

Errata and Comments for “Energy of knots and conformal geometry”

Jun O’Hara

Department of Mathematics, Tokyo Metropolitan University

e-mail : ohara@tmu.ac.jp

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Abstract

This article serves as errata of the book

“*Energy of knots and conformal geometry*”, Series on Knots and Everthing Vol. 33,
World Scientific, Singapore, 304 pages, (2003).

The updated version is available through web, linked from the author’s Home Page:

<http://www.comp.tmu.ac.jp/knotNRG/indices/indexe.html>

The list below would be far from being completed¹.

Suggestions and comments would be deeply appreciated.

Key words and phrases. Energy, knot, link, Möbius geometry, conformal geometry, cross ratio
1991 *Mathematics Subject Classification.* Primary 57M25, Secondary 53A30

- Throughout the book ${}_nC_p$ means the number of combination $\binom{n}{p} = \frac{n!}{p!(n-p)!}$. The author is sorry for the Japanese notation.

- Page 3. The second paragraph

“Let M denote \mathbb{R}^3 or S^3 . Two knots f and f' in M are called **isotopic** if there is an isotopy $h_t : M \rightarrow M$ ($t \in [0, 1]$) of the ambient space such that h_0 is equal to the identity map and that the map $(x, t) \mapsto (h_t(x), t)$ from $M \times [0, 1]$ to itself is a homeomorphism. Then two knots are isotopic if and only if there is an orientation preserving homeomorphism h of M that satisfies $f' = h \circ f$. A **knot type** $[K]$ a knot K is an isotopy class of K . ”

should be replaced by

“Let M denote \mathbb{R}^3 or S^3 . Two knots f and f' in M are called **isotopic** if there is an isotopy $h_t : M \rightarrow M$ ($t \in [0, 1]$) of the ambient space such that h_0 is equal to the identity map, that the map $(x, t) \mapsto (h_t(x), t)$ from $M \times [0, 1]$ to itself is a homeomorphism, **and that $h_1 \circ f = f'$** . Then two knots are isotopic if and only if there is an orientation preserving homeomorphism h of M that satisfies $f' = h \circ f$. **In this book, let us call an isotopy class of a knot K a knot type of $[K]$.** ”

- Page 15. The second line from the bottom.

$$E_\varepsilon^{(\alpha)}(K) = E_\varepsilon^{(\alpha)}(h) = \int_{S^1} V_\varepsilon^{(\alpha)}(K; x) dx = \int_{S^1} V_\varepsilon^{(\alpha)}(h; s) ds.$$

should be replaced by

$$E_\varepsilon^{(\alpha)}(K) = E_\varepsilon^{(\alpha)}(h) = \int_K V_\varepsilon^{(\alpha)}(K; x) dx = \int_{S^1} V_\varepsilon^{(\alpha)}(h; s) ds.$$

- Page 26. “then $E^{(\alpha)}$ ” in the first line of Theorem 2.4.1 (2) should be removed.

¹The author can find errors almost every time he has a look at the book.

- Page 26. The 5th line from the bottom, i.e. the second line in the proof of Theorem 2.4.1 (2).

“Let h be a knot with $|\mathbf{h}''| \equiv \mathbf{1}$ and $b = E^{(\alpha)}(h)$ ($b > 0$). ”

should be replaced by

“Let h be a knot with $|\mathbf{h}'| \equiv \mathbf{1}$ and $b = E^{(\alpha)}(h)$ ($b > 0$). ”

- Page 34. **Definition 3.1.** The first line of the formula holds if $X \neq P$:

$$I_{\Sigma}(X) = \begin{cases} P + \frac{r^2}{|X - P|^2}(X - P) & \text{if } X \in \mathbb{R}^3, \\ \infty & \text{if } X = P, \\ P & \text{if } X = \infty. \end{cases}$$

should be replaced by

$$I_{\Sigma}(X) = \begin{cases} P + \frac{r^2}{|X - P|^2}(X - P) & \text{if } X \in \mathbb{R}^3 \setminus \{\mathbf{P}\}, \\ \infty & \text{if } X = P, \\ P & \text{if } X = \infty. \end{cases}$$

- Page 42. Outline of proof of Theorem 3.5.1.

