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Non-Linear Analysis of Cable-Stiffened Structures

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In recent years, the finite element method has been extensively developed for solving many kinds of problems on the structural behavior of complicated systems with geometrical non-linearity.

In this paper, the authors deal with four kinds of analyses of cable members which stiffen plane-framed structures. In addition to cable members which can be replaced simply by axial members, the analyses include discussions of the effect of cable sag, the effect of cable sliding over a roller and the effects of both on cable members.

Non-linear cable equations, stiffness matrices and tangential stiffness matrices for cable members are newly proposed, extended or examined in order to directly introduce the Newton-Raphson method and the incremental procedure for the large deformation analysis of general framed structures. Then, for justification of these solutions, the results of numerical applications to some kinds of cable-stiffened structures are described and discussed.

I. INTRODUCTION

The recent development of high speed computers and numerical analysis techniques has enabled the finite element method to apply to the analysis of complicated structures with geometrical nonlinearity. In particular, the studies[1-7] of non-linear cable and frame interaction of cable-stiffened structures have developed rapidly, and those achievements have become a center of attention although they are still at an incomplete stage. The purposes of the present study are to investigate the problems in detail and then to propose more accurate and advantageous methods for the analysis of cable-stiffened structures.

In this paper, the authors deal with four kinds of analyses of cable members which stiffen plane-framed structures. In addition to cable members which can be replaced simply by axial members, the analyses include discussions of the effect of cable sag, the effect of cable sliding over a roller and the effect of both on cable members.

By replacing a sagging cable member with linking axial members, the effect of sag is taken into consideration fairly well. The approach, however, requires the calculation of a tangential stiffness matrix with a great band width in general. Therefore, the greatest difficulties in the method are to occupy a great part of the storage area of a computer and to spend long operating time. As alternative methods, Ernst,[1] Livesley,[2,3] Tang[4] and Lazar[5] each proposed a modified elastic modulus of a sagging cable, and on examination it was found that all of them agreed precisely. However, judging from the assumption used in the theory, the modified modulus holds only in limited cases, for example, systems with small non-linearity and cable members with a fairly low sag ratio. Moreover, Chu[6] proposed a different analytical procedure using a modified elastic modulus along with a non-linear cable equation. The method is one of the suitable methods of inquiring closely into the effect of sag. But, in order to be more reliable and reasonable, an actual tagential stiffness matrix should be used instead of the modified modulus according to the non-linear cable equation.

On the other hand, regarding a roller-supported cable member, Lazar[5] proposed a stiffness matrix by considering the cable as a particular bilinear element. Therefore, the subject for future studies is to make possible its general application in a similar way to Lazar's method, and then to introduce a tangential stiffness matrix. Furthermore, the problem of a roller-supported cable member, which requires that the effect of its sag be taken into consideration, should be solved. But no reports on this problem have apparently been published to date.

On the aforesaid four different problems of cable members, nonlinear cable equations, stiffness matrices and tangential stiffness matrices will be newly proposed, extended or examined[7] in this paper, in order to directly introduce the Newton-Raphson method and the incremental procedure[8,9] into the large deformation analysis of general framed structures. Then, for justification of these solutions, the results of numerical applications to some kinds of cable-stiffened structures will be presented. Finally, from all of the results, a general feature of the non-linear analysis of cable-stiffened structures by the finite element method will be discussed.

II. EQUILIBRIUM EQUATIONS AND SOLUTION PROCEDURE

Non-linear algebraic equivalents to equilibrium equations, for the large deformation analysis of general plane-framed structures, are obtained in terms of system coordinates as:

$$\boldsymbol{R}(\boldsymbol{D}) = \boldsymbol{F} \tag{1}$$

where $\boldsymbol{R}(\boldsymbol{D})$ is the internal resisting force vector and contains both linear and non-linear terms in the nodal displacement vector $\boldsymbol{D}$, and $\boldsymbol{F}$ is the externally applied force vector.

