

EVALUATION OF YIELD DEFORMATION IN R/C MEMBERS  
ACCOUNTING FOR PULL-OUT OF LONGITUDINAL BARS

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ABSTRACT

The yield deflection of rectangular beams was computed taking into account of the additional rotation at a beam critical section due to the beam bar slippage. The pull-out of beam bars from a joint was evaluated using the average bond stress along the beam reinforcing bars within a joint obtained by the regression analysis from the test results. The computed yield deflection underestimated the tests. This computation method could be applied to T-shaped beams with the slab cooperative width of 0.2 times the beam span to the beam flexural resistance.

1. INTRODUCTION

Bond situation along the beam reinforcement within a joint is severe in the reinforced concrete (R/C) frames designed by the weak-beam strong-column concept, since plastic hinges are permitted to develop at the beam ends. Bond deterioration is inevitable along the beam bars passing through an interior joint, which are in tension on one side of the column and in compression on the other side, and consequently the pull-out of the bars from a joint takes place. Test results indicated that the additional rotation due to beam bar slip at the critical section contributed to the approximately 40 percent of a beam deflection (Ref. 1).

Sugano's formula (Ref. 2), made by the regression method based on the tests of beams and columns without beam-column joints, is often used to predict the yield deflection, but does not include the component of the pull-out of the beam bar from a joint. This paper discusses the evaluation of the yield deflection in beams accounting for the pull-out of reinforcing bars from interior beam-column joints.

2. EVALUATION OF YIELD DEFLECTION

Beam yield deflection was assumed to consist of contribution of the elastic flexure along a beam and the additional rotation resulting from the pull-out of beam bars from a joint. The deformation due to shear was neglected, considering members in which the flexural deformation becomes

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dominant to the beam deflection.

2.1 Estimate of Flexural Deformation

The contribution of flexural deformation denoted by  $d_f$  to the beam yield deflection is estimated as given in Eq.(1), assuming that the curvature distribution along a beam at yielding in a column face is proportional to the moment diagram as shown in Fig.1.

$$d_f = L^2 \phi_y / 3 \tag{1}$$

where  $L$  : shear span, and  $\phi_y$  : yield curvature at the beam critical section. Equation(1) was used for its simple form although this hypothesis meant that Eq.(1) included indirectly the contribution of the beam bar pull-out and the shear distortion.

2.2 Estimate of Deformation from Additional Rotation

The strain distribution along a beam reinforcement passing through a joint is assumed as shown in Fig.2, reaching the tensile yield strain at one side of the column. The additional rotation  $\theta_p$  is expressed by Eq.(2), provided that the center of the additional rotation due to the beam bar slip is located on the neutral axis at the beam critical section as shown in Fig.3.