The convergence means the convergence with respect to the C^0 -topology.

- Page 43. The first line. “Kusner and J. Sullivan” should be replaced by “Kim and Kusner”.
- The book does not contain the proofs of Theorem 3.6.1 on page 44 and Theorem 3.7.7 on page 53, for which the reader is referred to [FHW] and [He1] respectively.
- Page 45. The right hand side of the 6th line from the bottom.

$$-\varepsilon [(1 - \xi) \{|\mathbf{f}'|'(s + \varepsilon\xi)\}]_0^1 + \varepsilon^2 \int_0^1 (1 - \xi) \{|\mathbf{f}'|'(s + \varepsilon\xi)\} d\xi$$

should be replaced by

$$-\varepsilon [(1 - \xi) \{|\mathbf{f}'|(s + \varepsilon\xi)\}]_0^1 + \varepsilon^2 \int_0^1 (1 - \xi) \{|\mathbf{f}'|'(s + \varepsilon\xi)\} d\xi$$

- Page 54. Section 3.8.

The main point of [KK] is as follows: As r approaches 0 or 1, the orbital configurations approach a p - or q -fold covered circle, and thus its energy $E = E^{(2)}$ must go to infinity. Since E is continuous, it takes on a minimum at some r_0 , and by the symmetric criticality argument, this is actually critical among all variations, not just the orbital ones.

- Page 57. The 4th line from the bottom.

$$\int_{|z|=1} \left(\frac{i(r^2 p^2 + (1 - r^2)q^2)}{r^2 z^{1-p}(z^p - 1)^2 + (1 - r^2)z^{1-q}(z^q - 1)^2} - \frac{i}{(z - 1)^2} \right) ds$$

should be replaced by

$$\int_{|z|=1} \left(\frac{i(r^2 p^2 + (1 - r^2)q^2)}{r^2 z^{1-p}(z^p - 1)^2 + (1 - r^2)z^{1-q}(z^q - 1)^2} - \frac{i}{(z - 1)^2} \right) dz$$

- Page 58, The 8th line in subsection 3.9.1.

$$\begin{aligned} \text{Conf}_{n,m}(K, \mathbf{S}^3) &= \{(x_1, \dots, x_{n+m}) \in K^n \times (\mathbf{S}^3)^m \mid x_j \neq x_k (j \neq k)\}, \\ U_D &= \{(x_1, \dots, x_{n+m}) \in \text{Conf}_{n,m}(K, \mathbf{S}^3) \mid x_1 \prec \dots \prec x_n\}. \end{aligned}$$

should be replaced by

$$\begin{aligned} \text{Conf}_{n,m}(K, \mathbb{R}^3) &= \{(x_1, \dots, x_{n+m}) \in K^n \times (\mathbb{R}^3)^m \mid x_j \neq x_k (j \neq k)\}, \\ U_D &= \{(x_1, \dots, x_{n+m}) \in \text{Conf}_{n,m}(K, \mathbb{R}^3) \mid x_1 \prec \dots \prec x_n\}. \end{aligned}$$

i.e. S^3 should be replaced by \mathbb{R}^3 , as we use Euclidean metric in this subsection.

- Page 59, in Definition 3.5.

$$E_{X,\cos}(K) = \int_{K^4; x_1 \prec \mathbf{x}_3 \prec \mathbf{x}_2 \prec x_4} \frac{(1 - \cos \theta_{13})(1 - \cos \theta_{24})}{|x_1 - x_3|^2 |x_2 - x_4|^2} dx_1 dx_2 dx_3 dx_4,$$

$$E_{X,\sin}(K) = \int_{K^4; x_1 \prec \mathbf{x}_3 \prec \mathbf{x}_2 \prec x_4} \frac{\sin \theta_{13} \sin \theta_{24}}{|x_1 - x_3|^2 |x_2 - x_4|^2} dx_1 dx_2 dx_3 dx_4,$$

should be replaced by

$$E_{X,\cos}(K) = \int_{K^4; x_1 \prec \mathbf{x}_2 \prec \mathbf{x}_3 \prec x_4} \frac{(1 - \cos \theta_{13})(1 - \cos \theta_{24})}{|x_1 - x_3|^2 |x_2 - x_4|^2} dx_1 dx_2 dx_3 dx_4,$$

$$E_{X,\sin}(K) = \int_{K^4; x_1 \prec \mathbf{x}_2 \prec \mathbf{x}_3 \prec x_4} \frac{\sin \theta_{13} \sin \theta_{24}}{|x_1 - x_3|^2 |x_2 - x_4|^2} dx_1 dx_2 dx_3 dx_4,$$

i.e. the order of x_2 and x_3 in “ $x_1 \prec x_3 \prec x_2 \prec x_4$ ” should be reversed.

