Kaonic deuterium from realistic antikaon-nucleon interaction

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$\bar{K}N$ interaction and potential

- Analysis with chiral SU(3) dynamics
  
  Y. Ikeda, T. Hyodo, W. Weise, PLB 706, 63 (2011); NPA 881 98 (2012)

- Realistic $\bar{K}N$ potentials
  
  K. Miyahara, T. Hyodo, PRC93, 015201 (2016)
  

Application to kaonic deuterium

- Prediction of shift and width

- Sensitivity to $|=\uparrow$ component

Two aspects of \(K(\bar{K})\) meson

- **NG boson** of chiral \(SU(3)_R \otimes SU(3)_L \rightarrow SU(3)_V\)
- Massive by strange quark: \(m_K \sim 496\) MeV

\[\rightarrow \text{Spontaneous/explicit symmetry breaking}\]

\(\bar{K}N\) interaction ...


- is coupled with \(\pi\Sigma\) channel
- generates \(\Lambda(1405)\) below threshold

- is fundamental building block for \(\bar{K}\)-nuclei, \(\bar{K}\)-atoms, ...
SIDDHARTA measurement

Precise measurement of the kaonic hydrogen X-rays


- Shift and width of atomic state $\leftrightarrow$ K-p scattering length


Quantitative constraint on the $\bar{K}N$ interaction at fixed energy
Best-fit results of chiral SU(3) dynamics

<table>
<thead>
<tr>
<th></th>
<th>TW</th>
<th>TWB</th>
<th>NLO</th>
<th>Experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td>ΔE [eV]</td>
<td>373</td>
<td>377</td>
<td>306</td>
<td>283 ± 36 ± 6</td>
</tr>
<tr>
<td>Γ [eV]</td>
<td>495</td>
<td>514</td>
<td>591</td>
<td>541 ± 89 ± 22</td>
</tr>
<tr>
<td>γ</td>
<td>2.36</td>
<td>2.36</td>
<td>2.37</td>
<td>2.36 ± 0.04</td>
</tr>
<tr>
<td>R_n</td>
<td>0.20</td>
<td>0.19</td>
<td>0.19</td>
<td>0.189 ± 0.015</td>
</tr>
<tr>
<td>R_c</td>
<td>0.66</td>
<td>0.66</td>
<td>0.66</td>
<td>0.664 ± 0.011</td>
</tr>
<tr>
<td>χ^2/d.o.f</td>
<td>1.12</td>
<td>1.15</td>
<td>0.96</td>
<td></td>
</tr>
</tbody>
</table>


cross sections

Y. Ikeda, T. Hyodo, W. Weise, PLB 706, 63 (2011); NPA 881 98 (2012)

Accurate description of all existing data (χ^2/d.o.f. ~ 1)
Subthreshold extrapolation

Uncertainty of $\bar{K}N \rightarrow \bar{K}N$ (I=0) amplitude below threshold

Y. Kamiya, K. Miyahara, S. Ohnishi, Y. Ikeda, T. Hyodo, E. Oset, W. Weise,

- c.f. without SIDDHARTA


Accurate data is essential to reduce theoretical uncertainty.
Remaining ambiguity

$
\overline{K}N$ interaction has two isospin components ($I=0$, $I=1$).

$$a(K^-p) = \frac{1}{2}a(I = 0) + \frac{1}{2}a(I = 1) + \ldots, \quad a(K^-n) = a(I = 1) + \ldots$$


Relatively large uncertainty in $|I=1|$ sector

- More constraints required (← kaonic deuterium?)
**PDG particle listing of Λ(1405)**


### Λ(1405) 1/2−

The nature of the Λ(1405) has been a puzzle for decades: it seems to be a quark state or hybrid; two poles or one. We cannot here survey the rather extensive literature. See, for example, CIEPLY 10, KISSLINGER 11, SEKIHARA 11, and SHEVCHENKO 12A for discussions and earlier references.