$$\theta_p = \Delta S / d_n \tag{2}$$

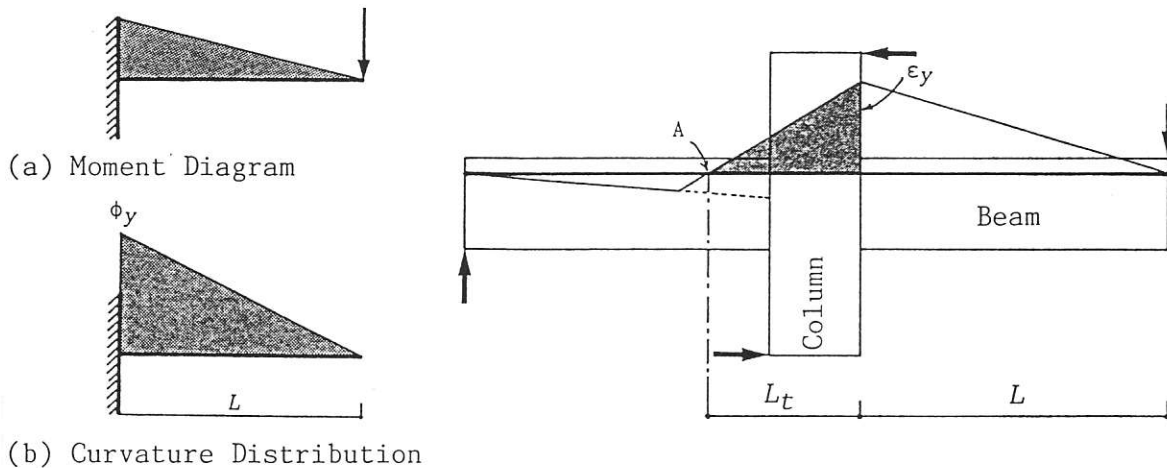


Fig. 1 Curvature Distribution      Fig. 2 Strain Distribution along Beam Bar

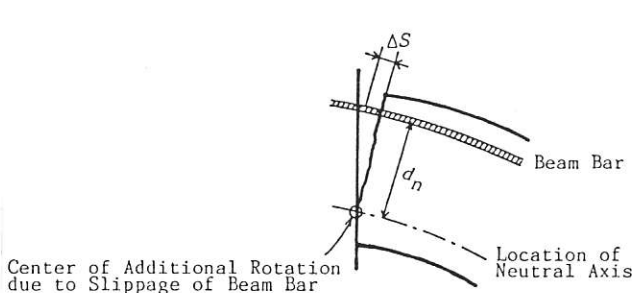


Fig. 3 Additional Rotation due to Beam Bar Slip

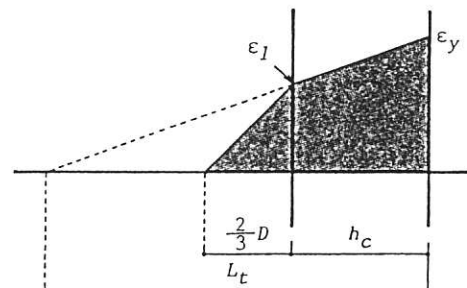


Fig. 4 Modified Strain Distribution along Beam Bar

where  $d_n$  : distance from a beam longitudinal bar to the neutral axis obtained<sup>n</sup> by the analysis assuming that plane sections remain plane, and  $\Delta S$  : beam bar pull-out from a joint as given by Eq.(3);

$$\Delta S = \epsilon_y L_t / 2 \quad (3)$$

where  $\epsilon_y$  : yield strain of beam bars, and  $L_t$  : distance from a beam critical<sup>y</sup> section to the point without any strain indicated by notation A in Fig.2.

Hence the deformation derived from the beam bar pull-out denoted by  $d_p$  is described by Eq.(4).

$$d_p = \theta_p L \quad (4)$$

$L_t$  in Eq.(3) is obtained as follows;

$$L_t = A_s f_y / (\tau_{av} \psi) \quad (5)$$

Where  $A_s$  : cross-sectional area of a beam longitudinal bar,  $f_y$  : yield strength<sup>s</sup> of a beam bar,  $\psi$  : perimeter of a beam bar, and  $\tau_{av}$  : average bond stress along a beam bar within a joint at reaching the yield strain in tension at a column face.  $L_t$  was limited as given by Eq.(6), since  $L_t$  increases unrestrictedly with the decrease in the average bond stress  $\tau_{av}$ .

$$L_t < h_c + (2/3)D \quad (6)$$

where  $h_c$  : column depth, and  $D$  : beam depth. If Eq.(6) is not satisfied, the strain distribution along a beam bar was modified as shown in Fig.4, and the beam bar pull-out  $\Delta S$  was replaced by the shaded area in Fig.4 as given by Eq.(7).

$$\Delta S = h_c (\epsilon_y + \epsilon_1) / 2 + D \epsilon_1 / 3 \quad (7)$$

$$\text{where } \epsilon_1 = (1 - h_c/L_t) \epsilon_y \quad (8)$$

### 2.3 Evaluation of Average Bond Stress $\tau_{av}$

The average bond stress  $\tau_{av}$  within a joint reaches the maximum value  $u_b$  under simultaneous tensile and compressive yielding at the column faces.

$$u_b = f_y (d_b / h_c) / 2 \quad (9)$$

where  $d_b$  : diameter of a beam bar. Note that the bond along a beam bar within a joint deteriorates with the increase in the  $u_b$  value (Ref. 3). Hence the  $u_b$  value can represent the beam bar bond condition within a joint.

The  $\tau_{av}/u_b - u_b/\sqrt{f_c}$  relationships are shown in Fig.5 with unit in  $\text{kgf/cm}^2$ . The maximum average bond stress  $u_b$  was normalized by the square root of the concrete compressive strength, since the bond strength may be proportional to the concrete tensile strength. The average bond stress  $\tau_{av}$  was evaluated as the difference in the stresses in the beam reinforcement at the two faces of a joint. Test results of fourteen plane beam-column joint specimens with about half-scale under cyclic load reversals at the University of Tokyo were used (Refs. 4, 5, and 6). The dimensions of

columns and beams were common in all specimens, 200x300 mm for beams and 300x300 mm for columns, and the shear span ratio of beams were 4.0. The tensile reinforcement ratio of beam sections was distributed from 0.7 to 1.9 percent, the diameter of a beam longitudinal bar from 10 to 16 mm, the yield strength of a beam bar from 3260 to 4250 kgf/cm<sup>2</sup>, and the concrete compressive strength  $f'_c$  from 245 to 293 kgf/cm<sup>2</sup>.

