = \begin{bmatrix} \cos R & -\sin R & 0 \\ \sin R & \cos R & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$\mathbf{e} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ -l_a \sin \theta_a^* & l_a \cos \theta_a^* & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & l_b \sin \theta_b^* & -l_b \cos \theta_b^* & 1 \end{pmatrix},$$

$$\mathbf{c} = \begin{bmatrix} \mathbf{c}_{ab} & \mathbf{o} \\ \mathbf{o} & \mathbf{c}_{ab} \end{bmatrix}, \quad \mathbf{c}_{ab} = \begin{bmatrix} \cos \alpha & -\sin \alpha & 0 \\ \sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$
(18)

Morever, an incremental form of Eq. (16) may be approximated by the following form:

$$\Delta \vec{F} = \Delta K(D) \cdot \Delta D = \bar{c}(e \cdot c \cdot \Delta f + \Delta e \cdot c \cdot f), \tag{19}$$

where the vector

$$D = \{U_a, V_a, \tan \Theta_a, U_b, V_b, \tan \Theta_b\}^T$$
(20)

indicates the end-displacement vector in the system coordinates. Therefore, the extended tangential stiffness matrix $\Delta K(D)$ for the non-wrapped parallel wire cable member is readily obtained in the system coordinates as follows:

$$\Delta K(D) = \bar{c} \{ e \cdot c \cdot \Delta k(d) \cdot c^T \cdot e^T + \Delta e_f \} \bar{c}^T, \tag{21}$$

where

and the second term is inherent to rotations of the band parts.

Hence, if definite values are assigned to l, l_a , l_b , R, α , θ_a , θ_b , θ_a^* , and θ_b^* from computation results of the end-displacement D, it goes without saying that the newly proposed and extended formulae in this paper can be evaluated easily. However, the following transcendental equations from the geometrical compatibility condition must be solved in advance:

$$\begin{split} X_{a} &= X_{a0} + U_{a}, \ Y_{a} = Y_{a0} + V_{a}, \\ X_{b} &= X_{b0} + U_{b}, \ Y_{b} = Y_{b0} + V_{b}, \\ L &= \sqrt{(X_{b} - X_{a})^{2} + (Y_{b} - Y_{a})^{2}}, \\ \tan R &= \frac{Y_{b} - Y_{a}}{X_{b} - X_{a}}, \\ \tan \theta_{a}^{*} &= \tan(\theta_{a} + R_{0} - R), \ \tan \theta_{b}^{*} = \tan(\theta_{b} + R_{0} - R), \\ l &= \sqrt{\frac{\{L - (l_{a}\cos\theta_{a}^{*} + l_{b}\cos\theta_{b}^{*})\}^{2}}{+ \{-(l_{a}\sin\theta_{a}^{*} + l_{b}\sin\theta_{b}^{*})\}^{2}}, } \\ \tan \alpha &= \frac{-(l_{a}\sin\theta_{a}^{*} + l_{b}\sin\theta_{b}^{*})}{L - (l_{a}\cos\theta_{a}^{*} + l_{b}\cos\theta_{b}^{*})} \end{split}$$

$$\tan \theta_{a} = \tan(\theta_{a}^{*} - \alpha), \ \tan \theta_{b} = \tan(\theta_{b}^{*} - \alpha),$$

$$T = \frac{E_{c}A_{c}}{l_{0}}(l - l_{0}) + \frac{E_{c}A_{c}}{2g_{n}l_{0}}(\tan^{2}\theta_{a} + \tan^{2}\theta_{b}),$$

$$Q = -\frac{E_{c}A_{c}}{g_{n}l_{0}^{2}}(l - l_{0})(\tan \theta_{a} + \tan \theta_{b}),$$

$$l_{a} = l_{a0}\left[1 + \frac{1}{E_{c}A_{c}}\left\{(T\cos\alpha - Q\sin\alpha)\cos\theta_{a}^{*} + (T\sin\alpha + Q\cos\alpha)\sin\theta_{a}^{*}\right\}\right],$$

$$l_{b} = l_{b0}\left[1 + \frac{1}{E_{c}A_{c}}\left\{(T\cos\alpha - Q\sin\alpha)\cos\theta_{b}^{*} + (T\sin\alpha + Q\cos\alpha)\sin\theta_{b}^{*}\right\}\right].$$

$$(23)$$

It has been assumed that the clamping band parts are extensibly different from the usual rigid bodies.