- Page 67. The 5th line

If $E^{\alpha,p}(h)$ with $\alpha p > 2$ is **finite** then h cannot have a sharp turn.

should be replaced by

If $E^{\alpha,p}(h)$ with $\alpha p > 2$ is **bounded** then h cannot have a sharp turn.

- Pages 95, Figure 6.7, page 97, Figure 6.11, page 99, Figure 6.15. The caption “Look with the **right** eye.” should be “Look with the **left** eye.” (You do not have to close your left eye.)
- There is a misunderstanding about the order of contact. The order of contact in the book should be reduced by 1. The errors can be found on pages 119, 120, 123-125, 184-185.

– Page 119. Definition 8.2

“(1) An **osculating circle** \dots is the circle which is tangent to K at x at least to the **third** order”

should be replaced by

“(1) An **osculating circle** \dots is the circle which is tangent to K at x at least to the **second** order”

and

“(2) An **osculating sphere** \dots is tangent to K at x at least to the **fourth** order. ”

should be replaced by

“(2) An **osculating sphere** \dots is tangent to K at x at least to the **third** order. ”

– Page 119.

“**Proposition 8.3.1**(1) *An osculating sphere is uniquely determined if the order of tangency of the osculating circle to the knot is just **3**.*

(2) *Suppose the order of tangency of the osculating circle to the knot is just **3**. Then \dots ”*

should be replaced by

“**Proposition 8.3.1**(1) *An osculating sphere is uniquely determined if the order of tangency of the osculating circle to the knot is just **2**.*

(2) *Suppose the order of tangency of the osculating circle to the knot is just **2**. Then \dots ”*

– Page 120. After Proposition 8.3.1.

“When the order of tangency of the osculating circle to the knot is greater than **3**, any 2-sphere through the osculating circle is tangent to the knot to the **fourth** order. But there might be a unique 2-sphere which is tangent to the knot with a higher order of tangency than **4**. ”

should be replaced by

“When the order of tangency of the osculating circle to the knot is greater than **2**, any 2-sphere through the osculating circle is tangent to the knot to the **third** order. But there might be a unique 2-sphere which is tangent to the knot with a higher order of tangency than **3**. ”

- Page 120. The 6th line from the bottom (after the formula (8.1)).
“Then C is tangent to K at 0 to the **fourth** order, i.e. $f^{(3)}(0) = C^{(3)}(0)$, if and only if $k' = 0$ and $k\tau = 0$. ”

should be replaced by

“Then C is tangent to K at 0 to the **third** order, i.e. $f^{(3)}(0) = C^{(3)}(0)$, if and only if $k' = 0$ and $k\tau = 0$. ”

- Pages 123-124. Proof of Proposition 8.4.2.

“Case I: ...

If the knot K is transversal to Σ at x and if the order of tangency of K and Σ at y is **2**, then K must intersect Σ (not necessarily transversally) at a third point z which is different from both x and y , and therefore $\Sigma = \sigma(x, z, y, y)$.

If the order of tangency of K and Σ at y is more than or equal to **3**, then $\Sigma = \sigma(x, y, y, y)$.

Case II: Suppose $x = y$. The order of tangency of K to Σ at x is more than or equal to **3**.

If it is **3** then K must intersect Σ (not necessarily transversally) at another point z ($z \neq x$). Then $\Sigma = \sigma(x, x, x, z)$.

If the order of tangency is more than or equal to **4** then $\Sigma = \sigma(x, x, x, x)$. ”

should be replaced by

“Case I: ...