The solid and dashed lines in Fig.5 were derived from the least squares method to fit the data, and expressed by Eqs.(10) and (11).

$$\tau_{av}/u_b = -0.146 u_b/\sqrt{f'_c} + 0.968 \quad \text{for beam top bars} \quad (10)$$

$$\tau_{av}/u_b = -0.126 u_b/\sqrt{f'_c} + 1.127 \quad \text{for beam bottom bars} \quad (11)$$

The  $u_b/\sqrt{f'_c}$  value may need to be restricted for the application of Eqs.(10) and (11).

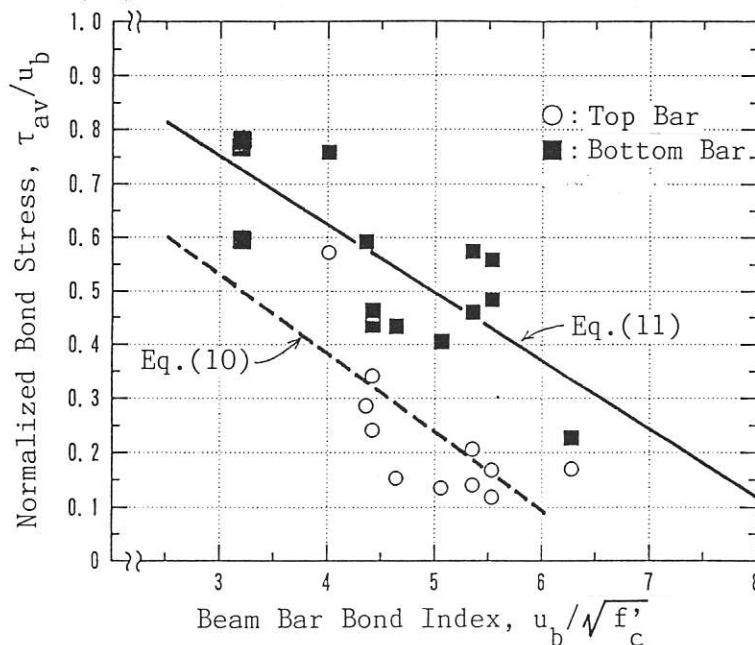
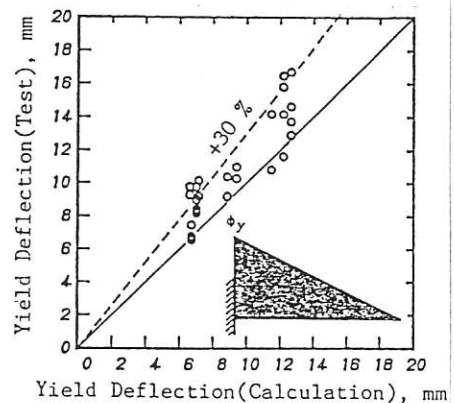


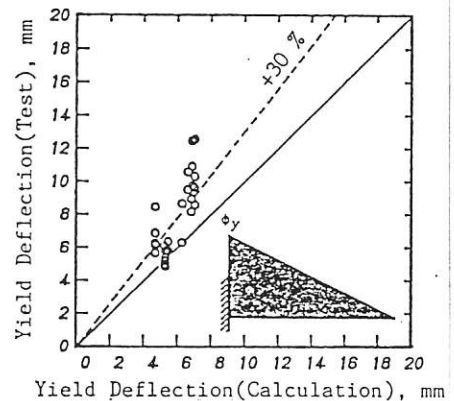
Fig. 5  $\tau_{av}/u_b - u_b/\sqrt{f'_c}$  Relations

### 3. COMPARISON WITH TEST RESULTS

The computed yield deflections for the top and bottom tension in a rectangular beam section are compared with test results in fourteen plane beam-column joint specimens in Fig.6. The yield deflection by the tests was determined as the point developing the abrupt stiffness degradation, when the deflection decided by the beam bar yielding at a column face was different remarkably from that defined by this stiffness degradation point. If two layers of longitudinal reinforcement were arranged at the beam top or bottom section, the curvature and the location of the neutral axis in a beam section corresponding to the yielding in intermediate beam bars were used for the calculation of yield deflection. The computation underestimated the test results as illustrated in Fig.6, especially for the



(a) Beam Top in Tension



(b) Beam Bottom in Tension

Fig. 6 Comparison with Test and Computation

bottom fiber of a beam section in tension, because;

- (a) the contribution of the deflection derived from the pull-out of beam bottom bars was underestimated in computation because of the large magnitude of average bond stress  $\tau_{av}$ , and,  
 (b) shear distortion was neglected in computation.

