

## RESTORING FORCE CHARACTERISTICS IN REINFORCED CONCRETE BEAM-COLUMN JOINTS

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### ABSTRACT

The model of the shear stress - distortion relation in an interior beam-column joint was made of the tri-linear envelope curve with the abrupt stiffness changes at shear cracking and failure. The elastic modulus was obtained from the secant modulus and Poisson's ratio of concrete. The shear stress at diagonal cracking was calculated from the stress condition that the principal stress reaches the tensile strength of concrete. The shear modulus after diagonal cracking was determined based on the test results using the concrete compressive strength, the amount of the joint lateral reinforcement and the column intermediate reinforcement, the column axial load and the effect of confinement provided by the transverse beams and slabs.

### 1. INTRODUCTION

The hysteresis model of beam-column joints subjected to cyclic shear is needed to calculate the earthquake response of reinforced concrete frames taking into account the shear distortion in beam-column joints. Therefore, first of all, the envelope curve of the shear stress - distortion relationship in an interior beam-column joint was constituted by the tri-linear curve in Fig. 1. The shear modulus was reduced by diagonal cracking in a joint panel and joint failure in shear. However joint shear strength can not be determined precisely. Then the joint input shear was assumed to be constant due to beam yielding prior to joint failure. The elastic modulus, the shear stress at diagonal cracking and the shear modulus after cracking in a joint were decided to establish the primary curve from the test results. The modified Takeda-slip model with stiffness degradation may be efficient as the hysteretic rules in a joint shear stress - distortion relation which are not dealt with in the paper. The stiffness degrading model proposed by Morita and Fujii (Ref. 1) was able to present the increase in the joint shear distortion after beam yielding without deciding the joint shear strength, but the unique skeleton curve based on their test results was used.

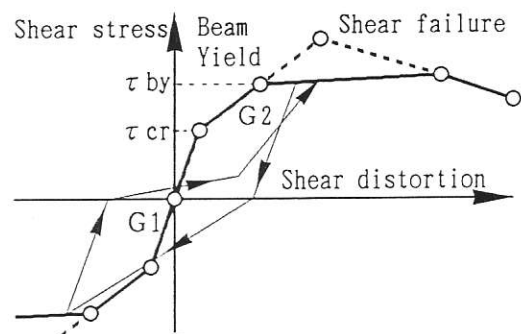


Fig. 1 : Model of shear stress - distortion relation in beam-column joint

There are some recent studies on the envelope curve. Teraoka et.al. described that the joint shear distortion at the abrupt change of the stiffness was almost constant (Ref. 2). Kokusho et.al. concluded that the shear modulus after joint cracking was one-fifth times the elastic modulus in a joint shear stress – distortion relation (Ref. 3).

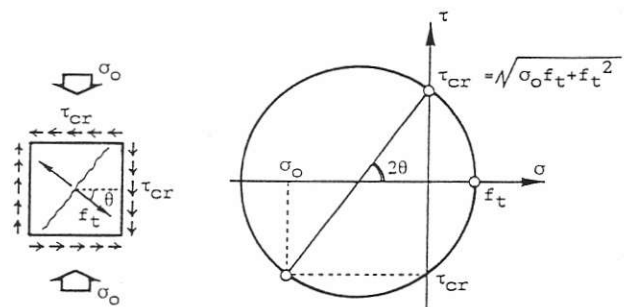
## 2. SHEAR STRESS AT CRACKING AND ELASTIC MODULUS $G_1$

### 2.1 SHEAR STRESS AT CRACKING IN JOINT

The joint cracks develop diagonally by shear when the principal stress reaches concrete tensile strength in a stress field shown in Fig. 2(a). Then the joint shear stress at cracking is derived as follows from the Mohr's stress circle in Fig. 2(b);

$$\tau_{cr} = \sqrt{\sigma_o f_t + f_t^2} \quad (1)$$

where  $\tau_{cr}$  is the shear stress at cracking in a joint in  $\text{kgf/cm}^2$ ,  $\sigma_o$  is the column axial stress in  $\text{kgf/cm}^2$  and  $f_t$  is the concrete tensile strength in  $\text{kgf/cm}^2$ . Several researchers have pointed out that the joint shear stress at cracking could be estimated by Eq. (1) (for instance, Ref. 4). The computed shear stress at cracking is compared with measured one in Fig. 3 for 29 plane interior beam-column joints tested at the University of Tokyo and Chiba University (Ref. 8 to 14). The concrete tensile strength was obtained from the splitting test of the cylinders with 10 cm diameter and 20 cm height. The effective joint area to resist shear was defined by the column depth and the average of the beam and column widths. The average ratio of observed to computed joint shear stress at cracking was 0.94. In other words, the average among the computed values was 1.06 times larger than that of the test results. The difference in the computed and measured stress at diagonal cracking was caused by the fact that the concrete tensile strength in reinforced concrete structures is actually smaller than that by the cylinder splitting test (Ref. 5). Therefore the joint shear stress at cracking can be predicted properly by Eq. (1).