If the knot K is transversal to Σ at x and if the order of contact of K and Σ at y is **1**, then K must intersect Σ (not necessarily transversally) at a third point z which is different from both x and y , and therefore $\Sigma = \sigma(x, z, y, y)$.

If the order of contact of K and Σ at y is more than or equal to **2**, then $\Sigma = \sigma(x, y, y, y)$.

Case II: Suppose $x = y$. The order of tangency of K to Σ at x is more than or equal to **2**.

If it is **3** then K must intersect Σ (not necessarily transversally) at another point z ($z \neq x$). Then $\Sigma = \sigma(x, x, x, z)$.

If the order of contact is more than or equal to **3** then $\Sigma = \sigma(x, x, x, x)$. ”

- Page 125. The 1st line.

“Since $\rho(x) > r(\Sigma)$ by the assumption, the knot K cannot have the tangency of order **3** with Σ at x , and hence K must lie in ...”

should be replaced by

“Since $\rho(x) > r(\Sigma)$ by the assumption, the knot K cannot have the tangency of order **2** with Σ at x , and hence K must lie in ...”

- Page 125. The 5th line.

“then the osculating sphere at x contains the osculating **sphere** at x as the great circle”

should be replaced by

“then the osculating sphere at x contains the osculating **circle** at x as the great circle”

- Page 141. The 7th and 8th lines from the bottom, i.e. the last two lines of (4) have two errors:

Since $u_5, v_5 > 1$ this implies $u_1v_1 + \dots + u_4v_4 \leq u_5v_5 - 1$ which means $\langle \mathbf{u}, \mathbf{v} \rangle < -1$.

should be replaced by

Since $u_5v_5 > 1$ this implies $u_1v_1 + \dots + u_4v_4 \leq u_5v_5 - 1$ which means $\langle \mathbf{u}, \mathbf{v} \rangle \leq -1$.

Furthormore, the proof that $\langle \mathbf{u}, \mathbf{v} \rangle \neq -1$ should be added:

Assume $\langle \mathbf{u}, \mathbf{v} \rangle = -1$. Since $\langle \mathbf{u}, \mathbf{u} \rangle = -1$ we have $\langle \mathbf{u}, \mathbf{u} - \mathbf{v} \rangle = 0$. Lemma 9.1.1 (3) implies that $\mathbf{u} - \mathbf{v}$ is either space-like or equal to $\mathbf{0}$. As $\langle \mathbf{u} - \mathbf{v}, \mathbf{u} - \mathbf{v} \rangle = 0$, $\mathbf{u} - \mathbf{v}$ cannot be space-like. Therefore, $\mathbf{u} - \mathbf{v} = \mathbf{0}$, which is a contradiction.

- Page 145. The 3rd line. S_{infy}^3 should be replaced by S_∞^3 .

- Page 145. The bijection in Theorem 9.3.2 can be considered as a modern version of the pentaspherical coordinates in [Dar].

- Page 151 last two lines to Page 152 line. (1a). The book misses the description of the case when $n = 1$.

When $n = 1$ $P^\perp \cap S_\infty^1 = \emptyset$ since P^\perp is a time-like line. In this case we may consider the “base sphere” to be $\emptyset = \partial(P^\perp \cap \mathbb{H}^2)$, where $\partial\mathbb{H}^2 = S_\infty^1$. Let us call \mathcal{P} or the set of corresponding spheres of dimension 0 a “space-like pencil”.

- Page 166. Three lines before **Lemma 9.8.2**.

$$\psi : \mathbb{R}_+^{n+1} \ni (\mathbf{X}, r) \mapsto \varphi^{-1} \circ p^{-1} (S_r^{n-1}(\mathbf{X})) \in \Lambda \setminus (\text{Span} \langle Q \rangle)^\perp$$

should be replaced by

$$\psi : \mathbb{R}_+^{n+1} \ni (\mathbf{X}, r) \mapsto \varphi^{-1} \circ p^{-1} (S_r^{n-1}(\mathbf{X})) \in \Lambda \setminus (\text{Span} \langle Q \rangle)^\perp.$$

- Page 167. The 8th line.