The stiffness degrading ratios evaluated by the method in this paper were 0.6 to 0.9 times smaller than those by Sugano's formula (Ref. 2) as shown in Fig.7, indicating that this method is more accurate than Sugano's formula.

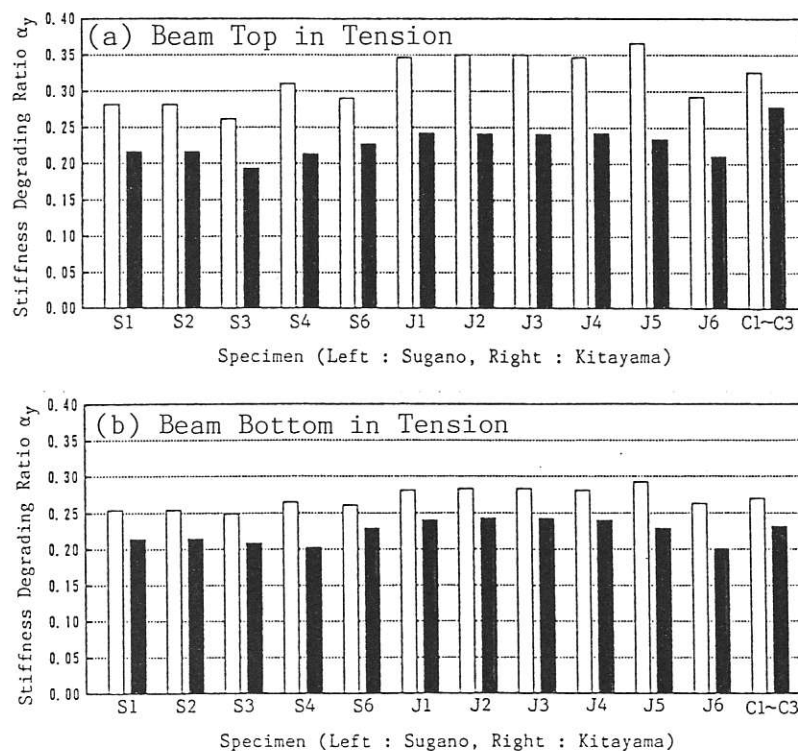


Fig. 7 Comparison of Stiffness Degrading Ratios by Sugano's and Kitayama's Methods

#### 4. APPLICATION TO T-SHAPED BEAMS

The slab cooperative width to the flexural resistance of T-shaped beams increases with the deformation (Ref. 7), accompanied with gradual increase in the resistance. Therefore, the abrupt stiffness degradation in the resistance-deformation relations may not occur. Hence, the yield deflection of T-shaped beams was computed assuming the slab effective width to be 0.1 or 0.2 times the beam span associated with the beam bar yielding.

##### 4.1 Evaluation of T-Shaped Beam Yield Deflection

The computed method of the yield deflection in T-shaped beams is similar to that in rectangular beams. The strength, curvature and neutral axis location at beam bar yielding, obtained by the section analysis assuming that plane sections remain plane and changing the slab cooperative

width to the beam flexural resistance from 0.0 to 0.2 times the beam span, were used.

4.2 Correlation with Test Results

The computed yield deflection is compared with the deflection in the test reaching the yield strength calculated by the analysis based on flexural theory for seven half-scale three-dimensional beam-column subassemblages with slabs (Refs. 8 to 11). The computed deflection under the top fiber of the beam critical section in tension increased with the slab participating width because the location of the neutral axis went down to the center of a beam critical section. On the contrary, the computed deflection under the beam bottom fiber in tension decreased with the slab effective width, since both the yield curvature and the additional rotation decreased due to the rising of the neutral axis to the slab top fiber. The computation in the cases of the slab cooperative width of below 0.1 times the beam span overestimated the test results.

The computed deflections assuming the slab cooperative width of 0.2 times the beam span distributed within 0.7 to 1.3 times the test results as shown in Fig.8. This indicates that the computation method reported herein can be applied to the T-shaped beams with the slab cooperative width of 0.2 times the beam span.