(a) Stress condition (b) Mohr's stress circle

Fig. 2 : Stress condition at cracking in joint

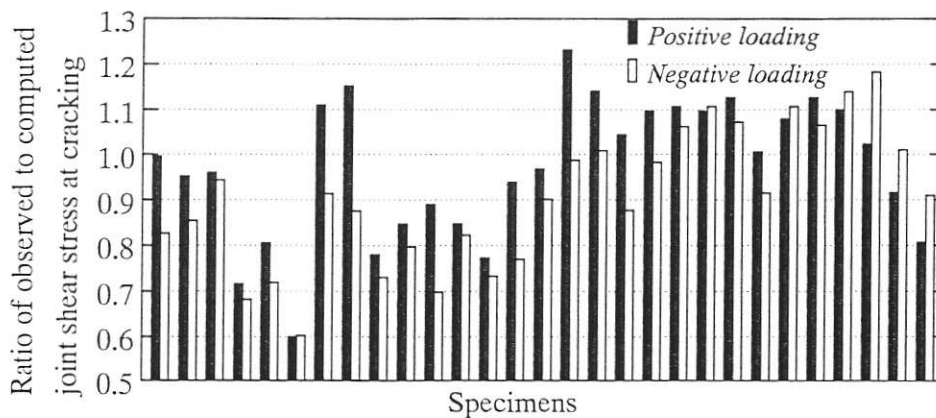


Fig. 3 : Shear stresses at diagonal cracking in joints

## 2.2 ELASTIC MODULUS $G_1$

Elastic modulus  $G_1$  in the joint shear stress - distortion relationship is expressed as Eq.(2).

$$G_1 = E_c / 2(1+\nu) \quad (2)$$

where  $E_c$  is the elastic modulus of concrete and  $\nu$  is the Poisson's ratio of concrete which was represented by Tomosawa et.al. (Ref. 6) as follows;

$$\nu = 4 \times 10^{-5} \cdot f_c' + 0.169 \quad (3)$$

where  $f_c'$  is the concrete compressive strength in  $\text{kgf/cm}^2$ . Fig. 4 shows the assessed elastic modulus by Eq.(2) in comparison to the test results of 15 interior beam-column joint specimens listed in Table 1 except for Specimens A3 and J5. Joint shear distortion in specimens was measured by two transducers mounted in diagonal or parallel direction on a joint panel. Joint shear was computed as the sum of the tensile and compressive forces acting on the beam sections at opposite column faces less the story shear. The coupled force on the beam section was derived from the beam flexural moment divided by 7/8 times the effective depth. The elastic modulus  $G_1$  in the test was determined as the secant modulus between the zero point and the point immediately before diagonal cracking. The concrete elastic modulus  $E_c$  was taken as the secant modulus at one-quarter of the compressive strength in the stress - strain relation of cylinder tests. The elastic moduli  $G_1$  obtained from the loading test in several specimens were larger than those computed. The reasons why the computed values became lower were that;

- the secant modulus at  $f_c'/4$  was used as the concrete elastic modulus  $E_c$ . When, however, the column axial stress  $\sigma_o$  was small, the compressive principal stress of  $(\sigma_o + f_t)$  at shear cracking in a joint did not reach the one-quarter of the concrete compressive strength,
- the contribution of the column longitudinal reinforcement to the elastic modulus  $G_1$  was ignored. The computed modulus, considering the effect of the elastic modulus of the steel and the sectional area of the column longitudinal bars, increased to approximately 1.2 times that without consideration.

Moreover, the use of the average of the beam and column widths as the effective width to resist joint shear increased apparently the elastic modulus  $G_1$  observed in the tests. If the column width is used since the joint diagonal cracks occur over the entire width of a column, the modulus  $G_1$  acquired from tests decreases to 0.8 times that using the average of the beam and column widths. Nevertheless the average was adopted in the paper as the effective joint width since there are few studies on the width to resist shear in a joint and it is difficult to vary the effective width with the distortion in a joint.