Most solution procedures of the non-linear equations (1) are based on the Newton-Raphson method and the incremental procedure. For example, employing the mixed method, linearized equations to be calculated, under the $(i+1)$-th interation step at the n-th incremental sted, are written as follows:

$$\Delta\boldsymbol{K}(\boldsymbol{D}_n^i) \cdot \Delta\boldsymbol{D}_n^{i+1} = \Delta\boldsymbol{Re}_n^i \tag{2}$$

where $\Delta\boldsymbol{D}$ is the incremental nodal displacement vector and $\Delta\boldsymbol{Re}$, called the residual force vector, is previously derived by calculating the following equations:

$$\Delta\boldsymbol{Re}_n^i = \boldsymbol{F}_n - \boldsymbol{R}(\boldsymbol{D}_n^i). \tag{3}$$

Namely, the solution procedure requires, in general, the tangential stiffness matrix $\Delta\boldsymbol{K}(\boldsymbol{D})$ and the internal resisting force vector $\boldsymbol{R}(\boldsymbol{D})$, expressed under any deformed state. The accuracy of both influences the dependability and reasonableness of the solution procedure.

III. FORMULATION FOR CABLE MEMBERS

This chapter is devoted to non-linear cable equations, stiffness matrices and tangential stiffness matrices for four different models of cable members, which are introduced into the solution procedure briefly explained in Section II. It goes without saying that internal resisting force vectors are derived from the former two.

1) Cable Member Replaced by Axial Member

Because studies on this model have been reported by many researchers, the formulation for this model is omitted here.

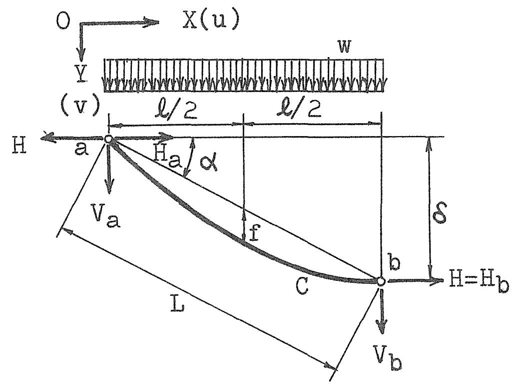


Fig. 1. Sagging cable member.

2) Sagging Cable Member

It may be assumed that the total dead weight of a cable member, instead of uniform distribution along its length, is nearly distributed uniformly along its total span. Namely, a curve which a perfectly flexible cable member takes, when freely suspended between two supports under its deformed state, can be a parabola.

(1) Non-linear cable equation and internal resisting force vector

Consider a cable member suspended as shown in Fig.1 between two supports a and b under its deformed state and loaded so that a vertical load w per unit length of the span can be constant. Let H be the horizontal component of cable tension; then it will be expressed from the parabola principle as follows:

$$H = \frac{wl^2}{8f}. \tag{4}$$

Therefore, equations of the sag ratio n, the length C and the elongation ΔC of the cable member are given by the following expressions, respectively:

$$n = \frac{f}{l}, \tag{5}$$

$$C = \int_0^l \sqrt{1 + \left(\frac{dY}{dX}\right)^2}\, dX$$

$$= \frac{l}{16n}\Big[(4n + m)\sqrt{1 + (4n + m)^2} + (4n - m)\sqrt{1 + (4n - m)^2}$$

$$+ \log_e \left\{ (\sqrt{1+(4n+m)^2} + 4n + m).(\sqrt{1+(4n-m)^2} + 4n - m) \right\} \Big] \tag{6}$$

and

$$\Delta C = \int_0^l \frac{H}{EA} \left\{ 1 + \left(\frac{dY}{dX} \right)^2 \right\} dX = \frac{Wl}{8nEA} \left(1 + \frac{16}{3} n^2 + m^2 \right) \tag{7}$$

where

$$m = \frac{\delta}{l}, \qquad W = wl\,(=\text{const}).$$

Then, denoting the length of the cable member with C_0 under its undeformed non-stress state, the compatibility equation is written as follows:

$$C - (C_0 + \Delta C) = 0. \tag{8}$$

Substituting Eqs. (6) and (7) into Eq. (8), and multiplying by n, the following non-linear equation $\Phi(n)$ for the sag ratio n is obtained:

$$\Phi(n) = \frac{l}{16} \Big[(4n+m)\sqrt{1+(4n+m)^2} + (4n-m)\sqrt{1+(4n-m)^2} + \log_e \left\{ (\sqrt{1+(4n+m)^2} + 4n + m)(\sqrt{1+(4n-m)^2} + 4n - m) \right\} \Big] - \frac{Wl}{8EA} \left(1 + \frac{16}{3} n^2 + m^2 \right) - nC_0 = 0. \tag{9}$$

This equation is what the authors call the cable equation. Namely, if a definite value is assigned to l and δ, Eq.(9) will be a function of the sag ratio n.

Consequently, from the values of l, δ and n, the member end-force vector $\boldsymbol{f}(H_a, V_a, H_b, V_b)$ is calculated by the following equations:

$$\left.\begin{aligned} H_a &= -\frac{W}{8n}, & V_a &= -\frac{W}{2}\left(1 + \frac{m}{4n}\right), \\ H_b &= \frac{W}{8n}, & V_b &= -\frac{W}{2}\left(1 - \frac{m}{4n}\right). \end{aligned}\right\} \tag{10}$$

Moreover, the internal resisting force vector $\boldsymbol{R}(\boldsymbol{D})$ under its deformed state is easily calculated by summarizing the member end-force vector $\boldsymbol{f}$ of all members.

(2) Tangential stiffness matrix

The compatibility equation (8) is approximated as follows, by expanding Eq.(6) in a series and omitting the terms higher than the fourth order for the small sag ratio n:

$$l\sqrt{1+m^2} + \frac{8l}{3(1+m^2)^{3/2}} n^2 - \frac{Wl}{8nEA}\left(1 + m^2 + \frac{16}{3} n^2\right) - C_0 = 0 \tag{11}$$

Since the increment of the sag ratio, Δn, is in relationship with Δl and $\Delta\delta$, the differentiation of Eq.(11) gives the following expression:

$$\Delta n = -\frac{8n^2}{W} k \left[\left\{\frac{C_0}{L} + \left(1 - 2\frac{C_0}{L}\right)\left(\frac{\delta}{L}\right)^2 - \frac{4W}{3EA}\left(\frac{l}{L}\right)\left(\frac{\delta}{L}\right)^2 n + \frac{40}{3}\left(\frac{l}{L}\right)^4\left(\frac{\delta}{L}\right)^2 n^2\right\}\Delta l \right.$$
$$\left. - \left\{\left(1 - 2\frac{C_0}{L}\right)\left(\frac{l}{L}\right)\left(\frac{\delta}{L}\right) - \frac{4W}{3EA}\left(\frac{l}{L}\right)^2\left(\frac{\delta}{L}\right) n + \frac{40}{3}\left(\frac{l}{L}\right)^5\left(\frac{\delta}{L}\right) n^2\right\}\Delta\delta\right] \tag{12}$$

where

$$k = \frac{EA}{L} \Big/ \left\{1 - \frac{16}{3}\left(\frac{l}{L}\right)^2 n^2 + \frac{128}{3}\frac{EA}{W}\left(\frac{l}{L}\right)^5 n^3\right\}, \quad L = \sqrt{l^2 + \delta^2}.$$

Similarly, by differentiating Eq. (10), the following equations are given:

$$\left.\begin{aligned} \Delta H_a &= -\Delta H_b = \frac{W}{8n^2}\Delta n, \\ \Delta V_a &= -\Delta V_b = \frac{W}{8n^2}\left(m\Delta n + \frac{mn}{l}\Delta l - \frac{n}{l}\Delta\delta\right) \end{aligned}\right\} \tag{13}$$

where $\Delta f(\Delta H_a, \Delta V_a, \Delta H_b, \Delta V_b)$ is the increment of the member end-force vector.