- Our analysis (+ 2 other groups) included
- Pole positions are now tabulated, prior to mass/width.

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**PDG changes**

- Our analysis (+ 2 other groups) included
- Pole positions are now tabulated, prior to mass/width.
Accurate scattering amplitude is now available.

- local $\bar{K}N$ potential in Schrödinger eq.

$\rightarrow$ device to be used in few-body calculations

Construction of equivalent potential

- single-channel $\bar{K}N$ potential
  

- coupled-channel $\bar{K}N-\pi\Sigma$ potential
  

- original (black) v.s. potential (red)

These potentials accurately reproduces data ($\chi^2$/d.o.f. $\sim$ 1)

$\rightarrow$ realistic $\bar{K}N$ potential
Kaonic deuterium: background

K-pn system with strong + Coulomb interaction

- Experiments are planned at J-PARC E57, SIDDHARTA-2

Theoretical requirements:

- Rigorous three-body treatment of strong + Coulomb
- Inclusion of SIDDHARTRA constraint (realistic $\bar{K}N$)
- c.f. advanced Faddeev calculations

Application to kaonic deuterium

Check of kaonic hydrogen

Kaonic hydrogen ($K$-$p$) in the present setup?
- Deser-type formula is based on (systematic) expansion.
- $\bar{K}N$ potential is formulated with isospin symmetry.

Two-body calculation with physical masses

\[
\left( \hat{T} + \hat{V}^{KN} + \hat{V}^{EM} \right) \left( \hat{T} + \hat{V}^{KN} + \Delta m \right) \left( \left| \bar{K}^{-}p \right> \right) = E \left( \left| \bar{K}^{-}p \right> \right)
\]

Result:
- consistent with SIDDHARTA constraint
- Resummed Deser-type formula works reasonably for $K$-$p$.

<table>
<thead>
<tr>
<th>Mass</th>
<th>$E$ dependence</th>
<th>$\Delta E$ (eV)</th>
<th>$\Gamma$ (eV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Physical</td>
<td>Self-consistent</td>
<td>283</td>
<td>607</td>
</tr>
<tr>
<td>Isospin</td>
<td>Self-consistent</td>
<td>163</td>
<td>574</td>
</tr>
<tr>
<td>Physical</td>
<td>$E_{KN} = 0$</td>
<td>283</td>
<td>607</td>
</tr>
<tr>
<td>Expt. [31,32]</td>
<td></td>
<td>283 $\pm$ 36 $\pm$ 6</td>
<td>541 $\pm$ 89 $\pm$ 22</td>
</tr>
</tbody>
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<table>
<thead>
<tr>
<th></th>
<th>$\Delta E$ (eV)</th>
<th>$\Gamma$ (eV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full Schrödinger equation</td>
<td>283</td>
<td>607</td>
</tr>
<tr>
<td>Improved Deser formula (18)</td>
<td>293</td>
<td>596</td>
</tr>
<tr>
<td>Resummed formula (19)</td>
<td>284</td>
<td>605</td>
</tr>
</tbody>
</table>
Formulation

Three-body calculation of $K$-$d$ with physical masses


\[
\begin{pmatrix}
\hat{H}_{K^-pn} & \hat{V}_{12}^{KN} + \hat{V}_{13}^{KN} \\
\hat{V}_{12}^{KN} + \hat{V}_{13}^{KN} & \hat{H}_{K^0nn}
\end{pmatrix}
\begin{pmatrix}
|K^-pn\rangle \\
|K^0nn\rangle
\end{pmatrix}
= E
\begin{pmatrix}
|K^-pn\rangle \\
|K^0nn\rangle
\end{pmatrix}
\]

\[
\hat{H}_{K^-pn} = \sum_{i=1}^{3} \hat{T}_i - \hat{T}_{cm} + \hat{V}_{23}^{NN} + \sum_{i=2}^{3} (\hat{V}_{1i}^{KN} + \hat{V}_{1i}^{EM}) \text{Coulomb}
\]

\[
\hat{H}_{K^0nn} = \sum_{i=1}^{3} \hat{T}_i - \hat{T}_{cm} + \hat{V}_{23}^{NN} + \sum_{i=2}^{3} \hat{V}_{1i}^{KN} + \Delta M \text{ threshold difference}
\]