$$\omega_{\mathbb{R}_+^{n+1}} = \frac{1}{\mathbf{x}_{n+2}} dx_1 \wedge \cdots \wedge dx_{n+1}$$

should be replaced by

$$\omega_{\mathbb{R}_+^{n+1}} = -\frac{1}{r^{n+1}} dX_1 \wedge \cdots \wedge dX_n \wedge dr$$

- Page 167. The 16th line.

$$\tilde{g} : \mathbb{R}_+^{n+1} \ni (\mathbf{X}, r) \mapsto \left(\frac{\mathbf{X}}{|\mathbf{X}|^2 - r^2}, \frac{r}{|\mathbf{X}|^2 - r^2} \right) \in \mathbb{R}_+^{n+1}.$$

should be replaced by

$$\tilde{g} : \mathbb{R}_+^{n+1} \ni (\mathbf{X}, r) \mapsto \left(\frac{\mathbf{X}}{|\mathbf{X}|^2 - r^2}, \frac{r}{||\mathbf{X}|^2 - r^2|} \right) \in \mathbb{R}_+^{n+1}.$$

- Page 175. **Definition 10.1** (2) should be replaced by

“(2) An oriented 2-sphere Σ is called a **non-trivial sphere in the strict sense** for a knot K if each connected component of $\mathbb{R}^3 \setminus \Sigma$ ($S^3 \setminus \Sigma$) contains at least 2 connected components of $K \setminus (K \cap \Sigma)$. ”

- Page 184. **Definition 10.6**. (1)

$$\mathcal{C}c^{(4)}(K) = \left\{ (s, s, s, s) \in \Delta^{(4)} \left| \begin{array}{l} \text{The osculating circle of } K \text{ at } f(s) \\ \text{is tangent to } K \text{ to the } \mathbf{fourth} \text{ order} \end{array} \right. \right\}.$$

should be replaced by

$$\mathcal{C}c^{(4)}(K) = \left\{ (s, s, s, s) \in \Delta^{(4)} \left| \begin{array}{l} \text{The osculating circle of } K \text{ at } f(s) \\ \text{is tangent to } K \text{ to the } \mathbf{third} \text{ order} \end{array} \right. \right\}.$$

- Page 184. **Proposition 10.3.3**. (1)

“The order of tangency of the osculating circle at $f(s)$ is exactly **3** if and only if $f(s) \wedge f'(s) \wedge f''(s) \wedge f'''(s) \neq \mathbf{0}$. ”

should be replaced by

“The order of tangency of the osculating circle at $f(s)$ is exactly **2** if and only if $f(s) \wedge f'(s) \wedge f''(s) \wedge f'''(s) \neq \mathbf{0}$. ”

- Page 185. The 6th line and the 10th line from the bottom (in the *Proof* of Proposition 10.3.3 (1)).

“(1) Suppose the osculating circle C is tangent to the knot at $f(s)$ to the **fourth** order. \cdots
 \cdots and hence the osculating circle C is tangent to the knot at $f(s)$ to the **fourth** order. ”

should be replaced by

“(1) Suppose the osculating circle C is tangent to the knot at $f(s)$ to the **third** order. \cdots
 \cdots and hence the osculating circle C is tangent to the knot at $f(s)$ to the **third** order. ”

- Page 191. The 7th line.

$$(I \times I)^* \lambda_{\mathfrak{R}} = \lambda_{\mathfrak{R}} + \frac{1}{2} d \log(|\mathbf{u}|^2)$$

should be replaced by

$$(I \times I)^* \lambda_{\mathfrak{R}} = \lambda_{\mathfrak{R}} + \frac{1}{2} d \log(\mathbf{u}^2 + \mathbf{v}^2)$$

- Page 193. The 4th line in the Proof of Lemma 11.2.2. ‘**bas**’ should be replaced by ‘**basis**’.
- Page 193. The 5th line in Lemma 11.2.3. ‘**this** local coordinate’ should be replaced by ‘**these** local coordinates’.
- Page 198. Theorem 11.2.7. As a corollary of this theorem, we have

Corollary The pull-back $\omega = \psi^* \omega_0$ of the canonical symplectic form ω_0 of $T^* S^n$ by $\psi : S^n \times S^n \setminus \Delta \rightarrow T^* S^n$ is invariant under any diagonal action of the Möbius group \mathcal{M} on $S^n \times S^n \setminus \Delta$.