The increment of the member end-displacement vector, $\Delta \boldsymbol{d}(\Delta U_a, \Delta V_a, \Delta U_b, \Delta V_b)$, is related to Δl and $\Delta\delta$ by the following equations:

$$\begin{aligned} \Delta U_b - \Delta U_a &= \Delta l \\ \Delta V_b - \Delta V_a &= \Delta\delta. \end{aligned} \tag{14}$$

Therefore, by eliminating the increment of the sag ratio, Δn, from Eqs. (12) and (13), and by substituting Eq. (14) into the equations, the relation between the increment of the member end-force vector, $\Delta \boldsymbol{f}$, and that of the member end-displacement vector, $\Delta \boldsymbol{d}$, may be derived as:

$$\Delta \boldsymbol{k}(n) \cdot \Delta \boldsymbol{d} = \Delta \boldsymbol{f} \tag{15}$$

in which the tangential stiffness matrix $\Delta \boldsymbol{k}(n)$ in system coordinates under its deformed state is concretely shown by the following form:

$$\Delta \boldsymbol{k}(n) = \boldsymbol{C} \cdot \Delta \boldsymbol{k}^* \cdot \boldsymbol{C}^T \tag{16}$$

where $\Delta \boldsymbol{k}^*$, shown as follows, corresponds to the tangential stiffness matrix in member coordinates:

$$\left.\begin{array}{l} \Delta \boldsymbol{k}^* = \begin{bmatrix} \boldsymbol{k}' & -\boldsymbol{k}' \\ -\boldsymbol{k}' & \boldsymbol{k}' \end{bmatrix}, \quad \boldsymbol{k}' = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}, \\ a = \dfrac{C_0}{L}, \quad b = \left(1 - \dfrac{C_0}{L}\right) - \dfrac{4W}{3EA}\left(\dfrac{l}{L}\right) n + \dfrac{40}{3}\left(\dfrac{l}{L}\right)^4 n^2 \end{array}\right\} \tag{17}$$

and $\boldsymbol{C}$, shown as follows, corresponds to the coordinate transformation matrix:

$$\boldsymbol{C} = \begin{bmatrix} \boldsymbol{C}' & \boldsymbol{O} \\ \boldsymbol{O} & \boldsymbol{C}' \end{bmatrix}, \quad \boldsymbol{C}' = \begin{bmatrix} \cos\alpha, & -\sin\alpha \\ \sin\alpha, & \cos\alpha \end{bmatrix}. \tag{18}$$

Hence, the tangential stiffness matrix of the entire structure, $\Delta\boldsymbol{k}(\boldsymbol{D})$, is calculated by summarizing the matrix for each member, $\Delta\boldsymbol{k}(n)$.

3) Roller-supported Cable Member

It is assumed that the effects of frictional resistance between a cable and rollers and eccentricity due to the diameters of rollers supporting the cable member at some supports may be negligible. Then, suppose that the total dead weight of the cable member may be approximately concentrated at the supports instead of being distributed. In other words, in the case of this model, suppose that the effect of cable sag can be neglected.

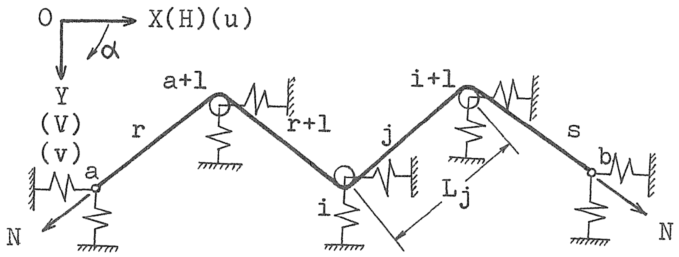


Fig. 2. Roller-supported cable member.

