Resonances in hadron physics

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Contents

Part I: Compositeness of hadron resonances


Part II: Universal thee-pion physics

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Exotic structure of hadrons

Various excitations of baryons

**conventional**

**exotic**

**energy**

**internal excitation**

**q\bar{q} pair creation**

Physical state: superposition of 3q, 5q, MB, ...

$$\left| \Lambda(1405) \right\rangle = N_{3q} \left| uds \right\rangle + N_{5q} \left| uds \, q\bar{q} \right\rangle + N_{\bar{K}N} \left| \bar{K}N \right\rangle + \cdots$$

Find out the dominant component among others.
Introduction (Part I)

**Structure of resonances?**

Excited states: finite width (unstable against strong decay)

- stable (ground) states
- unstable states

Most of hadrons are unstable!

State vector of resonance?

\[ | \Lambda(1405) \rangle = N_{3q} | uds \rangle + N_{5q} | uds \, q\bar{q} \rangle + N_{KN} | \bar{K}N \rangle + \cdots \]

We need a classification scheme applicable to resonances.
Compositeness of bound states

Compositeness approach: decompose Hamiltonian

\[ H = H_0 + V \]

Complete set for free Hamiltonian: bare \(|B_0\rangle + \text{continuum}\)

\[ 1 = |B_0\rangle\langle B_0| + \int dp |p\rangle\langle p| \]

Physical bound state \(|B\rangle\)

\[ H|B\rangle = -B|B\rangle, \quad \langle B|B\rangle = 1 \]

\[ 1 = \langle B|B_0\rangle\langle B_0|B\rangle + \int dp \langle B|p\rangle\langle p|B\rangle \]

\(Z: \text{elementariness} \quad X: \text{compositeness}\)

\[ Z, X: \text{real and nonnegative} \implies \text{probabilistic interpretation} \]

\[ 0 \leq Z \leq 1, \quad 0 \leq X \leq 1 \]
Field renormalization constant $\mathcal{Z}$ and compositeness (Part I)

**Weak binding limit**

In general, $\mathcal{Z}$ depends on the choice of the potential $\mathcal{V}$.
- $\mathcal{Z}$ : model-(scheme-)dependent quantity

\[
1 - \mathcal{Z} = \int dp \frac{|\langle p | V | B \rangle|^2}{(E_p + B)^2}
\]

In the **weak binding** limit, $\mathcal{Z}$ is related to observables


\[
a = \frac{2(1 - \mathcal{Z})}{2 - \mathcal{Z}} R + \mathcal{O}(R_{typ}), \quad r_e = \frac{-\mathcal{Z}}{1 - \mathcal{Z}} R + \mathcal{O}(R_{typ}),
\]

$a$ : scattering length, $r_e$ : effective range
$R = (2\mu B)^{-1/2}$ : radius (binding energy)
$R_{typ}$ : typical length scale of the interaction

Criterion for the structure:

\[
\begin{align*}
&\begin{cases} 
a \sim R_{typ} \ll -r_e \quad &\text{(elementary dominance),} \quad \mathcal{Z} \sim 1 \\
a \sim R \gg r_e \sim R_{typ} \quad &\text{(composite dominance).} \quad \mathcal{Z} \sim 0
\end{cases}
\end{align*}
\]
Field renormalization constant $\tilde{Z}$ and compositeness (Part I)

**Interpretation of negative effective range**

For $Z>0$, effective range is always **negative**.

$$a = \frac{2(1 - Z)}{2 - Z} R + \mathcal{O}(R_{\text{typ}}), \quad r_e = \frac{-Z}{1 - Z} R + \mathcal{O}(R_{\text{typ}}),$$

\[
\begin{aligned}
    a &\sim R_{\text{typ}} \ll -r_e & \text{(elementary dominance)}, \\
    a &\sim R \gg r_e \sim R_{\text{typ}} & \text{(composite dominance)}.
\end{aligned}
\]

**Simple attractive potential:** $r_e > 0$

--> only “composite dominance” is possible.