V. EXAMINATION OF JUSTIFICATION AND EFFICIENCY

For the purpose of verifying the justification and efficiency of the newly proposed and extended formulae, a numerical example for a non-wrapped parallel wire cable-structure with bands, illustrated in Fig.4, is computed by introducing these formulae into a program for an ordinary displacement method. In the computation, the principal dimensions are summarized in Table 1, and two loading cases are employed as shown in Fig.4. The ordinary analysis neglecting the secondary stresses (Analysis-1) as well as the proposed analysis are performed.

The secondary stresses at the right end of the freely slipping part of the member 3

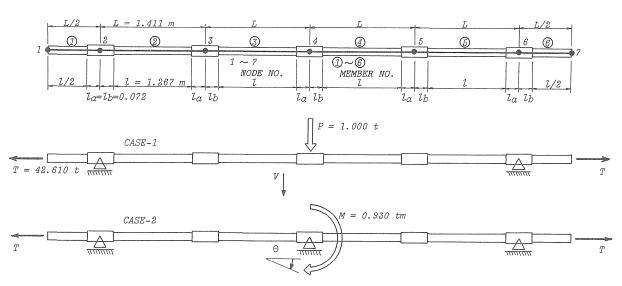


Fig. 4. Cable-structure.

Commence of the commence of th			
NO. OF WIRE	$n_{\mathcal{W}}$		217
DIAMETERS	đ	(mm)	5.000
DIAMETERS	D	(mm)	78.612
	$A_{\mathcal{C}}$	(m^2)	0.004261
SECTIONAL	$I_{\mathcal{C}}$	(m ⁴)	0.156×10 ⁻⁵
VALUES	I_n	(m ⁴)	0.666×10 ⁻⁸
	E_{α}	(t/m^2)	2.0×10 ⁷

Table 1. Summary of dimensions.

are illustrated in each case in Fig.5. The displacements at the nodes 3 and 4 are tabulated for each case in Table 2. For reference, the analytical and experimental values quoted from Ref. 3 are included in Fig.5 and Table 2. Then, Table 3 shows the number of iterations using the Newton-Raphson method. In this table, the values obtained by using the tangential stiffness matrix with no addition of the proper term to rotations of the band parts are given for comparison.

From the results of the numerical computations, it can be seen that the newly proposed and extended formulae are accurate enough for practical usage and satisfy a higher degree

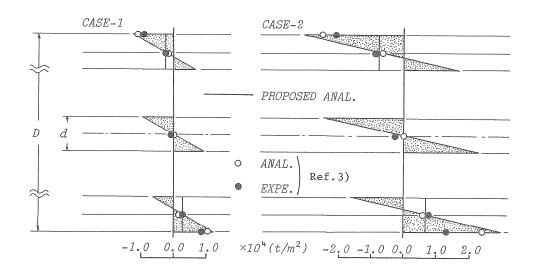


Fig. 5. Secondary stresses for each case.

Table 2. Displacements for each case.

		V	(m)	Θ (rad)		
			NODE 3	NODE 4	NODE 3	NODE 4
ANALYSIS		-1	0.0149	0.0298		
CASE-1 PROPOSED Ref.3)	PROPOSED	ANAL.	0.0134	0.0263		
	Pof 3)	ANAL.	0.013	0.026		
	EXPE.	0.013	0.027			
CASE_2	PROPOSED	ANAL.	-0.0010	0.0	0.0104	0.0255
	Ref.3)	ANAL.	-0.001	0.0	0.011	0.024
		EXPE.	-0.001	0.0	0.009	0.026

		INCREMENTAL STEP	NO. OF ITERATION	
	ANALYSIS-1	1	5	
CASE-1	PROPOSED	7	g	
	ANALYSIS	1		
	NO-ADDITION	1	15	
	PROPOSED	7	7	
CASE-2	ANALYSIS	. <i>I</i>	′	
	NO-ADDITION	1	32	

Table 3. Number of iteration.

of convergence. Moreover, it seems that these formulae can be easily introduced into a program using the ordinary displacement method and enable more efficient analysis without an increase in the number of nodes.

VI. FINITE DISPLACEMENT ANALYSIS OF ACTUAL STRUCTURE

As a representative example of an actual framed structure stiffened with non-wrapped parallel wire cable members, a suspension bridge, of which superstructures are under erection, illustrated in Fig.6, is treated, and principal dimensions of the model are summarized in Table 4. The erection process is divided into six steps as shown in Fig.6, in which each horizontal bar indicates the range of suspended structures already erected. After completion of the erection, the suspended structures are loaded uniformly throughout the spans by the deck weight.