(1) Stiffness matrix and internal resisting force vector

Consider that the form shown in Fig.2 is a configuration of the roller-supported cable member in a deformed state. The cable member consists of axial elements, from the r-th to the s-th, which are supported by sliding over rollers at the supports, from the a-th to the b-th.

By adding the following condition of continuity of the elements:

$$N_j = N_{j+1} = N \qquad (j = r, r+1, \cdots, s-1), \tag{19}$$

the compatibility equation of the cable member is written as follows:

$$\sum_{j=r}^{s} \Delta L_j = \frac{N}{EA} \sum_{j=r}^{s} L_j \tag{20}$$

where N and ΔL_j are the tension and elongation of the j-th element, respectively.

The member end-displacement vector $\boldsymbol{d}(U_a, V_a, U_{a+1}, V_{a+1}, \ldots, U_b, V_b)$ is related to the elongation ΔL_j by the following equation:

$$\sum_{j=r}^{s} \Delta L_j = \boldsymbol{c}^T \cdot \boldsymbol{d} \tag{21}$$

where the vector $\boldsymbol{c}$, shown as follows, is a kind of coordinate transformation matrix:

$$\boldsymbol{c} = \{-\cos\alpha_r, -\sin\alpha_r, (\cos\alpha_r - \cos\alpha_{r+1}), (\sin\alpha_r - \sin\alpha_{r+1}), \\ \ldots, (\cos\alpha_{s-1} - \cos\alpha_s), (\sin\alpha_{s-1} - \sin\alpha_s), \cos\alpha_s, \sin\alpha_s\}^T. \tag{22}$$

Thus, from Eqs. (20) and (21), the tension N is related to the member end-displacement vector $\boldsymbol{d}$ as follows:

$$N = k \cdot \boldsymbol{c}^T \cdot \boldsymbol{d} \tag{23}$$

where

$$k = EA \,/\, \sum_{j=r}^{s} L_j.$$

Similarly, by using the transformation vector $\boldsymbol{c}$, the relation between the member end-force vector $\boldsymbol{f}(H_a, V_a, H_{a+1}, V_{a+1}, \ldots, H_b, V_b)$ and the tension N is given as:

$$\boldsymbol{f} = N \cdot \boldsymbol{c}. \tag{24}$$

Therefore, by substituting Eq. (23) into Eq.(24), the relation between the member end-force vector $\boldsymbol{f}$ and the member end-displacement vector $\boldsymbol{d}$ may be derived as:

$$\boldsymbol{k}_0 \cdot \boldsymbol{d} = \boldsymbol{f} \tag{25}$$

in which the stiffness matrix $\boldsymbol{k}_0$ in system coordinates is concretely shown in terms of the following expression:

$$\boldsymbol{k}_0 = k \cdot \boldsymbol{c} \cdot \boldsymbol{c}^T. \tag{26}$$

Consequently, if a definite value is assigned to the member end-displacement vector $\boldsymbol{d}$, the corresponding value of the member end-force vector $\boldsymbol{f}$ is immediately calculated from the above equation (25). Moreover, the internal resisting force vector $\boldsymbol{R}(\boldsymbol{D})$ under its deformed state is easily calculated by summarizing the member ene-force vector $\boldsymbol{f}$ of all members.

(2) Tangential stiffness matrix

By using the relationship equation expressed as:

$$\Delta\alpha_j = \{\sin\alpha_j(\Delta u_i - \Delta u_{i+1}) - \cos\alpha_j(\Delta V_i - \Delta V_{i+1})\}/L_j \quad (j = r, r+1, \ldots, s) \tag{27}$$

and by differentiating Eq. (22), the increment of the transformation vector, $\Delta\boldsymbol{c}$, is related to the one of the member end-displacement vectors, $\Delta \boldsymbol{d}(\Delta u_a, \Delta v_a, \Delta u_{a+1}, \Delta v_{a+1}, \ldots, \Delta u_b, \Delta v_b)$, as follows:

$$\Delta \boldsymbol{c} = \boldsymbol{h} \cdot \Delta \boldsymbol{d} \tag{28}$$

where

$$\boldsymbol{h} = \begin{pmatrix} \boldsymbol{h}_r & -\boldsymbol{h}_r & & & & \\ -\boldsymbol{h}_r & (\boldsymbol{h}_r + \boldsymbol{h}_{r+1}) & -\boldsymbol{h}_{r+1} & & & \\ & -\boldsymbol{h}_{r+1} & (\boldsymbol{h}_{r+1} + \boldsymbol{h}_{r+2}) & -\boldsymbol{h}_{r+2} & & \\ & & \ddots & \ddots & \ddots & \\ & & -\boldsymbol{h}_{s-1} & (\boldsymbol{h}_{s-1} + \boldsymbol{h}_s) & -\boldsymbol{h}_s \\ & & & -\boldsymbol{h}_s & \boldsymbol{h}_s \end{pmatrix}$$

$$\boldsymbol{h}_j = \frac{1}{L_j}\begin{bmatrix} \sin^2 \alpha_j & -\cos \alpha_j \sin \alpha_j \\ -\cos \alpha_j \sin \alpha_j & \cos^2 \alpha_j \end{bmatrix}$$
$$(j = r, r+1, \ldots, s).$$

The increment of the member end-force vector, $\Delta \boldsymbol{f}\,(\Delta H_a, \Delta V_a, \Delta H_{a+1}, \Delta V_{a+1}, \ldots, \Delta H_b, \Delta V_b)$, by differentiating Eq.(25) and neglecting higher-order terms, is given by the following equation:

$$\Delta \boldsymbol{f} = (k \cdot \boldsymbol{c} \cdot \boldsymbol{c}^T) \cdot \Delta \boldsymbol{d} + (k \cdot \boldsymbol{c}^T \cdot \boldsymbol{d})\, \Delta \boldsymbol{c} \tag{29}$$

Therefore, by substituting Eqs. (23), (26) and (28) into Eq. (29), the relation between the increment of the member end-force vector, $\Delta \boldsymbol{f}$, and that of the member end-displacement vector, $\Delta \boldsymbol{d}$, may be derived as:

$$\Delta \boldsymbol{k}(N) \cdot \Delta \boldsymbol{d} = \Delta \boldsymbol{f} \tag{30}$$

in which the tangential stiffness matrix $\Delta \boldsymbol{k}(N)$ in system coordinates under its deformed state is concretely expressed by the following form:

$$\Delta \boldsymbol{k}(N) = \boldsymbol{k}_0 + N \cdot \boldsymbol{h}. \tag{31}$$

Hence, the tangential stiffness matrix of the entire structure, $\Delta \boldsymbol{K}(\boldsymbol{D})$, is calculated by summarizing the matrix for each member, $\Delta \boldsymbol{k}(N)$.

4) Combined Cable Member

As a convenient and temporary method (although not completely accurate), the model obtained by combining the above two cases of models, as explained in Fig.3, deals with the roller-supported cable member taking into account the effect of its sag.

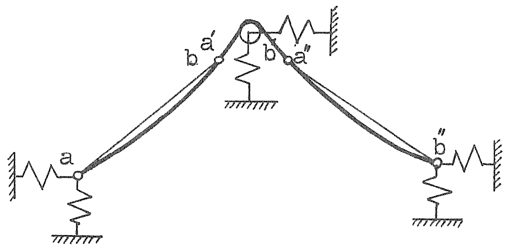


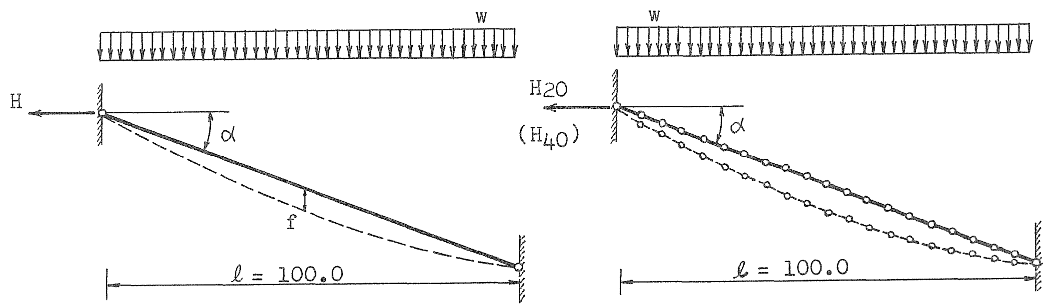
Fig. 3. Combined cable member.