It is concluded that the elastic modulus  $G_1$  can be evaluated by Eq.(2) if the elastic modulus of the concrete  $E_c$  is settled appropriately accounting for the column axial load.

## 3. SHEAR MODULUS $G_2$ AFTER DIAGONAL CRACKING

The reduced modulus  $G_2$  by diagonal shear cracks in the joint shear stress - distortion relationship depends on a) concrete compressive strength, b) amount of the joint lateral reinforcement, c) amount of the column intermediate reinforcement, d) column axial load, e) confinement due to transverse members, f) amount of the beam intermediate reinforcement, g) bi-directional input shear to a joint and h) the ratio of beam depth to column depth. The

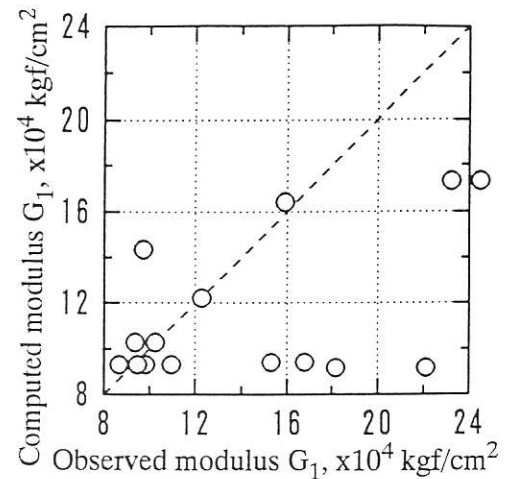


Fig. 4 : Elastic modulus  $G_1$

Table 1 : Properties of specimens

Specimen	G2	Concrete	Joint Rein-	Column Inter-	Column Ax-	Masking	G2i	Failure	Reference
	(Test)	Strength	forcement	mediate Bar	ial Stress	Ratio	(Computed)		
	kgf/cm <sup>2</sup>	$f_c'$ kgf/cm <sup>2</sup>	$\rho_{jh}$ %	$\rho_{ci}$ %	$\sigma_o$ kgf/cm <sup>2</sup>	$\alpha$	kgf/cm <sup>2</sup>	Mode	Number
I 1	31545	962	0.41	1.91	36.0	0.000	18136.2	J	8
I 3	19596	422	0.37	1.33	10.8	0.000	15880.8	BJ	
I 4	22115	405	0.37	1.33	10.8	0.000	18164.8	BJ	12
I 5	21954	870	0.42	1.33	20.0	0.000	14052.9	B	9
I 6	25179	870	0.42	1.33	20.0	0.000	16117.2	B	
A 1	15631	312	0.38	1.33	20.0	0.000	14280.4	J	
A 3	15774	312	0.38	1.33	20.0	0.744	14411.0	B	10
A 4	18478	312	0.38	1.33	20.0	0.233	—	J	
B 1	14339	250	0.35	1.33	20.0	0.000	14270.9	BJ	
B 2	20886	250	0.35	1.33	20.0	0.000	20786.9	BJ	10
B 3	17185	250	0.88	0.85	20.0	0.000	12498.7	BJ	
B 4	13372	250	0.88	0.85	20.0	0.000	9725.5	BJ	
J 1	16233	262	0.27	0.85	20.0	0.000	18366.3	BJ	
J 2	18229	245	0.54	0.85	20.0	0.000	18355.4	BJ	
J 3	38313	245	1.27	0.85	20.0	0.000	18371.3	BJ	11
J 4	13478	262	0.27	0.85	80.0	0.000	18366.6	BJ	
J 5	13101	293	0.27	0.00	20.0	0.000	18355.4	BJ	
							(Average)	16260.0	

Failure mode B : Beam yielding. BJ : Joint shear failure after beam yielding and J : Joint shear failure

shear modulus  $G_2$  was determined by quantifying the influence of the factors from a) to e) by the same manner as Fujii et.al. (Ref. 7). The factors from f) to h) were not included due to the uncertainty.

### 3.1 QUANTIFICATION OF SHEAR MODULUS $G_2$

The modulus  $G_2$  after cracking can be defined as Eq.(4).

$$G_2 = G_{2i} \cdot K_0 K_1 K_2 K_3 K_4 \quad (4)$$

where  $G_{2i}$  is the standard shear modulus after cracking when the coefficients of  $K_0$  to  $K_4$  are assumed to be unity, and  $K_0$ ,  $K_1$ ,  $K_2$ ,  $K_3$  and  $K_4$  are the coefficients to consider the effects of the concrete compressive strength, the joint lateral reinforcement, the column intermediate reinforcement, the column axial load and the transverse members framing into a joint, respectively. Equation (4) is based on the assumption that each of the factors operates independently on the modulus.