$r_e < 0$: energy- (momentum-)dependence of the potential


<-- pole term/Feshbach projection of coupled-channel effect

Negative $r_e$ --> **Something other** than $|p>$ : CDD pole
Application to near-threshold resonances (Part I)

Application to resonances

Compositeness approach at the weak binding:

- Model-independent (no potential, wavefunction, ... )
- Related to experimental observables
- Only for bound states with small binding

Application to general resonances


- $Z$ and $X$ are in general complex. Interpretation?

\[
\langle R | R \rangle \rightarrow \infty, \quad \langle \tilde{R} | R \rangle = 1
\]

\[
1 = \langle \tilde{R} | B_0 \rangle \langle B_0 | R \rangle + \int dp \langle \tilde{R} | p \rangle \langle p | R \rangle
\]

What about near-threshold resonances (~ small binding)?
Near-threshold phenomena: effective range expansion

Application to near-threshold resonances (Part I)


\[ f(p) = \left( -\frac{1}{a} - pi + \frac{r_e}{2} p^2 \right)^{-1} \]

\[ p^\pm = \frac{i}{r_e} \pm \frac{1}{r_e} \sqrt{\frac{2r_e}{a} - 1} \]

Pole trajectories with a fixed \( r_e < 0 \)

Resonance pole position \( \leftrightarrow (a, r_e) \)
Example of resonance: $\Lambda_c(2595)$

Pole position of $\Lambda_c(2595)$ in $\pi\Sigma_c$ scattering

- central values in PDG

\[ E = 0.67 \text{ MeV}, \quad \Gamma = 2.59 \text{ MeV} \]

- deduced threshold parameters

\[ a = -\frac{p^+ + p^-}{ip^+ p^-} = -10.5 \text{ fm}, \quad r_e = \frac{2i}{p^+ + p^-} = -19.5 \text{ fm} \]

- field renormalization constant: complex

\[ Z = 1 - 0.608i \]

Large negative effective range

\[ \text{ <-- substantial elementary contribution other than } \pi\Sigma_c \text{ (three-quark, other meson-baryon channel, or ... )} \]

$\Lambda_c(2595)$ is not likely a $\pi\Sigma_c$ molecule
Renormalization constant $Z$ measures elementariness of a stable bound state.

In general, $Z$ of a resonance is complex.

Negative effective range $r_e$: CDD pole

Near-threshold resonance: pole position is related to $r_e$ --> elementariness

Universal phenomena in hadron physics

Universal few-body physics \(--\) large scattering length

S-wave $\pi\pi$ scattering length
- $a_{l=0} \sim -0.31$ fm, $a_{l=2} \sim 0.06$ fm / QCD scale $\sim 1$ fm
- $l=0$ component can be increased by $m_\pi \uparrow$ or $f_\pi \downarrow$


- Realizable by lattice QCD / nuclear medium

$\Rightarrow$ Three-pion system with a large scattering length
Isospin symmetric three pions

Pion has an internal degree of freedom: isospin \( I = 1 \)

- S-wave two-body amplitude: \( I = 0 \) and \( I = 2 \)

\[
\begin{align*}
it_0(p) &= \frac{8\pi}{m} \frac{i}{\frac{1}{a} - \sqrt{\frac{p^2}{4} - mp_0 - i0^+}}, \quad \text{and} \quad it_2(p) = 0
\end{align*}
\]

S-wave three-pion system in total \( I = 1 \)

\[
\left( \begin{array}{c}
| \pi \otimes [\pi \otimes \pi]_{I=0} \rangle \langle I=1 \\
| \pi \otimes [\pi \otimes \pi]_{I=2} \rangle \langle I=1 
\end{array} \right) = \left( \begin{array}{cc}
1/3 & \sqrt{5}/3 \\
\sqrt{5}/3 & 1/6
\end{array} \right) \left( \begin{array}{c}
| [\pi \otimes \pi]_{I=0} \otimes \pi \rangle \langle I=1 \\
| [\pi \otimes \pi]_{I=2} \otimes \pi \rangle \langle I=1
\end{array} \right)
\]