IV. NUMERICAL APPLICATIONS

For the purpose of verifying justification of the formulation in Section III, several numerical examples of cable-stiffened structures are illustrated with calculation results in this Section.

1) Individual Cable

In order to justify the examined non-linear cable equation for a sagging cable member, an individual cable, subjected to an uniformly distributed load along its span as illustrated in Fig.4(a), is calculated. At this calculation, an ordinary method is also applied by replacing the sagging individual cable with linking axial members as shown in Fig.4(b). Then, by comparing the two cases, an accuracy of the non-linear cable equation will be scrutinized.



(a) Sagging Cable Member (b) Linking Axial Members

Fig. 4. Individual cable.

(1) Conditions for numerical application (input data)

In the calculation, the values of extensional rigidity EA of the cable is assumed to be 0.00001, and 30° and 60° are taken for the angle of inclination α.

To obtain the sag ratio f/l close to 0.2, which value has been said to be the upper limit available for the assumption based on the principle of parabola, the uniformly distributed load w is determined. Namely, at both angles, each load is increased with a constant increment 15.0 up to 75.0 or 1.0 up to 5.0.

On the other hand, in case of replacement with the linking axial members, the cable is divided into two kinds of values, 20 and 40.

(2) Results of Calculation (output data)

The sag ratio f/l and the horizontal component of tension H are shown in Table 1. In this table, numeral 20 or 40 indicates the number of division in case of replacing with the linking axial members.

Table 1. Results of calculation.

Angle	Load		Sag Ratio	Horiz. Compo. of Tension		
α	No.	w	f/l	H	H_{20}	H_{40}
30°	1	15.0	0.11479	1633.3	1631.9	1633.0
	2	30.0	0.14604	2567.7	2565.3	2567.1
	3	45.0	0.16859	3336.4	3333.3	3335.6
	4	60.0	0.18701	4010.5	4006.7	4009.6
	5	75.0	0.20293	4619.8	4615.4	4618.7
60°	1	1.00	0.11430	109.36	109.25	109.33
	2	2.00	0.14388	173.76	173.57	173.71
	3	3.00	0.16457	227.87	227.61	227.81
	4	4.00	0.18099	276.25	275.92	276.17
	5	5.00	0.19483	320.79	320.39	320.69

Although the difference between the sagging cable and the linking axial members tends to increase gradually as the value of the load increases, the former is in satisfactory agreement with the latter.

Therefore, it can be assumed that the sagging parabolic cable also agrees very well with an actual cable, and the examined nonlinear cable equation is accurate enough for practical usage.

2) Cable-stiffened Tower

Calculations by the newly proposed, extended or examined formulae for the four models of cable members are performed for the analysis of a cable-stiffened tower illustrated in Fig.5. As two cable members, X1 and X2, four models, namely, axial members, sagging cable members, roller-supported ones and combined ones, are applied.