- Page 200. The condition (ii) of Theorem 11.2.9 is not necessary.
- Page 203. A remark on Definition 11.4:

The real part of the infinitesimal cross ratio of a knot K is a smooth 2-form on $K \times K \setminus \Delta$, but it is not the case with the imaginary part. Since the conformal angle $\theta_K(x, y)$ is not a smooth function of x and y (see Figure 10.1 on page 183), **the imaginary part of the infinitesimal cross ratio may have singularity** at a pair of points $(x, y) \in K \times K \setminus \Delta$ where the conformal angle $\theta_K(x, y)$ vanishes.

- Page 206. The 2nd and 3rd lines

$$((T_0 \circ f)^* dx_3)(s_0, t_0) = \left(\frac{d}{ds} (T_0 \circ f)_3 \right) (s_0),$$

$$((T_0 \circ f)^* dy_3)(s_0, t_0) = \left(\frac{d}{dt} (T_0 \circ f)_3 \right) (t_0)$$

should be replaced by

$$((T_0 \circ f)^* dx_3)(s_0, t_0) = \left(\frac{d}{ds} (T_0 \circ f)_3 \right) (s_0) = \mathbf{0},$$

$$((T_0 \circ f)^* dy_3)(s_0, t_0) = \left(\frac{d}{dt} (T_0 \circ f)_3 \right) (t_0) = \mathbf{0}$$

i.e. = 0 should be added at the end of the both formulae.

- Page 211. The 2nd and 3rd lines

$$\begin{aligned} \frac{1}{2} E_{\circ}^{(2)}(L_r) = E_{\circ}^{(2), mut}(L_r) &= \int_0^{2\pi} \int_0^{2\pi} \frac{ds dt}{2 + 2 \cos(t - s)} \\ &= 4\pi \int_0^{\frac{\pi}{2}} \frac{d\xi}{1 - r \cos 2\xi} \quad (\xi = \tan \eta) \end{aligned}$$

should be replaced by

$$\begin{aligned} \frac{1}{2} E_{\circ}^{(2)}(L_r) = E_{\circ}^{(2), mut}(L_r) &= \int_0^{2\pi} \int_0^{2\pi} \frac{ds dt}{2 - 2r \cos(t - s)} \\ &= 4\pi \int_0^{\frac{\pi}{2}} \frac{d\xi}{1 - r \cos 2\xi} \quad (\eta = \tan \xi) \end{aligned}$$

- Page 218. The 3rd line in the Proof of Lemma 12.3.2

$$V_{\sin \theta}(K; f(\pm t)), V_{\sin \theta}(K; f(\delta \pm t)) \geq \frac{1}{100} \cdot \frac{1}{d + t}$$

should be replaced by

$$V_{\sin \theta}(K; f(\pm t)), V_{\sin \theta}(K; f(\delta \pm t)) \leq \frac{1}{100} \cdot \frac{1}{d + t}$$

i.e. the inequality should be reversed.

- Page 218. The 8th line in the Proof of Lemma 12.3.2, i.e. just above Figure 12.2.

$$\lim_{K \setminus \{x_+\} \ni y \rightarrow x_+} \pi_{x_+}(\mathbf{y}) = \hat{x}_+.$$

should be replaced by

$$\lim_{K \setminus \{x_+\} \ni y \rightarrow x_+} \pi_{x_+}(\mathbf{I}_{x_+}(\mathbf{y})) = \hat{x}_+.$$

- Page 218. The right picture of Figure 12.2 should be replaced by Figure 1. The radius of the circle, which