### 3.2 DECISION OF COEFFICIENTS

The coefficients of  $K_0$  to  $K_4$  were estimated through the test results of interior beam-column joint specimens (Ref. 8 to 12) listed in Table 1. Most of the joints failed in cyclic shear after the beam reinforcement yielded at the column faces. Then the modulus  $G_2$  after joint cracking in the test

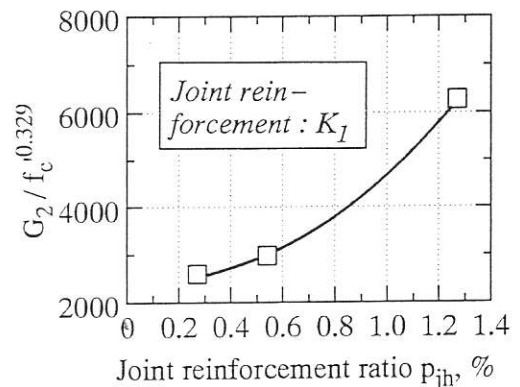


Fig. 5 : Effect of joint lateral reinforcement

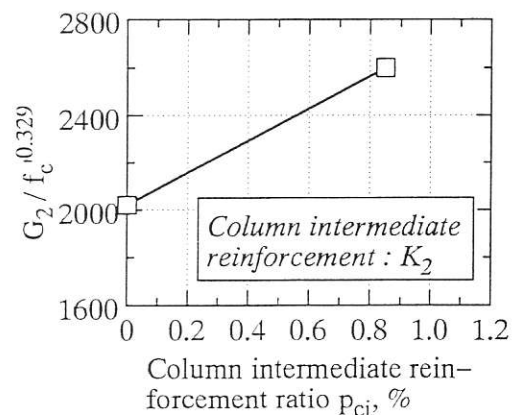


Fig. 6 : Effect of column intermediate reinforcement

was defined as the secant modulus connecting the cracking shear stress with the three-fourths of the shear stress  $\tau_{by}$  at beam bar yielding, and the larger one in the positive and negative cycles was adopted.

The moduli  $G_2$  obtained from Specimens J1, J2 and J3, which had the same reinforcing details except for the different amount of the joint lateral reinforcement, were associated with the joint lateral reinforcement ratio  $p_{jh}$  in Fig. 5 to identify the coefficient  $K_1$ . The modulus increased with the ratio  $p_{jh}$ .

The column intermediate reinforcement ratio  $p_{ci}$  - the modulus  $G_2$  relations are shown in Fig. 6 for Specimens J1 and J5 with and without the column intermediate bars respectively. The column intermediate bars enhanced the modulus.

The column axial load - the modulus  $G_2$  relations are shown in Fig. 7 to identify the coefficient  $K_3$  for Specimens J1 and J4 applied different axial load. The modulus decreased with the increase in the column axial load.

The masking ratio to a joint - the modulus  $G_2$  relations are shown in Fig. 8 for plane joint Specimen A1, Specimen A4 with slabs and Specimen A3 with transverse beams and slabs loaded cyclically to beam yielding. The masking ratio was defined as the ratio of the sectional area of transverse beams and slabs framing into a joint to the area of a joint panel surrounded by the longitudinal beams and the columns. Joint shear distortion in a three-dimensional Specimen A3 was computed as the story drift less the contribution from the beam and column deflections. The slabs without cracks at the slab-joint interface enhanced the modulus. On the contrary, the transverse beams and slabs with cracks over the entire sections at opposite column faces did not affect the modulus. Flexural cracks at the critical section of the transverse beam develop inevitably during earthquakes since the two-way frames with the beam collapse mechanism are used commonly in Japan. Then the effect of the transverse members on the modulus was neglected to determine the coefficient  $K_4$  of unity.