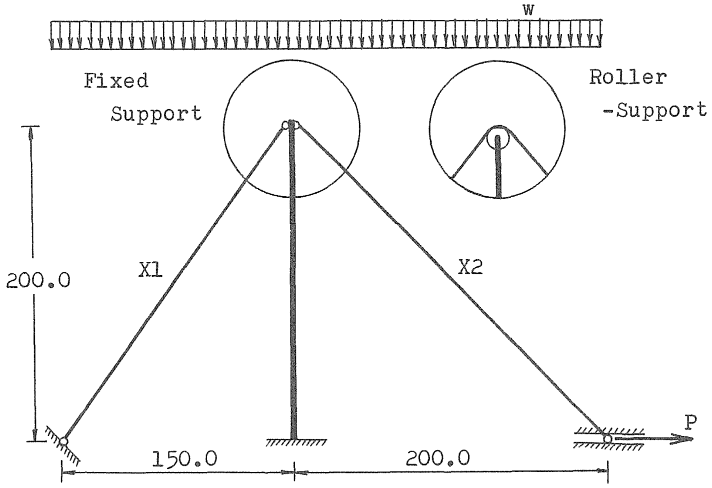


Fig. 5. Cable-stiffened tower.

Similarly, in order to justify these solutions for sagging cable members or combined ones, calculation by replacing with linking axial members is carried out at the same time.

(1) Conditions for numerical application (input data)

The following values are used for sectional area, moment of inertia and Young's modulus, respectively:

$$A_c = 0.01, \quad I_c = 0.0, \quad E_c = 2.0 \times 10^7,$$
$$A_t = 1.00, \quad I_t = 10.0, \quad E_t = 2.1 \times 10^7$$

where suffix c or t indicates a value of the cable members or the tower, respectively.

The tower is divided into 20 parts and the horizontal concentrated load P applied at the free-end and the uniformly distributed load w take the values, 500.0 and 1.0, respectively.

On the other hand, in case of replacement with the linking axial members, each cable is divided into 30 parts.

(2) Results of calculation (output data)

For the four models of cable members, several values indicative of their structural behavior are shown in Table 2. Corresponding values in case of replacement with the linking axial members are added in brackets.

Table 2. Results of calculation.

Calculated Item \ Cable Mem.			Axial Member	Sagging Cable Mem.	Roll.-Supp. Cable Mem.	Combined Cable Mem.
Horiz. Displ.	at Free-End of Cable		2.82	1.93 (1.93)	2.84	1.96 (1.95)
	at Top of Tower		1.40	1.17 (1.17)	1.06	0.95 (0.95)
Sag Ratio	X1-Cable		.0	.045	.0	.042
	X2-Cable		.0	.050	.0	.048
Tension	X1	Left Side	663	633 (637)	704	659 (663)
		Right Side	663	752 (752)	704	777 (779)
	X2	Left Side	705	780 (780)	704	777 (779)
		Right Side	705	639 (643)	704	639 (643)
Moment	at Fixed-End of Tower		21699	18103 (18065)	16446	14743 (14655)

In case of sagging cable members or combined ones, the calculation results agree very well with those obtained by replacement with the linking axial members.

In the case of roller-supported cable members or combined ones, it has been found that the converged values satisfy the condition of continuity and the compatibility equation under its deformed state.

Therefore, it can be assumed that the newly proposed, extended or examined formulae are solutions appropriate for inquiring into nonlinear cable and frame interaction of several kinds of cable-stiffened structures closely and advantageously.

V. CONCLUSIONS

From the results of the numerical calculations for cable-stiffened structures, the following conclusions may be drawn:

(1) Insofar as the assumption based on the principle of the parabola can be permitted, the examined non-linear cable equation and the newly proposed tangential stiffness matrix for the sagging cable member are accurate enough for practical use and, therefore, should be accepted for wide application.

(2) By using the extended stiffness matrix and the newly introduced tangential stiffness matrix, it is possible to analyze plane-framed structures stiffened with roller-supported cable members fairly accurately and rationally; these have never been solved until the present.

(3) As a convenient and temporary method until a complete one is found, the model of a roller-supported cable member taking into consideration the effect of its sag, newly assumed in this paper, will give fairly good accuracy in practical application. However, the development of the complete method is a problem to be solved in the future.

(4) By directly introducing these solutions into the Newton-Raphson method and the in cremental procedure, the non-linear analysis of plane-framed structures stiffened by cable members becomes more reliable and reasonable than the previous analyses by the finite element method.

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