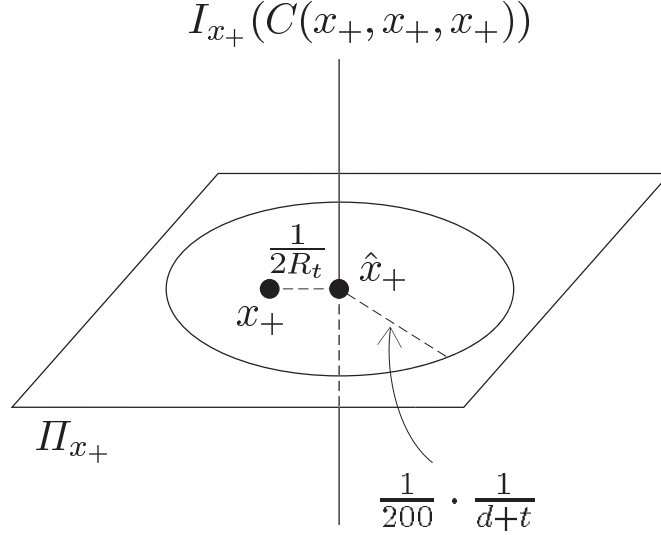


Figure 1: The right picture of Figure 12.2.

was $\frac{1}{400} \cdot \frac{1}{d+t}$ in the book, should be replaced by $\frac{1}{200} \cdot \frac{1}{d+t}$. Also, the point x_+ should be closer to the center of the circle, \hat{x}_+ , as $|x_+ - \hat{x}_+| \leq \frac{1}{400} \cdot \frac{1}{d+t}$, which is a half of the radius.

- Page 218. The 3rd line from the bottom.

$\pi_{x_+}(\tilde{K}_{x_+})$ lies inside the circle on Π_{x_+} with center x_+ and radius $1/(200(d+t))$.
should be replaced by

$\pi_{x_+}(\tilde{K}_{x_+})$ lies inside the circle on Π_{x_+} with center \hat{x}_+ and radius $1/(200(d+t))$.

- Page 219. The 8th line (just above Figure 12.3).

N_t whose meridian disc has radius $(400(d+t))/3$, as illustrated in Figure 12.3.

should be replaced by

N_t whose meridian disc has radius $(200(d+t))/3$, as illustrated in Figure 12.3.

- Page 219. Figure 12.3 should be replaced by Figure 2. Namely, the radius of the inner circle of the left picture, which was $\frac{1}{400(d+t)}$ in the book, should be $\frac{1}{200(d+t)}$, and the radius of the ‘degenerate solid torus’ in the right picture, which was $\frac{400(d+t)}{3}$ in the book, should be $\frac{200(d+t)}{3}$.

- Page 219. The 7th line from the bottom

$$|f(t) - f(-t)| \leq 2t \ll \frac{400}{3}(d+t)$$

should be replaced by

$$|f(t) - f(-t)| \leq 2t \ll \frac{200}{3}(d+t)$$

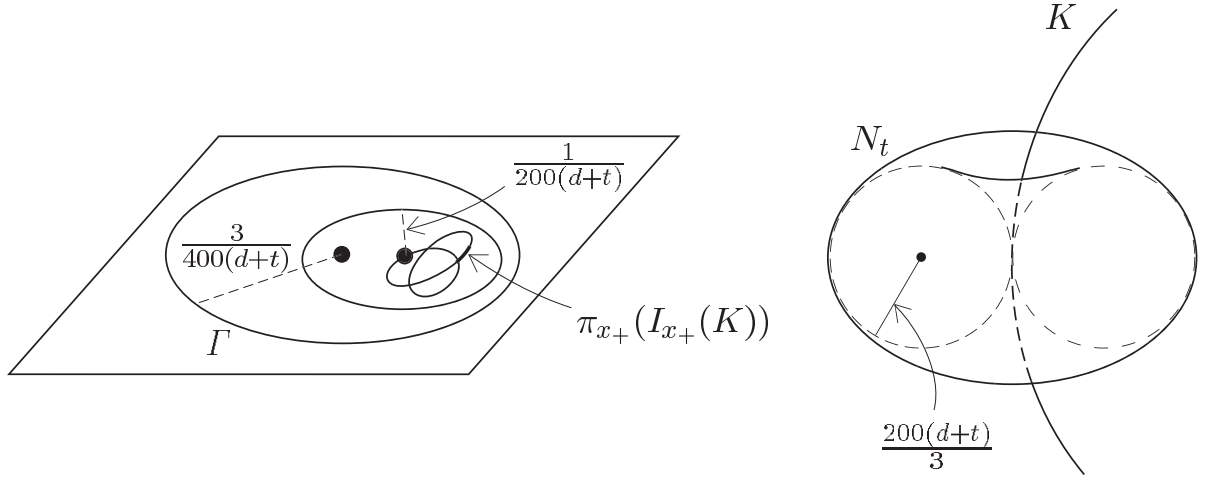


Figure 2: Figure 12.3.