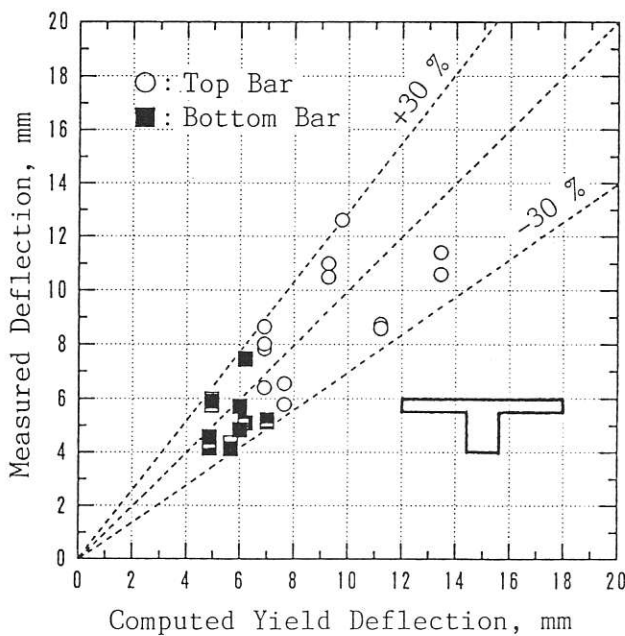
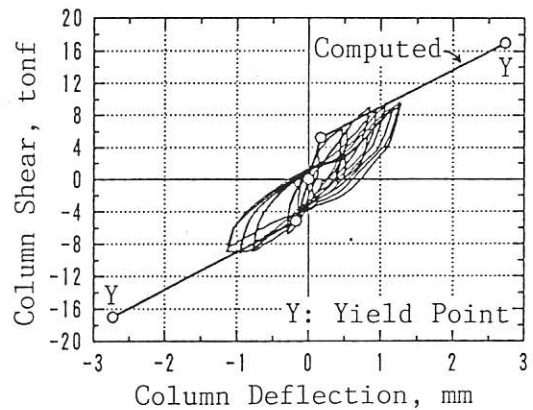


Fig. 8 Yield Deflections Computed and Measured in T-shaped Beams

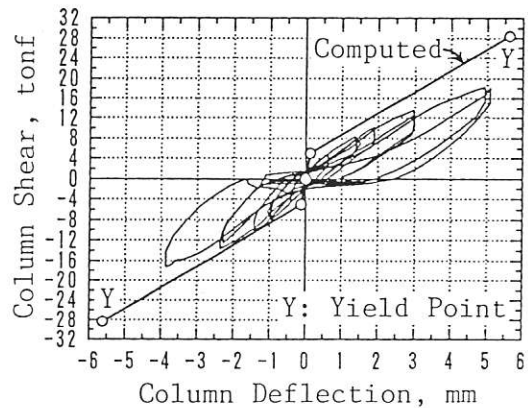
5. APPLICATION TO COLUMNS

Columns are different from beams as follows;

(a) columns are subjected to the axial load, and,



(a) Specimen C3



(b) Specimen A2

Fig. 9 Skeleton Curves in Column Shear - Deflection Relations



(b) the direction of the concrete casting is parallel to the longitudinal reinforcing bars in the column.

However, if the column bars are supposed to yield prior to the beam bars, the stress condition in the concrete surrounding the beam and column longitudinal reinforcement is considered almost similar since the diagonal compression strut is formed along the main diagonal within a joint panel. The difference in the concrete compressive strength at the top and bottom of the joint is negligible. Therefore, the yield deflection of a column may be evaluated by the same method as developed for a beam, using  $\tau_{av}$  value for the beam bottom bars described in Eq.(11) as the average bond stress along a column bar within a joint.

The column deflection-shear relations by the tests and the skeleton curves by the computation are shown in Fig.9 for interior beam-column joint specimens with the beam yielding before the column yielding, and with the compressive column axial stress of  $20 \text{ kgf/cm}^2$  (Refs. 6 and 11). The computed second stiffness after the column flexural cracking was almost coincident with the envelope curve of the test. The cracking deformation and shear was computed using the resistance and the curvature based on the section analysis, and the yield point in computation was defined as the yielding of column reinforcing bars in the most outer layer. The comparison of the yield deflection with test results and the computed, moreover, must be done directly.

## 6. CONCLUDING REMARKS

The following conclusions were drawn from the study reported;

(1) The yield deflection of rectangular beams was computed taking into account of the additional rotation at a beam critical section due to the beam bar slippage. The pull-out of beam bars from a joint was evaluated using the average bond stress along the beam reinforcing bars within a joint as expressed by Eqs.(10) and (11) obtained by the regression analysis from the test results. The yield deflection by the computation was somewhat less than that by the test.

(2) The computed deflections of T-shaped beams assuming the slab cooperative width of 0.2 times the beam span to the flexural resistance were agreed with the test results within the error of  $\pm 30$  percent.

(3) The evaluation method studied here for the beam yield deflection may be applied to the columns with low axial load in compression.

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