- (single-channel) realistic $\overline{KN}$ potential


Few-body technique

- stochastic variational method + correlated gaussian basis

Application to kaonic deuterium

Kaonic deuterium: shift and width

Results of the three-body calculation

- energy convergence
- large number of basis

<table>
<thead>
<tr>
<th>$N$</th>
<th>Re[$E$] (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1677</td>
<td>$-2.211689436$</td>
</tr>
<tr>
<td>2194</td>
<td>$-2.211722964$</td>
</tr>
<tr>
<td>2377</td>
<td>$-2.211732072$</td>
</tr>
<tr>
<td>2511</td>
<td>$-2.211735493$</td>
</tr>
<tr>
<td>2621</td>
<td>$-2.211737242$</td>
</tr>
<tr>
<td>2721</td>
<td>$-2.211737609$</td>
</tr>
<tr>
<td>2806</td>
<td>$-2.211737677$</td>
</tr>
<tr>
<td>2879</td>
<td>$-2.211737682$</td>
</tr>
</tbody>
</table>

Shift-width of the $1S$ state:

$$\Delta E - i\Gamma/2 = (670 - i508)\,\text{eV}$$

- No shift in $2P$ state is shown by explicit calculation.
- Deser-type formula does not work accurately for K-d


<table>
<thead>
<tr>
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<th>$\Delta E$ (eV)</th>
<th>$\Gamma$ (eV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full Schrödinger equation</td>
<td>670</td>
<td>1016</td>
</tr>
<tr>
<td>Improved Deser formula (18)</td>
<td>910</td>
<td>989</td>
</tr>
<tr>
<td>Resummed formula (19)</td>
<td>818</td>
<td>1188</td>
</tr>
</tbody>
</table>
Study sensitivity to \( |I|=1 \) interaction

- introduce parameter \( \beta \) to control the potential strength

\[
\text{Re } \hat{V}KN(I=1)(r) \rightarrow \beta \text{[Re } \hat{V}KN(I=1)(r)]
\]

Vary \( \beta \) within SIDDHARTA uncertainty of \( K-p \)

- allowed region: \(-0.17 < \beta < 1.08\)

(negative \( \beta \) may contradict with scattering data)

<table>
<thead>
<tr>
<th>( \beta )</th>
<th>( K^-p )</th>
<th>( K^-d )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \Delta E )</td>
<td>( \Gamma )</td>
</tr>
<tr>
<td>1.08</td>
<td>287</td>
<td>648</td>
</tr>
<tr>
<td>1.00</td>
<td>283</td>
<td>607</td>
</tr>
<tr>
<td>-0.17</td>
<td>310</td>
<td>430</td>
</tr>
</tbody>
</table>

- deviation of \( \Delta E \) of \( K-d \) \( \sim 170 \) eV

- Planned precision: 60 eV (30 eV) at J-PARC (SIDDHARTA-2)

Measurement of \( K-d \) will provide strong constraint on \( |I|=1 \)
Realistic $\overline{K}N$ potentials ($\chi^2$/d.o.f. $\sim 1$) based on NLO chiral SU(3) dynamics are now available, thanks to precise kaonic hydrogen data.

Y. Ikeda, T. Hyodo, W. Weise, PLB 706, 63 (2011); NPA 881 98 (2012)
K. Miyahara, T. Hyodo, PRC93, 015201 (2016)

We study kaonic dueterium as

- Prediction of shift and width
  \[ \Delta E - i\Gamma/2 = (670 - i508) \text{ eV} \]

- sensitive to $I=\frac{1}{2}$ component