Primary and secondary stresses arising at the specific points in the main cable are traced at each step, as shown in Fig.7. Then, the horizontal and vertical displacements at the extreme end of the suspended structures already erected in the center span are tabulated at

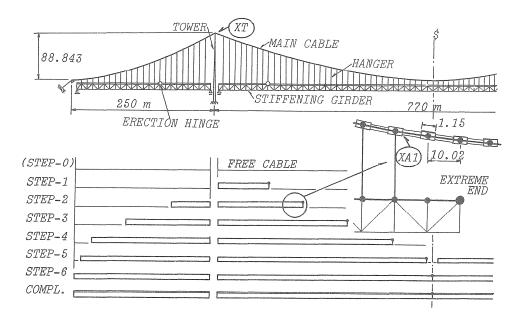


Fig. 6. Actual structure during erection.

Table 4. Dimensions of members.

SPAN			SIDE	CENTER	
NO. OF WIRE		$n_{\mathcal{W}}$		11,557	
DIAMETERS		d	(mm)	5.	. 170
		D	(mm)	618.000	
		$A_{\mathcal{C}}$	(m^2)	0.2426	
	MAIN		(m ⁴)	0.580×10 ⁻²	
	CABLE	In	(m ⁴)	0.405×10 ⁻⁶	
		$E_{\mathcal{C}}$	(t/m^2)	2.0×10 ⁷	
	HANGER	Ah	(m^2)	0.0061	
SECT.		Ih	(m ⁴)	0.0	
VALUES		Eh	(t/m^2)	1.4×10 ⁷	
	STIFF. GIRDER	Ag	(m^2)	0.0943	0.1223
		\underline{Ig}	(m ⁴)	1.9102	2.4764
		\underline{Eg}	(t/m^2)	2.1×10 ⁷	
	TOWER	Αt	(m^2)	0.7526	,0.8995
		Ιt	(m ⁴)	2.1027,3.701	
		Et	(t/m^2)	2. :	1×107

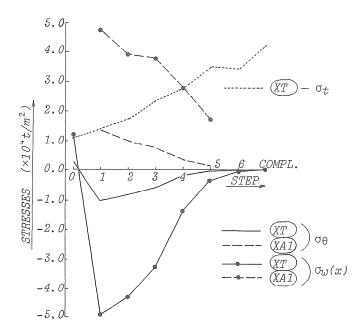


Fig. 7. Primary and secondary stresses at each step.

Table 5. Displacements at each step.

				(m)
	NEGLECTE)	CONSIDERE	:D
	SECONDARY	Z STRESSES	SECONDARY	STRESSES
	HORIZ.	VERTI.	HORIZ.	VERTI.
STEP-1	-1.613	4.347	-1.575	4.197
STEP-2	-1.509	1.535	-1.493	1.488
STEP-3	-0.836	-2.716	-0.829	-2.729
STEP-4	-0.193	-3.445	-0.191	-3.453
STEP-5	0.023	-0.688	0.023	-0.696
STEP-6	0.0	-1.087	0.0	-1.086
COMPL.	0.0	0.0	0.0	0.0

each step in Table 5. For reference, the values obtained by neglecting the secondary stresses are given in this table.

From the results of the finite displacement analysis, it can be judged that the secondary stresses have large values not only in the main cable nearest the tower, but also in the one near the extreme end of the suspended structures already erected. Moreover, the effects of the secondary stresses are clearly observed in displacements at the extreme end of the suspended structures already erected.

VII. CONCLUSIONS

From the afore-mentioned results of the numerical applications, the following conclusions may be drawn:

- (1) The newly proposed and extended formulae are accurate enough for practical use, and the tangential stiffness matrix satisfies a higher degree of convergence by involving the proper term to rotations of the band parts.
- (2) It seems that these formulae can be easily introduced into a program using the ordinary displacement method without an increase in the number of modes, so that the finite displacement analysis of plane framed structures, which are stiffened with non-wrapped parallel wire cable members during the erection, can be performed more accurately and efficiently.
- (3) It is confirmed that the secondary stresses due to clamping of bands and local bending of wires are not negligible at the time of evaluating not only a safety factor for cables, but also of standard displacements of superstructures, and some countermeasures are required to ensure quality during erection.
- (4) Because there is a great possibility that analytical solutions of certain design problems due to the effects of secondary stresses will be indispensable for the plan of actual framed structures stiffened with parallel wire cables, programming of the formulae in this paper is expected to be accepted for wide application in no distant future.

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