Note that the concrete compressive strengths  $f'_c$  were different among Specimens J1, J2, J3 and J5 which were used to determine the coefficients  $K_1$  and  $K_2$ . Therefore the procedure as follows was taken to eliminate the influence of the concrete compressive strength;

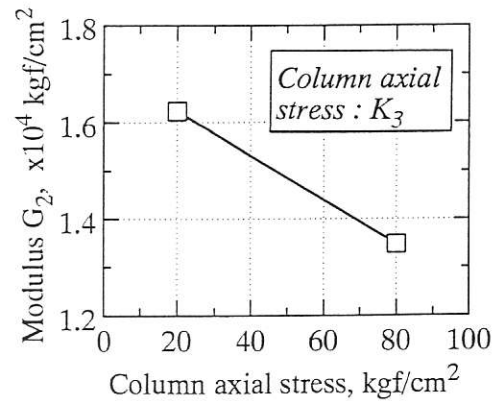


Fig. 7 : Effect of column axial load

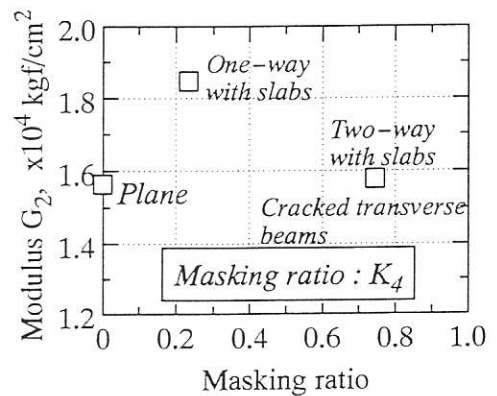


Fig. 8 : Effect of transverse members

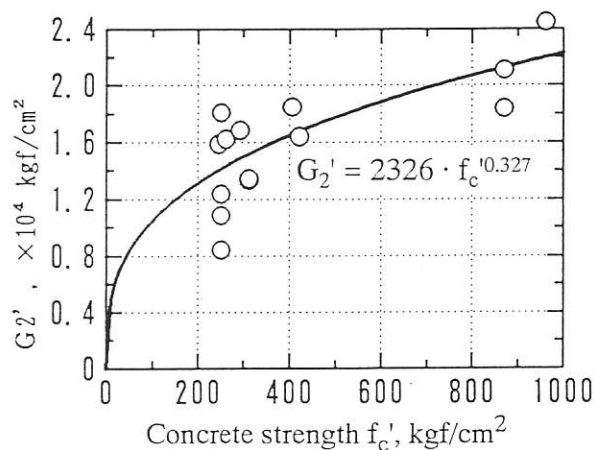


Fig. 9 : Effect of concrete compressive strength

- (a) The moduli  $G_2$  in these specimens were normalized by  $f_c^{10.4}$  to determine the coefficients  $K_1$  and  $K_2$  tentatively.
- (b) The modified modulus  $G_2'$  was calculated by dividing the modulus  $G_2$  by the coefficients  $K_1, K_2, K_3$  and  $K_4$ .
- (c) The modified modulus  $G_2'$  was related with concrete compressive strength  $f_c'$  by the multiple regression analysis as shown in Fig. 9. The coefficient  $K_0$  could be expressed by the form of  $f_c'^n$ .
- (d) The moduli  $G_2$  in Specimens J1, J2, J3 and J5 were normalized by  $f_c'^n$  obtained above to determine the new coefficients  $K_1$  and  $K_2$ .
- (e) After the iteration from (b) to (d) was carried out twice, the coefficients  $K_0, K_1$  and  $K_2$  were determined finally.

At last, the standard modulus  $G_{2i}$  was determined as the average of the moduli  $G_2$  normalized by the coefficients from  $K_0$  to  $K_4$  in all specimens. These coefficients are expressed below;

$$G_{2i} = 16260 \text{ kgf/cm}^2 \quad (5)$$

$$K_0 = 0.143 \cdot f_c'^{10.327} \quad \text{when } f_c' < 962 \text{ kgf/cm}^2 \quad (6.1)$$

$$K_0 = 1.35 \quad \text{when } f_c' \geq 962 \text{ kgf/cm}^2 \quad (6.2)$$

$$K_1 = 1.00 \quad \text{when } p_{jh} < 0.27 \% \quad (7.1)$$

$$K_1 = 1.184 \cdot p_{jh}^2 - 0.411 \cdot p_{jh} + 1.025 \quad \text{when } 0.27 \% \leq p_{jh} < 1.27 \% \quad (7.2)$$

$$K_1 = 2.41 \quad \text{when } 1.27 \% \leq p_{jh} \quad (7.3)$$

$$K_2 = 0.261 \cdot p_{ci} + 0.778 \quad \text{when } p_{ci} < 0.85 \% \quad (8.1)$$