Eigenvalue equation (eigenvalue \( B_3 \) for eigenfunction \( z(|p|) \))

\[
z(|p|) = \frac{2}{3\pi} \int_0^\infty dq \left| \frac{q}{p} \right| \ln \left( \frac{q^2 + p^2 + |q||p| + mB_3}{q^2 + p^2 - |q||p| + mB_3} \right) \frac{z(|q|)}{\sqrt{\frac{3}{4}q^2 + mB_3 - \frac{1}{a}}}
\]

Factor \( 1/3 \) difference from the identical boson case
Universal physics (Part II)

Spectrum in the isospin symmetric limit

Result: one universal three-pion bound state

\[ B_3 = \frac{1.04391}{ma^2} \quad \text{for } 1/a > 0 \]

\[ \text{c.f. } B_2 = \frac{1}{ma^2} \]

Resonances?

- phase rotation of binding energy = phase rotation of \( a \)

\[ B_3 \rightarrow B_3 e^{i\theta} \iff \frac{1}{a} \rightarrow \frac{1}{a} e^{-i\theta/2} \]

Negative \( a \): virtual state

\(-- \text{ rotation of } B_3 \text{ by } 2\pi = \text{ sign flip of } a \)

No resonance for all \( a \)

\(-- \text{ interchange of Riemann sheet } = \text{ sign flip of } a \)
In nature, $m_{\pi^\pm} = m_{\pi^0} + \Delta$ with $\Delta > 0$

- In the energy region $E \ll \Delta$, heavy $\pi^\pm$ can be neglected.

Identical three-boson system with a large scattering length

$\rightarrow$ Efimov effect

$$z(|p|) = \frac{2}{\pi} \int_0^\infty |q| d|q| \left| \frac{q}{p} \right| \ln \left( \frac{q^2 + p^2 + |q||p| + mB_3}{q^2 + p^2 - |q||p| + mB_3} \right) \text{sgn}(E) \text{Im}E/\kappa^*_2 \frac{1}{4}$$

$$\times \frac{z(|q|)}{\sqrt{\frac{3}{4}q^2 + mB_3 - \frac{1}{a}}} f_\Lambda(|q|)$$

Universal physics at $E \ll (2m\Lambda)^{1/2}$

$\leftrightarrow$ Efimov parameter $\kappa^*$
Efimov resonances

Resonance solution is now possible.

- phase rotation of binding energy = phase rotation of $a$ and $\Lambda$ + proper treatment of singularity in $f_{\Lambda}(|q|)$

\[ B_3 \rightarrow B_3 e^{i\theta} \iff \frac{1}{a} \rightarrow \frac{1}{a} e^{-i\theta/2} \quad \text{and} \quad \Lambda \rightarrow \Lambda e^{-i\theta/2} \]

Efimov bound state $\rightarrow$ resonance
Discussion (Part II)

Interpolation by model

A model with finite mass difference \( \Delta = m_{\pi^\pm} - m_{\pi^0} \)

\[
\mathcal{L} = \sum_{i=0,\pm} \pi_i^\dagger \left( i\partial_t + \frac{\nabla^2}{2m_i} - m_i \right) \pi_i + \frac{g}{4} \frac{\pi_0^\dagger \pi_0^\dagger - 2 \pi_+^\dagger \pi_-^\dagger}{\sqrt{3}} \frac{\pi_0 \pi_0 - 2 \pi_- \pi_+}{\sqrt{3}}
\]

- \( \mathcal{E} \ll \Delta : \text{Efimov states, } (\wedge \gg) \) \( \mathcal{E} \gg \Delta : \text{single bound state} \)
- cutoff for the Efimov effect is introduced by \( \Delta \).

Lowest Efimov level --> universal bound state

universal (isospin breaking)

universal (isospin symmetry)
Large $\pi\pi$ scattering length ($l=0$) can be realized by \( m_\pi \uparrow \) or \( f_\pi \downarrow \).

With isospin symmetry: single three-body bound state for \( l=1, J=0 \).

--> turns into virtual state

With isospin breaking: Efimov states for three neutral pions.

--> turn into resonances

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