- Page 220. The 2nd line from the bottom

$$|f(0) - \mathbf{g}(t)| \geq \int_0^t (1 - \kappa s) ds = t \left(1 - \frac{\kappa}{2}t\right) > d,$$

should be replaced by

$$|f(0) - \mathbf{f}(t)| \geq \int_0^t (1 - \kappa s) ds = t \left(1 - \frac{\kappa}{2}t\right) > d,$$

- Page 226. The last three lines of Remark 13.2.2 should be replaced by
This is because $1 \leq {}_n C_2 \leq {}_{2n} C_4$ when $n \geq 2$.
- Page 229. Line 6 up.
“subarc of $C_3(r)$ between $Q_{12}(r)$ and $P_{23}(r)$ ”
should be replaced by
“subarc of $C_3(r)$ between $Q_{12}(r)$ and $P_{13}(r)$ ”

- Page 229. The last line. Page 230. The second line.

The domain of integration “ $\left[\frac{1}{2 \sin \theta_K}, \infty\right)$ ” should be replaced by “ $\left[\frac{1}{2 \sin \theta}, \infty\right)$ ”

- Page 231. Lines 3-5. “ $\dots + o(|z|)$ ” should be replaced by “ $\dots + O(|z|)$ ”.
- Page 236. The 13th line around the middle of the page

$$\int_{\Sigma_r(X,Y,Z) \cap K_d \neq \emptyset} \#(\Sigma_r(X,Y,Z) \cap K_d) dX dY dZ = 2\pi r^2 (\bar{K}_d),$$

should be replaced by

$$\int_{\Sigma_r(X,Y,Z) \cap K_d \neq \emptyset} \#(\Sigma_r(X,Y,Z) \cap K_d) dX dY dZ = 2\pi r^2 (\bar{K}_d),$$

- Page 236. The 5th line from the bottom

$$E_{\text{mnts}}(K_d) = \int_{S(K_d)} C_2^{\#(\Sigma_r(X,Y,Z) \cap K_d)} \cdot \frac{1}{r^4} dX dY dZ dr$$

should be replaced by

$$E_{\text{mnts}}(K_d) = \int_{S(K_d)} \frac{1}{2} \#(\Sigma_r(X,Y,Z) \cap K_d) C_2 \cdot \frac{1}{r^4} dX dY dZ dr$$

This implies the 4th line from the bottom because ${}_m C_2 \geq m - 1$ for $m \geq 0$.

- Page 256. The 8th line (just before Example A.1) should be replaced by
We show that I_{tv} can detect the unknot.
Namely, it is a conjecture whether $|csl|$ can detect the unknot or not.
- Page 273.
[CKS2] J. Cantarella, R. B. Kusner, and J. M. Sullivan, *On the minimum ropelength of knots and links*, Preprint.
should be updated to
[CKS2] J. Cantarella, R. B. Kusner, and J. M. Sullivan, *On the minimum ropelength of knots and links*, *Invent. Math.* **150** (2002), no. 2, 257–286.
[CKS] should be removed as it is same as [CKS1].
- Page 276.
[GLP] M. Gromov, J. Lafontaine, and P. **Pnau**, *Structures métriques pour les variétés riemanniennes*, Cedic/Fernand Nathan, Paris, 1981.
should be replaced by
[GLP]Gr-La-Pn M. Gromov, J. Lafontaine, and P. **Pansu**, *Structures métriques pour les variétés riemanniennes*, Cedic/Fernand Nathan, Paris, 1981.
- Page 279.
[Lin] X.-S. Lin, *Knot energies and knot invariants*, **J. Differential Geom.** **44** (1996), 74–95.
should be replaced by
[Lin] X.-S. Lin, *Knot energies and knot invariants*. *Knot theory and its applications*, **Chaos Solitons Fractals** **9** (1998) 645–655
- Page 285, Index. The “average crossing number” also appears on page 44.

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