$$K_2 = 1.00 \quad \text{when } p_{ci} \geq 0.85 \% \quad (8.2)$$

$$K_3 = -2.83 \cdot 10^{-3} \cdot \sigma_o + 1.057 \quad \text{when } \sigma_o < 80 \text{ kgf/cm}^2 \quad (9.1)$$

$$K_3 = 0.83 \quad \text{when } \sigma_o \geq 80 \text{ kgf/cm}^2 \quad (9.2)$$

$$K_4 = 1.00 \quad (10)$$

where  $p_{jh}$  (unit in %) is the ratio of the sum of the sectional area of the joint lateral reinforcement between the top and bottom beam reinforcing bars to the product of the column width and 7/8 times the beam effective depth under the bottom tension,  $p_{ci}$  (unit in %) is the ratio of the sum of the sectional area of the column intermediate reinforcement to the column gross section and  $\sigma_o$  is the axial stress in kgf/cm<sup>2</sup> to the column gross section.

The computed moduli by the proposal agreed well with those obtained by the tests as shown in Fig. 10. The correlative coefficient was 0.85. The study is needed to predict the modulus  $G_2$  more accurately since the number of specimens available in the paper was limited.

### 3.3 COMPARISON WITH OTHER TESTS

The proposed method was applied to other test results of 23 interior beam-column joint specimens (Ref. 1, 13 to 19). Concrete compressive strength ranged from 296 to 828 kgf/cm<sup>2</sup>, joint lateral reinforcement ratio from 0 to 1.63 %, column intermediate reinforcement ratio from 0 to 1.77 % and column axial stress from 20 to 133 kgf/cm<sup>2</sup>. The computed

moduli are compared with test results in Fig. 11. The moduli in the tests were calculated according to the definition of  $G_2$  from the joint shear stress - distortion relations reported in the references 1 and from 13 to 19. The analysis could predict the modulus  $G_2$  after cracking in a joint within an error of 30 %.

#### 4. CONCLUSIONS

The envelope curve of the shear stress - distortion relation in an interior beam-column joint was established. The elastic modulus was obtained from the secant modulus and Poisson's ratio of concrete. The joint shear stress at cracking was found from the stress condition that the principal stress reaches the tensile strength of concrete. The modulus after diagonal shear cracking was determined by quantifying the influences of the concrete compressive strength, the amount of the joint lateral reinforcement and the column intermediate reinforcement, the column axial load and the confinement provided by the transverse beams and slabs. Hence the envelope curve suggested in the paper should be refined through many experimental researches. The hysteretic rules are also necessitated to consider the increase in the joint shear distortion caused by the cyclic load reversals after beam yielding.

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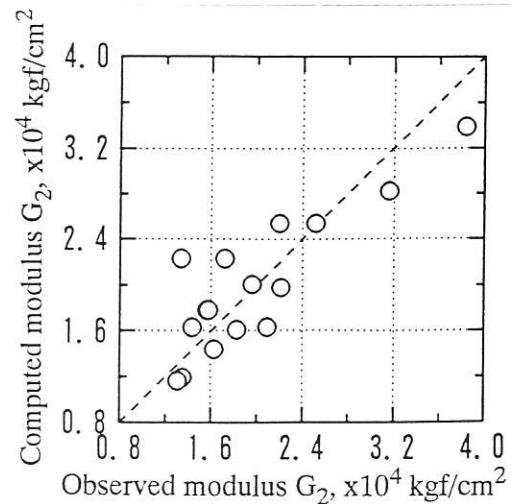


Fig. 10 : Modulus  $G_2$  after cracking

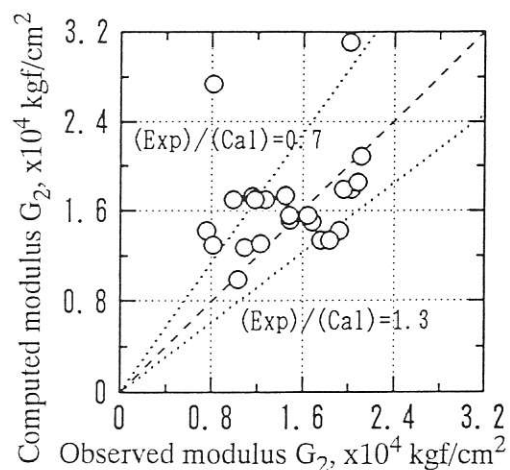


Fig. 11 : Application of proposal to other specimens

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